$e^{-\frac{S_A}{k}} + e^{-\frac{S_B}{k}} = 1$ $\delta S_A + \delta S_B = 0$ $\delta e^{-\frac{S_A(x)}{k}} = \frac{d e^{-\frac{S_A(x)}{k}}}{dx} dx = -$

$\delta e^{-\frac{S_A}{k}} +$

$e^{-x} + e^{-y} = 1$ $x + y$ const

$+ e^{-x} dx + e^{-y} dy = 0$ $dx + dy = 0$

$e^{-x} = e^{-y}$ $x = y$

$$\int \frac{z\,dz}{e^{z}-1} = \int z\, d\ln(e^{-z}-1) = z\cdot\ln(e^{-z}-1) - \int \ln(e^{-z}-1)\, dz$$

$$\frac{d\ln(e^{z}-1)}{dz} = \frac{-e^{z}}{e^{z}-1} = \frac{-1}{1-e^{+z}} = \frac{1}{e^{z}-1}$$

$$\frac{d\ln(e^{-z}-1)}{dz} = -\frac{e^{-z}}{e^{-z}-1} = -\frac{1}{1-e^{z}} = \frac{1}{e^{z}-1}$$

$$\frac{z}{ce^{-z}-1} =$$

$$z : ce^{z}-1 = c$$

$$\frac{1}{a+bx} = 1 : a+bx = f(x)$$

$$1 = f(x)\times[a+bx]$$

$$= \frac{1}{a}[a+bx] = 1+\frac{b}{a}x$$

$$= \left(\frac{1}{a} - \frac{bx}{a^2}\right)(a+bx) = 1 + \cancel{\frac{bx}{a}} - \cancel{\frac{bx}{a}} - \left(\frac{bx}{a}\right)^2$$

$$\mathfrak{U}(\alpha\varrho) = e^{-\alpha} + \frac{e^{-2\alpha}}{2^{\varrho+1}} + \frac{e^{-3\alpha}}{3^{\varrho+1}}$$

$$\int_0^\infty z^{\varrho}(e^{\alpha+z}-1)^{-1}dz = \int_0^\infty e^{-\alpha}e^{-z}z^{\varrho}dz + \int_0^\infty e^{-2\alpha}e^{-2z}z^{\varrho}dz = e^{-\alpha}\int_0^\infty z^{\varrho}e^{-z}dz$$

$$= e^{-\alpha}\,\Gamma(\varrho+1)$$

$$n-1=\varrho,\ n=\varrho+1$$

$$\int_0^\infty \frac{z\,dz}{e^{\alpha+z}-1} = \int_0^\infty z(e^{\alpha+z}-1)^{-1}dz = \int_0^\infty z e^{-\alpha}e^{-z}dz + \int_0^\infty z e^{-2\alpha}e^{-2z}dz +$$

$$= e^{-\alpha} + \frac{e^{-2\alpha}}{2^2} + \frac{\alpha^{-3\alpha}}{3^2} = e^{-\alpha} + \frac{(e^{-\alpha})^2}{2^2} + \frac{(e^{-\alpha})^3}{3^2} = \varepsilon + \frac{\varepsilon^2}{2^2} + \frac{\varepsilon^3}{3^2}$$

$$\frac{1}{n^2} - \frac{1}{(n+1)^2} = \frac{n^2+2n+1-n^2}{n^2(n+1)^2} = \frac{2n+1}{n^2(n+1)^2} = \frac{n+(n+1)}{n^2(n+1)^2}$$

1)

change of impuls in scattering not predictable in individual case
only probability distribution as in stastical thermodynamics (fluctuation)
Bohr, Naturwiss. 16, 245 (1928)

Anfangsbedingungen auch in Q.Th. nicht genau angebbar (second quant.)
Zeit genau angebbar?
Lagebestimmung innerhalb Compton-λ erzeugt pairs.

On Quantumtheory ~~from the Viewpoint of~~ and Thermodynamics

The expression for the entropy S of a system in Q.th.
is

$$S = -k \sum_{i}^{N?} w_i \ln w_i \quad (k \text{ Boltzmann's konstant})$$

where w_i

This entropy

However this quantity is in general <u>not</u> identical with the thermodynamic entropy and cannot be used as such in thermodynamic derivations. It is a special kind of entropy connected with the process of measurment. Therefore we shall call it in the following the m-entropy (speak ementropy).

This distinction plays already a role in the classical theory as can be seen from the following example

$$dn = \mathcal{C}\frac{d\varepsilon}{e^{\alpha+\beta\varepsilon}-1} \qquad n = \mathcal{C}\int_0^\infty \frac{d\varepsilon}{e^{\alpha+\beta\varepsilon}-1} = -\frac{\mathcal{C}}{\beta}\ln(1-e^{-\alpha})$$

$$\mathcal{C} \sim \frac{2\pi l}{h^2}$$

$$\frac{d\ln(e^{-x}-1)}{dx} = \frac{-e^{-x}}{e^{-x}-1} = \frac{-1}{1-e^{+x}} = \frac{1}{e^{x}-1}$$

$$\frac{d\ln[e^{-(\alpha+\beta\varepsilon)}-1]}{d\varepsilon} = \frac{-\beta e^{-(\ )}}{e^{-(\alpha+\beta\varepsilon)}-1} = \frac{-\beta}{1-e^{+(\alpha+\beta\varepsilon)}} = \frac{\beta}{e^{(\alpha+\beta\varepsilon)}-1}$$

$$\frac{1}{\beta}d\ln[e^{-(\alpha+\beta\varepsilon)}-1] = \frac{d\varepsilon}{e^{\alpha+\beta\varepsilon}-1}$$

$$\int_0^\infty \frac{d\varepsilon}{e^{\alpha+\beta\varepsilon}-1} = \frac{1}{\beta}\ln[e^{-(\alpha+\beta\varepsilon)}-1]\Big|_0^\infty = \frac{1}{\beta}\left[\ln(-1)-\ln(e^{-\alpha}-1)\right] = \frac{1}{\beta}\ln\frac{e^{-\infty}-1}{e^{-\alpha}-1}$$

$$= \frac{1}{\beta}\ln\frac{1}{1-e^{-\alpha}} = -\frac{1}{\beta}\ln(1-e^{-\alpha})$$

$$\ln(1-e^{-\alpha}) = -\frac{n\beta}{\mathcal{C}}, \quad 1-e^{-\alpha} = e^{-\frac{n\beta}{\mathcal{C}}}, \quad e^{-\alpha} = 1-e^{-\frac{n\beta}{\mathcal{C}}}, \quad e^{\alpha} = \frac{1}{1-e^{-\frac{n\beta}{\mathcal{C}}}} = \frac{1}{1-e^{-\frac{n}{\mathcal{C}kT}}}$$

$$\frac{d\ln(e^{-x}+1)}{dx} = \frac{-e^{-x}}{e^{-x}+1} = \frac{-1}{1+e^{x}} = -\frac{1}{e^{x}+1} \qquad T\to\infty, \ \frac{n}{\mathcal{C}kT} << 1, \ e^{\alpha} = \frac{1}{1-1+\frac{n}{\mathcal{C}kT}} = \frac{\mathcal{C}kT}{n}$$

$$\frac{d\ln[e^{-(\alpha+\beta\varepsilon)}+1]}{d\varepsilon} = \frac{-\beta e^{-(\ )}}{e^{-(\ )}+1} = -\frac{\beta}{e^{\alpha+\beta\varepsilon}+1}$$

$$\int_0^\infty \frac{d\varepsilon}{e^{\alpha+\beta\varepsilon}+1} = -\frac{1}{\beta}\ln[e^{-(\alpha+\beta\varepsilon)}+1]\Big|_0^\infty = \frac{1}{\beta}\left[\ln(e^{-\alpha}+1) - \overset{0}{\cancel{\ln(e^{-\infty}+1)}}\right] = \frac{1}{\beta}\ln(e^{-\alpha}+1)$$

$$\frac{n\beta}{\mathcal{C}} = \ln(e^{-\alpha}+1), \quad e^{-\alpha}\cancel{+1} = e^{\frac{n\beta}{\mathcal{C}}}-1, \quad e^{\alpha} = \frac{1}{e^{\frac{n\beta}{\mathcal{C}}}-1} = \frac{1}{e^{\frac{n}{\mathcal{C}kT}}-1}, \quad T\to\infty \quad e^{\alpha} = \frac{\mathcal{C}kT}{n}$$

$\psi = a_1 u_1 + a_2 u_2$

$|\psi|^2 = a_1^2 u_1^2 + a_2^2 u_2^2 + a_1 a_2 \cancel{\int u_1 u_2 dq}$ 0

$\bar{q} = \int q |\psi|^2 dq =$

$\psi = \sum_i a_i e^{-i\left(\frac{E_i}{\hbar}t + \alpha_i\right)} u_i \qquad P_{mn} = a_n a_m e^{-i\left(\frac{E_n - E_m}{\hbar}t + \alpha_m - \alpha_n\right)} = a_n a_m e^{-i\nu_{mn} t + \alpha_{mn}}$

$a_1^2 \quad a_2 a_1 e^{-i\nu_{12} t + \alpha_{21}}$

$\bar{F} = \sum_n (FP)_{nn} \qquad (qP)_{11} = q_{11} P_{11} + q_{21} P_{21} \qquad a_1 a_2 e^{+i\nu_{21} t + \alpha} \qquad a_2^2$

$(qP)_{22} = q_{21} P_{12} + q_{22} P_{22}$

$\bar{q} = (qP)_{11} + (qP)_{22} = a_1^2 q_{11} + a_2^2 q_{22} + a_1 a_2 q_{12} \left(e^{i\nu t + \alpha} + e^{-(i\nu t + \alpha)}\right)$

$= a_1^2 q_{11} + a_2^2 q_{22} + a_1 a_2 q_{12}\, 2\cos(\nu t + \alpha)$

$\psi = \sum_n a_n e^{i\alpha_n} e^{-i\frac{E_n}{\hbar}t} u_n(q) = \sum c_n u_n(q) \qquad P_{mn} = c_n^* c_m \qquad \bar{F} = \sum (FP)_{nn} = \int \psi^* (F\psi)\, dq$

$P = \sum \omega_i P_i$

$\psi = a_1 e^{i\delta_1} e^{-i\frac{E_1}{\hbar}t} u_1 + a_2 e^{i\delta_2} e^{-i\frac{E_2}{\hbar}t} u_2$

$\psi^*\psi = |\psi|^2 = a_1^2 u_1^2 + a_2^2 u_2^2 + 2 a_1 a_2 u_1 u_2 \cos\left(\frac{E_1 - E_2}{\hbar}t + \delta_1 - \delta_2\right)$

$\bar{q} = \int \psi^* q \psi\, dq = a_1^2 q_{11}^2 + a_2^2 q_{22}^2 + 2 a_1 a_2 q_{12} q_{21} \cos(\qquad)$

$\bar{q} = \int \psi^* q \psi\, dq = \sum c_n^* c_m q_{nm} = \sum_n (qP)_{nn}, \quad q_{nm} = \int u_n^* q u_m\, dq. \quad P_{mn} = c_m^*(t) c_n(t)$

$a_1 e^{-i\left(\frac{E_1}{\hbar}t + \alpha\right)} u_1 \quad P \quad a_1^2$

$\begin{vmatrix} c_1^2 - \lambda & c_1^* c_2 \\ c_2^* c_1 & c_2^2 - \lambda \end{vmatrix} = (c_1^2 - \lambda)(c_2^2 - \lambda) - c_1^2 c_2^2 = 0, \quad \lambda^2 - \lambda(c_1^2 + c_2^2) = 0 = \lambda^2 - \lambda \quad (c_1^2 + c_2^2 = 1)$

$\lambda = \lambda^2 \quad \lambda = \mp 1$

$$k \ln \frac{N!}{\prod n_i!}$$

$$= k\left[N \ln N \cancel{- N} - \sum n_i \ln n_i \cancel{+ \sum n_i}\right]$$

$$= k N\left[\ln N - \sum x_i (\ln x_i + \ln N)\right]$$

$$= k N\left[\ln N - \sum x_i \ln x_i - \ln N\right]$$

$$= - R \sum x_i \ln x_i$$

$$\psi_p = \sum_i a_i \psi_i \qquad \sum a_i^2 = 1$$

$$\sum \omega_i = 1$$

$$\psi_p^* \psi_p = \left(\sum a_i \psi_i\right)^2 \qquad a_i a_k \neq 0$$

$$\psi_m^* \psi_m = \sum_i \omega_i |\psi_i|^2$$

$$S = - N k \sum \omega_i \ln \omega_i$$

$$\psi = a_1 e^{-i\frac{E_1}{\hbar}t} u_1 + a_2 e^{-i\frac{E_2}{\hbar}t} u_2 \qquad (AB)_{nm} = \sum_k A_{nk} B_{km}$$

$$P : \begin{matrix} a_1^2 & a_1 a_2 e^{-i\frac{E_1-E_2}{\hbar}t} \\ a_2 a_1 e^{+i\frac{E_1-E_2}{\hbar}t} & a_2^2 \end{matrix} = \begin{matrix} a_1^2 & a_1 a_2 e^{-i\omega t} \\ a_1 a_2 e^{i\omega t} & a_2^2 \end{matrix} \qquad \bar{E} = \sum_n (PE)_{nn} \qquad E : \begin{matrix} E_1 & 0 \\ 0 & E_2 \end{matrix}$$

$$(PE)_{11} = a_1^2 E_1 + 0, a_1 a_2 e^{+i\omega t} \quad (PE)_{22} = a_1 a_2 e^{-i\omega t} \times 0 + a_{22}^2 E_2 \quad \bar{E} = a_1^2 E_1 + a_2^2 E_2$$

$$q : \begin{matrix} \int q u_1^2 dq & \int q u_1 u_2 dq \\ \int q u_2 u_1 dq & \int q u_2^2 dq \end{matrix} : \begin{matrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{matrix} \qquad \bar{q} = \sum (qP)_{nn} = a_1^2 q_{11} + a_2^2 q_{22} + a_1 a_2 q_{12} (e^{i\omega t} + e^{-i\omega t})$$

$2a_1 a_2 q_{12} \cos\omega t$

$q_{21} = q_{12}$

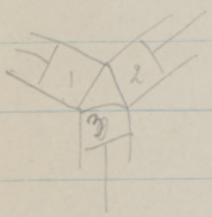
$$(qP)_{11} = q_{11} P_{11} + q_{12} P_{21} = a_1^2 q_{11} + a_1 a_2 e^{i\omega t} q_{12} \quad (qP)_{22} = q_{21} P_{12} + q_{22} P_{22} = a_1 a_2 e^{-i\omega t} q_{21} + a_2^2 q_{22}$$

$$k \ln Z \qquad Z = w_1 Z_1 + w_2 Z_2 \quad \ln Z = \ln$$

$$Z = \frac{N!}{n_1! n_2!}, \quad \ln Z = \ln N! - \ln n_1! - \ln n_2! = N \ln N \cancel{-N} - n_1 \ln n_1 \cancel{+n_1} - n_2 \ln n_2 \cancel{+n_2}$$

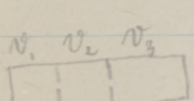
$$= n_1 \ln N - n_1 \ln n_1 + n_2 \ln N - n_2 \ln n_2 = -n_1 \ln \frac{n_1}{N} - n_2 \ln \frac{n_2}{N}$$

$$= -N x_1 \ln x_1 - N x_2 \ln x_2 = -N(x_1 \ln x_1 + x_2 \ln x_2)$$



$$e^{-\frac{S}{k}} = \frac{1}{2} \qquad \frac{S}{k} = \ln 2 \qquad k$$

$$\frac{n_1}{n} V \qquad n_1 k \ln \frac{Vn}{n_1 V} \qquad -\sum n_i \ln \frac{n_i}{n}$$



$$n_1 : n_2 : n_3 = v_1 : v_2 : v_3 \qquad n_i = c v_i$$

On the entropy of measurement and quantumtheory

I believe that there is a close ~~and deep~~ connection between statistical thermodynamics and the fundaments of quant.-theory. ~~The~~ Following the work of Szilard and v. Neumann, I shall ~~considerations are the first steps in an~~ attempt ~~in trying~~ to gain a clearer insight in the nature of this connection.

Qu.-theory is in the main part a theory of measurement. The ground for a theory of the thermodynamics of measurement was laid in 1929 by Szilard. He showed on a simple example: ~~A~~ measurement to decide which one of two possibilities is realized requires at least $k \ln 2$ increase of entropy. A short time later v. Neumann derived an expression for the entropy of a qu. theoretical system which can be considered as a „mixture" of „pure" cases.

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \qquad \psi = \sum a_k(t)\, u(p_k, q)$$

$$i\hbar \sum \dot{a}_k(t)\, u(p_k, q) = \sum a_k(t)\, H u(p_k, q) \;\Big|\; u^*(p_k, q) \int$$

$$i\hbar\, \dot{a}_k(t) = \sum a_k(t) \int u^* H u \, d\tau$$

Wahrscheinlichkeitstemperatur ϑ: $\frac{\vartheta}{k}$ entspricht T

$\vartheta \sum w_i \ln w_i > \frac{h}{t}$ d.h. je kleiner ϑ, um so länger hat $\sum w_i \ln w_i$ Sinn

$$\psi = a_1 u_1 e^{-i\frac{E_1}{\hbar}t + \delta_1} + a_2 u_2 e^{-i\frac{E_2}{\hbar}t + \delta_2}$$

$$i\hbar \frac{\partial \psi}{\partial t} = H(\psi)$$

Wie hängt die Entropie von der Genauigkeit der Messung ab?

Energiebestimmungsgenauigkeit $\Delta E \sim \frac{\hbar}{\Delta t}$

Wenn ich die Energie einordnen kann, ist die Entropie $k \ln n$, wobei n die Anzahl der Eigenwerte ist, die in ΔE liegt. Bei Atomen im einem anderen Zustand als [illegible] klein selbst für sehr kleine Δt, bei [illegible] ungestört. Ich muss also Modell kennen.

Wenn ich einen reinen Fall superponiert aus zwei Energiezuständen in Phase, wie hängt ~~dann~~ von Genauigkeit der Phasenmessung ab?

Um einen reinen Fall bzgl. Energie zu erhalten, muss ich ∞ lange Zeit zur Messung verwenden. Wenn ich nur endliche Zeit t habe, wird bestimmt einen reinen Fall? die Phase? Aber nur Phasen-differenzen sind messbar. Wenn $\Delta E \sim \frac{\hbar}{\Delta t}$ mehrere Energiezustände enthält, sind doch offenbar die Phasendifferenzen zwischen diesen Energiezuständen, wenn nicht, ist $S = 0$.

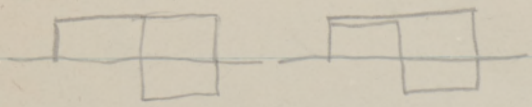
Reiner Fall nicht realisierbar (Nernst Th.)

Die Energie genau $t = \infty$. Nernst?

Reinen Fall nicht scharf realisieren, sehr ähnliche Nachbarzustände (→ Gibbs' Paradoxon). Die Energie muß Zeit länger sehen als $\Delta t \sim \frac{\hbar}{\Delta E}$, wenn ΔE Energiedifferenz benachbarter Zustände ist. Klassisch Messung reversibel, d. h. Entropiezuwachs und beim Messen gleich Entropieabnahme des gemessenen Objekts (mindestens!). Quantenph. Entropiezuwachs prinzipiell größer, ∞ für reinen Fall? Richtig, falls ich Zustand durch ψ bestimmen, weil zur Messung von $|\psi|^2$ ∞ viele Beobachtungen nötig sind. Wieso bei $t = \infty$? Oder bei $T \to 0$?

Individual Partikel und Atome

Der klassische Term $k \ln n!$ muß bei hoher Temp. die Häufigkeit des Austausches hängt von Wechselwirkungsenergie E ab, ist Null, falls $E = 0$. Wenn ich bei $E = 0$ abkühle zu $T \to 0$, bleibt Term erhalten (wegen exclusion?) (Ausgangszustand) (Boltzm. Lösg. Ich kühle ab $\to T = 0$, Entropie $-k\sum x_i \ln x_i$, Betrag von Entropie die ich mindestens aufwenden muß, um Anordnung festzustellen. Nicht thermodynamische Entropie, da ich nicht reversibel zur Zustandsgrößen-Entropie (Mischung) gehen kann, würde Nernst Th. widersprechen. Gilt für jede Messung, da von T nicht die Rede, d. h. $T = 0$. Also Entropie eines reinen Falles, oder überhaupt eines Quantenzustandes aus Superposition gebildet, nicht thermodyn. Entropie. Denn Fermigas nicht so einfach abkühlen,



$\psi = \sum a_i u_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)}$ $\quad \mathcal{H} u_i = E_i u_i \quad \mathcal{H}\psi = \sum a_i E_i u_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)}$

$\frac{d\psi}{dt} = \sum a_i u_i \left(-\frac{i}{\hbar}E_i\right) e^{-\frac{i}{\hbar}(E_i t + \alpha_i)}, \quad -\frac{\hbar}{i}\frac{d\psi}{dt} = \sum a_i E_i u_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)} = \mathcal{H}\psi$

$i\hbar \frac{d\psi}{dt} = \mathcal{H}\psi \quad \mathcal{H} = \mathcal{H}_0 + V \quad \psi = \sum a_i(t) u_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)} \quad \mathcal{H}_0 u_i = E_i u_i$

$i\hbar \sum \left[\dot{a}_i(t) u_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)} - \frac{i}{\hbar} a_i(t) E_i e^{-\frac{i}{\hbar}(E_i t + \alpha_i)}\right] = \sum a_i(t)(E_i u_i + V u_i) e^{-\frac{i}{\hbar}(E_i t + \alpha_i)} \Big| u_k^* \int$

$i\hbar \dot{a}_k(t) e^{-\frac{i}{\hbar}(E_k t + \alpha_k)} \cancel{+ a_k(t) E_k e^{-\frac{i}{\hbar}(E_k t + \alpha_k)}} = \cancel{a_k(t) E_k e^{-\frac{i}{\hbar}(E_k t + \alpha_k)}} + \sum a_i(t) \int u_k^* V u_i d\tau \, e^{-\frac{i}{\hbar}(E_i t + \alpha_i)}$

$i\hbar \dot{a}_k(t) = \sum a_i(t) V_{ki} e^{-\frac{i}{\hbar}[(E_i - E_k)t + (\alpha_i - \alpha_k)]}$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \;,\quad \psi = \sum a_n(t)\, u_n^0\, e^{-\frac{iE_n^0}{\hbar}t} \;,\quad H = H_0 + H' \;,\quad H_0(u_n^0) = E_n^0 u_n^0$$

$$i\hbar\sum \dot{a}_n(t) u_n^0 e^{-\frac{iE_n^0 t}{\hbar}} + \cancel{\sum E_n^0 a_n(t) u_n^0 e^{-\frac{iE_n^0}{\hbar}t}} = \cancel{\sum a_n(t) E_n^0 u_n^0 e^{-\frac{iE_n^0 t}{\hbar}}} + \sum a_n(t) H'(u_n^0) e^{-\frac{iE_n^0 t}{\hbar}} \Big| \bar{u}_m \int$$

$$i\hbar\, \dot{a}_m(t)\, e^{-\frac{iE_m^0 t}{\hbar}} = \sum a_n(t)\, e^{-\frac{iE_n^0 t}{\hbar}} \int \bar{u}_m^0 H' u_n^0\, d\tau$$

$$\dot{a}_m(t) = \frac{1}{i\hbar}\sum a_n(t)\, e^{-\frac{i(E_n^0-E_m^0)}{\hbar}t}\, H'_{mn} = \frac{1}{i\hbar}\sum H'_{mn} a_n(t)\, e^{i\omega_{mn}t} \qquad E_m^0 - E_n^0 = \hbar\omega_{mn}$$

$$H = \underbrace{\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}a^2 x_1 + \frac{1}{2}a^2 x_2}_{H_0} + \underbrace{\frac{1}{2}b^2(x_1-x_2)^2}_{H'}$$

$$u_n^0 = u_k^1 u_l^2 \qquad H'_{mn} = \int u_k^1 u_l^2\, \frac{1}{2}b^2(x_1^2 + x_2^2 - 2x_1x_2)\, u_{k'}^1 u_{l'}^2\, dx_1 dx_2$$

$$u_m^0 = u_{k'}^1 u_{l'}^2 \qquad = \frac{1}{2}b^2\left[\int_{-\infty}^{+\infty} u_k^1 x_1^2 u_{k'}^1\, dx_1 \int_{-\infty}^{+\infty} u_l^2 u_{l'}^2\, dx_2 + \int x_2^2 - 2\int_{-\infty}^{+\infty} u_k^1 x_1 u_{k'}^1\, dx_1 \int_{-\infty}^{+\infty} u_l^2 x_2 u_{l'}^2\, dx_2\right]$$

$$(X)_{nm} = \begin{cases} \frac{1}{\alpha}\sqrt{\frac{n+1}{2}} & m = n+1 \\ \frac{1}{\alpha}\sqrt{\frac{n}{2}} & m = n-1 \end{cases}$$

[illegible] $= \frac{1}{2}b^2\left[(X^2)_{kk'}\,\delta_{ll'} + (X^2)_{ll'}\,\delta_{kk'} - 2(X)_{kk'}(X)_{ll'}\right]$

$$(X^2)_{nm} = \begin{cases} \frac{1}{2\alpha^2}\sqrt{(m+1)(m+2)} & n = m+2 \\ \frac{1}{2\alpha^2}(2m+1) & n = m \\ \frac{1}{2\alpha^2}\sqrt{m(m-1)} & n = m-2 \end{cases}$$

[illegible]

$$\alpha^2 = \frac{m\omega}{\hbar} \quad \frac{1}{\alpha^2} = \frac{\hbar}{m\omega} \quad \mathscr{C} = \frac{\hbar}{\sqrt{m_1\omega_1 m_2\omega_2}} \times \frac{1}{2}$$

$$k' = k,\; l' = l \quad \omega_{mn} = 0 \qquad H_{\substack{k,k+1\\ l,l+1}} = \mathscr{C}(k+1)(l+1) \quad H_{\substack{k,k+1\\ l,l-1}} = \mathscr{C}(k+1)l \quad H_{\substack{k,k-1\\ l,l+1}} = \mathscr{C}k(l+1) \quad H_{\substack{k,k-1\\ l,l-1}} = \mathscr{C}kl$$

2 gekoppelte Resonatoren: $\omega_1 = \omega$, $\omega_2 = \omega + \Delta\omega$, $m_1 = m$, $m_2 = m - \Delta m$

$$E_a = k\hbar\omega_1 + l\hbar\omega_2 = k\hbar\omega + l\hbar\omega + l\hbar\Delta\omega = (k+l)\hbar\omega + l\hbar\Delta\omega$$

$$E_b = l\hbar\omega_1 + k\hbar\omega_2 = l\hbar\omega + k\hbar\omega + k\hbar\Delta\omega = (k+l)\hbar\omega + k\hbar\Delta\omega$$

$$E_b - E_a = (k-l)\hbar\Delta\omega$$

$$H' = \tfrac{1}{2}b^2(x_1-x_2)^2 = \tfrac{1}{2}b^2x_1^2 + \tfrac{1}{2}b^2x_2^2 - b^2x_1x_2$$

$$H_{ab} = \int \bar{u}_1^k \bar{u}_2^l \tfrac{1}{2}b^2(x_1-x_2)^2 u_1^l u_2^k \, dx_1 dx_2$$

$k \neq l$
$$= \tfrac{1}{2}b^2 \int_{-\infty}^{+\infty} \bar{u}_1^k x_1^2 u_1^l \, dx_1 \underbrace{\int_{-\infty}^{+\infty} \bar{u}_2^l u_2^k \, dx_2}_{0} + 0 - b^2 \int_{-\infty}^{+\infty} \bar{u}_1^k x_1 u_1^l \, dx_1 \int_{-\infty}^{+\infty} \bar{u}_2^l x_2 u_2^k \, dx_2$$

$$= b^2 (x_1)_{kl}(x_2)_{lk} \qquad k-l=1,\ k=l+1,\ l=k-1 \qquad H_{ab} = b^2\frac{1}{\alpha_1\alpha_2}\frac{1}{2}\sqrt{k(l+1)} = \frac{b^2k}{\alpha_1\alpha_2} \qquad k-l = \pm 1$$

$$l-k=1,\ l=k+1,\ k=l-1 \qquad H_{ba} \quad '' \quad \sqrt{(k+1)l} = \frac{b^2 l}{\alpha_1\alpha_2}$$

$$\alpha_1\alpha_2 = \sqrt{\frac{m_1\omega_1m_2\omega_2}{\hbar^2}} = \sqrt{\frac{m\omega(m-\Delta m)(\omega+\Delta\omega)}{\hbar^2}} = \sqrt{\frac{m^2\omega^2}{\hbar^2}(1-\mu)(1+\varepsilon)} = \frac{m\omega}{\hbar}(1-\mu+\varepsilon) \qquad \mu = \frac{\Delta m}{m} \quad \varepsilon = \frac{\Delta\omega}{\omega}$$

$$(1-\varepsilon)$$
$$(1-\tfrac{\mu}{2})$$

$$a^2 = m_1\omega_1^2 = m_2\omega_2^2 = m\omega^2 = m\omega^2(1-\mu+2\varepsilon),\ 1 = 1-\mu+2\varepsilon,\ \mu = 2\varepsilon,\ -\mu+\varepsilon = -2\varepsilon+\varepsilon = -\varepsilon = -\frac{\mu}{2}$$

$$\omega_2 = \omega\left(1+\frac{\Delta\omega}{\omega}\right) = \omega(1+\varepsilon),\quad \frac{\Delta\omega}{\omega} = \varepsilon = \frac{\mu}{2}$$

Zustand durch Energie der einzelnen Resonatoren gemessen in Zeit t_0

$t_0 \gg \frac{1}{\omega}$ $\quad \Delta E \sim \frac{\hbar}{t_0} = \hbar\omega$ um zu unterscheiden zwischen k u. $k\pm1$, auch l u. $l\pm1$

$t_0 \ll \frac{1}{\Delta\omega}$ um zwischen Res. 1 u. 2 zu unterscheiden, weil Energie mit Periode $\frac{1}{\Delta\omega}$ wechselt.

nur möglich, falls $\frac{\Delta\omega}{\omega} = \varepsilon \ll 1$

Wenn $\mu = 0$, $\varepsilon = 0$ Unterscheidung der Resonatoren nur durch Geschichte, $\Delta\omega = 0$

$$\bar{H}_{ab} = \frac{b^2k}{\alpha_1\alpha_2} = \frac{b^2k\hbar}{m\omega}\left(1+\frac{\mu}{2}\right), \qquad \dot{a}_{ab}(t) = \frac{1}{i\hbar}a_{ba}(t)\,e^{-i\Delta\omega t}\frac{b^2k\hbar}{m\omega}\left(1+\frac{\Delta\omega}{\omega}\right)$$

w

$$H'_{mm} = \int \bar{\psi}_m H' \psi_m \, d\tau \qquad H' = -\frac{1}{2} b^2 x_1^2 - \frac{1}{2} b^2 x_2^2 + b^2 x_1 x_2 = -\frac{1}{2} b^2 \left(x_1^2 + x_2^2 - 2x_1 x_2\right)$$

$$\psi_m = u_k(x_1)\, u_l(x_2)$$

$$H'_{mm} = -\frac{1}{2} b^2 \Big[\underbrace{\int_{-\infty}^{+\infty} |u_k(x_1)|^2 x_1^2 dx_1}_{\frac{2k+1}{2\alpha^2} = \frac{(2k+1)\hbar}{2m\omega_1}} \underbrace{\int_{-\infty}^{+\infty} |u_l(x_2)|^2 dx_2}_{1} + \cdots - 2 \underbrace{\int |u_k(x_1)|^2 x_1 dx_1}_{0} \underbrace{\int |u_l(x_2)|^2 x_2 dx_2}_{0} \Big]$$

$$E_1 = \overline{H'_{mm}} = -\frac{1}{2} b^2 \hbar \left[\left(k+\frac{1}{2}\right) \frac{\omega_1^0}{a^2} + \left(l+\frac{1}{2}\right) \frac{\omega_2^0}{a^2} \right] = -\frac{1}{2} \beta \hbar \left[\left(k+\frac{1}{2}\right) \omega_1^0 + \left(l+\frac{1}{2}\right) \omega_2^0 \right]$$

$$E = \left(k+\frac{1}{2}\right) \hbar \omega_1 + \left(l+\frac{1}{2}\right) \hbar \omega_2$$

$$\omega_1 = \omega_1^0 \left(1 - \frac{1}{2}\beta - \frac{1}{2}\beta^2 \frac{\omega_2^{0\,2}}{\omega_2^{0\,2} - \omega_1^{0\,2}}\right) \qquad \omega_2 = \omega_2^0 \left(1 - \frac{1}{2}\beta + \frac{1}{2}\beta^2 \frac{\omega_1^{0\,2}}{\omega_2^{0\,2} - \omega_1^{0\,2}}\right)$$

$$\frac{1}{\hbar} E = \underbrace{\left(k+\frac{1}{2}\right) \omega_1^0 + \left(l+\frac{1}{2}\right) \omega_2^0}_{E_0} - \underbrace{\frac{\beta}{2} \left[\left(k+\frac{1}{2}\right) \omega_1^0 + \left(l+\frac{1}{2}\right) \omega_2^0\right]}_{E_1 = H_{mm}} - \underbrace{\frac{1}{2}\beta^2 \frac{\omega_1^0 \omega_2^0}{\omega_2^{0\,2} - \omega_1^{0\,2}} \left[\left(k+\frac{1}{2}\right) \omega_2^0 - \left(l+\frac{1}{2}\right) \omega_1^0\right]}_{E_2 = \sum' \frac{|H'_{mn}|^2}{E_m^0 - E_n^0}}$$

$$\frac{1}{\hbar} E = \left(k+\frac{1}{2}\right) \omega_1^0 + \left(l+\frac{1}{2}\right) \omega_2^0 - \frac{\beta}{2} \left[\left(k+\frac{1}{2}\right) \omega_1^0 + \left(l+\frac{1}{2}\right) \omega_2^0\right] - \frac{1}{2}\beta^2 \frac{\omega_1^0 \omega_2^0}{\omega_2^{0\,2} - \omega_1^{0\,2}} \left[\left(k+\frac{1}{2}\right) \omega_2^0 - \left(l+\frac{1}{2}\right) \omega_1^0\right]$$

$$H'_{mn} = b^2 x_{k,k\pm1} x_{l,l\pm1} \qquad x_{k,k+1} = \sqrt{\frac{k+1}{2\alpha_1}}, \; x_{k,k-1} = \sqrt{\frac{k}{2\alpha_1}}, \qquad |H'_{mn}|^2 = \frac{b^4}{4\alpha_1\alpha_2} k\, l \qquad \alpha_1 = \frac{m_1\omega_1}{\hbar} \quad \alpha_2 = \frac{m_2\omega_2}{\hbar}$$

		$\lvert H'_{mn}\rvert^2$	$E_m^0 - E_n^0$
$k, k+1$	$l, l+1$	$\frac{b^4}{4\alpha_1\alpha_2}(k+1)(l+1)$	$\hbar(-\omega_1^0 - \omega_2^0)$
$k, k+1$	$l, l-1$	" $(k+1)l$	" $-\omega_1^0 + \omega_2^0$
$k, k-1$	$l, l+1$	" $k(l+1)$	" $\omega_1^0 - \omega_2^0$
$k, k-1$	$l, l-1$	" kl	" $\omega_1^0 + \omega_2^0$

$$E_2 = \frac{b^4}{4\alpha_1\alpha_2} \frac{1}{\hbar} \left[-\frac{(k+1)(l+1)}{\omega_1^0 + \omega_2^0} + \frac{(k+1)l}{\omega_2^0 - \omega_1^0} - \frac{k(l+1)}{\omega_2^0 - \omega_1^0} + \frac{kl}{\omega_2^0 + \omega_1^0} \right]$$

$$= \frac{b^4 \hbar^2}{4 m_1 m_2 \omega_1 \omega_2} \frac{1}{\hbar} \left[-\frac{k+l+1}{\omega_1^0 + \omega_2^0} + \frac{l-k}{\omega_2^0 - \omega_1^0} \right]$$

$$= -\frac{\beta^2 \hbar}{4} \frac{\omega_1^0 \omega_2^0}{\omega_2^{0\,2} - \omega_1^{0\,2}} \left[(k+l+1)(\omega_2^0 - \omega_1^0) + (k-l)(\omega_2^0 + \omega_1^0) \right]$$

$$= -\frac{1}{2} \beta^2 \hbar \frac{\omega_1^0 \omega_2^0}{\omega_2^{0\,2} - \omega_1^{0\,2}} \left[\left(k+\frac{1}{2}\right) \omega_2^0 - \left(l+\frac{1}{2}\right) \omega_1^0 \right]$$

$$k\omega_2^0 \cancel{+ l\omega_2^0} + \omega_2^0 + k\omega_2^0 \cancel{+ k\omega_1^0} \quad = (2k+1)\omega_2^0$$

$$\cancel{- k\omega_1^0} - l\omega_1^0 - \omega_1^0 \cancel{- l\omega_2^0} - l\omega_1^0 \quad -(2l+1)\omega_1^0$$

$$V = a(e^{a\xi}-1) \quad \xi=0,\ V=0,\ \frac{dV}{d\xi} = ae^{a\xi} \quad \frac{p^2}{2m} = e^{a\xi_0}-1$$

$$\int e^{-\frac{e^{a\xi}-1}{kT}} d\xi \qquad x = e^{a\xi}-1 \quad dx = ae^{a\xi}d\xi \qquad p = -\frac{\partial V}{\partial x}$$

$$V = a\xi^n \quad \frac{dV}{d\xi} = an\xi^{n-1} \quad \int e^{-\frac{a\xi^n}{kT}} d\xi \quad , x = a\xi^n \quad dx = an\xi^{n-1}d\xi$$

$$\int \frac{\partial V}{\partial \xi} e^{-\frac{V(\xi)}{kT}} d\xi = \int e^{-\frac{V}{kT}} dV = kT\int_0^{\frac{V}{kT}} e^{-x}dx = kT(1-e^{-\frac{V}{kT}})$$

$$\frac{p^2}{2m} + V = E,\ p=0,\ V=E$$

$$V_0 = \frac{p^2}{2m}$$

$$\overline{p} = -\int \frac{dV}{dx} e^{\frac{V}{kT}} e^{-\frac{E}{kT}} dp\,dx \qquad e^{-\frac{V}{kT}} = \int e^{-\frac{E}{kT}} dp\,dx$$

$$= -e^{\frac{V}{kT}} \int_{-\infty}^{+\infty} dp\, e^{-\frac{p^2}{2mkT}} \int_0^l \frac{\partial V}{\partial x} e^{-\frac{V}{kT}} dx = -e^{\frac{V}{kT}} \int_{-\infty}^{+\infty} dp\, e^{-\frac{p^2}{2mkT}} \left[\int_0^{l-\xi} 0\,dx + \int_0^{\xi_1} \frac{\partial V}{\partial \xi} e^{-\frac{V}{kT}} d\xi\right]$$

$$= -e^{\frac{V}{kT}} \int_{-\infty}^{+\infty} dp\ e^{-\frac{p^2}{2mkT}} kT(1-e^{-\frac{p^2}{2mkT}}) = -e^{-\frac{V}{kT}} kT(2\pi mkT)^{\frac{1}{2}}(1-\frac{1}{\sqrt{2}})$$

$$e^{-\frac{V}{kT}} = \int e^{-\frac{E}{kT}} dp\,dx = \int_{-\infty}^{+\infty} dp\, e^{-\frac{p^2}{2mkT}} \int_0^l e^{-\frac{V}{kT}} dx = \int_{-\infty}^{+\infty} dp\, e^{-\frac{p^2}{2mkT}} \left[\int_0^l dx + \int_0^{\xi_0} e^{-\frac{V(\xi)}{kT}} d\xi\right]$$

$$V(\xi) = a\xi^n \quad dV = an\xi^{n-1}d\xi = \frac{n}{\xi} V d\xi,\ \xi = \left(\frac{V}{a}\right)^{\frac{1}{n}},\ d\xi = \frac{\xi}{nV} dV = $$

$$V = a\xi \quad \int_0^{\xi_0} e^{-\frac{a\xi}{kT}} d\xi = \frac{kT}{a} \int_0^{\frac{p^2}{2mkT}} e^{-x}dx = \frac{kT}{a}(1-e^{-\frac{p^2}{2mkT}})$$

$$\iint e^{-\frac{E}{kT}}\,dp\,dx = l(2\pi mkT)^{\frac{1}{2}}\left(1+\frac{2kT}{al}-\frac{\sqrt{2}\,kT}{al}\right) \qquad \begin{array}{l} 2.0000 \\ \sqrt{2} = 1.4142 \end{array} \quad \begin{array}{l} \delta \\ = 0.5858 \sim 0.6 \end{array}$$

$$\simeq l(2\pi mkT)^{\frac{1}{2}}\left(1+\delta\frac{kT}{al}\right)$$

$a\xi_0 \simeq kT$, $\frac{kT}{a} \simeq \xi_0$, $\delta\frac{kT}{al} \simeq \delta\frac{\xi_0}{l}$

$$\delta\frac{\xi_0}{l}$$

$$\psi = -kT\ln\iint e^{-\frac{E}{kT}}\,dp\,dx = -kT\left[\ln l + \ln(2\pi mkT)^{\frac{1}{2}} + \ln\left(1+\delta\frac{\xi_0}{l}\right)\right]$$

$$p = -\frac{\partial\psi}{\partial l} = \frac{kT}{l} + kT\frac{-\delta\frac{\xi_0}{l^2}}{1+\delta\frac{\xi_0}{l}} \cong \frac{kT}{l}\left(1-\delta\frac{\xi_0}{l}\right)$$

$$p = -\frac{\partial E}{\partial x} \qquad \bar{p} = \iint -\frac{\partial E}{\partial x}\, e^{\frac{\psi-E}{kT}}\,dp\,dx \qquad \int e^{\frac{\psi-E}{kT}}\,dp\,dx = 1 = e^{\frac{\psi}{kT}}\iint e^{-\frac{E}{kT}}\,dp\,dx$$

$\frac{\partial E}{\partial x} = 0$ für x im Intervall l, $\frac{\partial E}{\partial x} = a$ für $0<\xi<\infty$

$$\bar{p} = -a\int_{-\infty}^{+\infty}dp\int_0^{\xi_0} e^{\frac{\psi}{kT}}\, e^{-\frac{p^2}{2mkT}}\, e^{-\frac{a\xi}{kT}}\,d\xi = -kT\int_{-\infty}^{+\infty}dp\; e^{-\frac{p^2}{2mkT}}\, e^{\frac{\psi}{kT}}\, 2\int_0^{\frac{a\xi_0}{kT}} e^{-\frac{a\xi}{kT}}\,d\frac{a\xi}{kT}$$

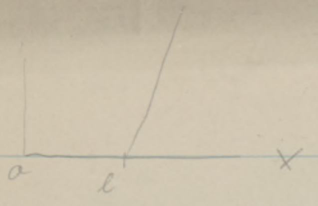
$$p = -kT\, e^{\frac{\psi}{kT}}\int_{-\infty}^{+\infty} e^{-\frac{p^2}{2mkT}}\; 2\left(1-e^{-\frac{p^2}{2mkT}}\right)dp \qquad \int_0^{\xi_0} e^{-\frac{a\xi}{kT}}\,d\xi = \frac{kT}{a}\int_0^{\frac{p^2}{2mkT}} e^{-x}\,dx = \frac{kT}{a}\left(1-e^{-\frac{p^2}{2mkT}}\right)$$

$$= -kT\, e^{\frac{\psi}{kT}}(2\pi mkT)^{\frac{1}{2}}\,\delta$$

$$\int e^{-\frac{E}{kT}}\,dp\,dq = \int_{-\infty}^{+\infty}dp\int_0^{l} e^{-\frac{p^2}{2mkT}}\, e^{-\frac{a\xi}{kT}}\,d\xi = \int_{-\infty}^{+\infty}dp\; e^{-\frac{p^2}{2mkT}}\, l\left[1+2\frac{kT}{al}\left(1-e^{-\frac{p^2}{2mkT}}\right)\right]$$

$$e^{-\frac{\psi}{kT}} = l(2\pi mkT^{\frac{1}{2}})\left[1+\ldots(2-\sqrt{2})\right. \qquad p = -\frac{kT}{l}\,\frac{2-\sqrt{2}}{2-\frac{1}{l}(\sqrt{2}-2)}$$

VI



$\mathcal{E}_p = g(x-l) = g\xi$ für $\xi > 0$, $\mathcal{E}_p = 0$ für $\xi < 0$ $\qquad \xi = x - l$

$-p = -\frac{\partial \mathcal{E}_p}{\partial x} = -g$ " , $p = 0$ "

$$-\bar{p} = \frac{\iint_{-\infty\ 0}^{+\infty\ \infty} -\frac{\partial \mathcal{E}_p}{\partial x} e^{-\frac{\mathcal{E}}{kT}} dp\, dx}{\iint dp\, dx} = \frac{\int_0^\infty -g e^{-\frac{g(x-l)}{kT}} dx}{\int_0^\infty e^{-\frac{g(x-l)}{kT}} dx} = \frac{\int_0^l 0\, e^{-0} dx + \int_l^\infty -g e^{-\frac{g(x-l)}{kT}} dx}{\int_0^l e^{-0} dx + \int_l^\infty e^{-\frac{g(x-l)}{kT}} dx}$$

$$-\bar{p} = \frac{1}{l}\int_0^\infty -g e^{-\frac{g\xi}{kT}} d\xi = \frac{kT}{l}\int_0^\infty e^{-\frac{g\xi}{kT}} d\left(-\frac{g\xi}{kT}\right) = \frac{kT}{l}\int_0^{-\infty} e^y dy = \frac{kT}{l} e^{y}\Big|_0^{-\infty} = -\frac{kT}{l}$$

$$-\bar{p} = \left(\frac{\partial P}{\partial l}\right)_T = -kT\frac{\partial}{\partial l}\ln l = -\frac{kT}{l}$$

$P = \bar{\mathcal{E}} - TS$, $S = \frac{\bar{\mathcal{E}}}{T} - \frac{P}{T}$ $\qquad$ idealen Gas: $J = (2\pi m kT)^{\frac{3n}{2}} V^n$

$\ln J = \frac{3n}{2}\ln(2\pi m kT) + n\ln V$

$$S = \frac{\bar{\mathcal{E}}}{T} - \frac{P}{T} = kT\frac{d\ln J}{dT} + k\ln J = \underbrace{kT\frac{3n}{2}\frac{1}{T}}_{\frac{3n}{2}k} + k\frac{3n}{2}\ln(2\pi m kT) + kn\ln V$$

$S_1 = kn_1\ln V_1 + C_1$, $S_2 = kn_2 \ln V_2 + C_2$, $S_1 + S_2 = k(n_1\ln V_1 + n_2 \ln V_2) + C_1 + C_2$

$S_{tot} = k[(n_1+n_2)\ln(V_1+V_2)] + C_1 + C_2$

$\Delta S = S_{tot} - (S_1 + S_2) = k[(n_1+n_2)\ln(V_1+V_2) - n_1\ln V_1 - n_2 \ln V_2]$

$n_1 = n_2 = n$, $V_1 = V_2 = V$, $\Delta S = 2n\ln 2V - 2n\ln V = 2n\ln 2$

$180^\circ = 3{,}14 \quad 1 = \frac{180^\circ}{3{,}14} = 57{,}3 \quad \cos \overset{32.7}{57{,}3} = \sin 32.7^\circ = 0.5403 \quad \cos 1 = 0.54 \quad \frac{1}{\cos 1} = 1.85$

$\varphi_0 \cong \sqrt{ma} \quad \frac{1}{2}mv^2 = \frac{a}{\xi_0^2} = \frac{1}{2}kT, \; a = \frac{1}{2}kT \times \xi_0^2 \cong 2\times 10^{-14} \times 0.5\times 10^{-16} = 10^{-30}$

$\sqrt{ma} \cong \sqrt{10^{-22} \times 10^{-30}} = 10^{-26} \quad \varepsilon_0 = \frac{v}{l} 2\varphi_0 \quad \mathcal{E} = \frac{1}{2}mv^2 + 2.10^{-26}\frac{v}{l}.$

$V = a\xi \quad \frac{1}{2}mv^2 + a\xi = \frac{1}{2}mv_0^2 \quad v = 0 \quad a\xi_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\dot\xi^2 + a\xi$

$v_0^2 = \dot\xi^2 + \frac{2a}{m}\xi \quad \frac{d\xi}{dt} = \sqrt{v_0^2 - \frac{2a}{m}\xi} \quad \int_0^t a\xi\, dt = \int_0^{\xi_0} \frac{a\xi\, d\xi}{\sqrt{v_0^2 - \frac{2a}{m}\xi}} = \frac{a}{v_0}\int_0^{\xi_0} \frac{\xi\, d\xi}{\sqrt{1 - \frac{2a}{mv_0^2}\xi}}$

$\sqrt{1 - c\xi} = x, \; 1 - c\xi = x^2, \; -c\,d\xi = 2x\,dx, \; \xi = \frac{1+x^2}{c}, \; \xi\, d\xi = -\frac{(1+x^2)2x\,dx}{c^2} \quad c = \frac{2a}{mv_0^2}$

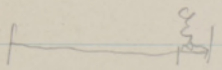
$\int_0^{\xi_0} \frac{\xi\, d\xi}{\sqrt{1-c\xi}} = \int_{\sqrt{1-c\xi_0}}^{1} \frac{(1+x^2)2x\,dx}{c^2 x} = \frac{1}{c^2}\int_{\sqrt{1-c\xi_0}}^{1} (1+x^2)\,dx^2 = \frac{1}{c^2}\int_{1-c\xi_0}^{1} (1+y)\,dy = \frac{1}{c^2}\left(y + \frac{1}{2}y^2\right)\Big|_{1-c\xi_0}^{1}$

$= \frac{1}{c^2}\left[\frac{3}{2} - 1 + c\xi_0 - \frac{1}{2}(1 - 2c\xi_0 + c^2\xi_0^2)\right] = \frac{1}{c^2}\left[\cancel{\tfrac{1}{2}} + c\xi_0 - \cancel{\tfrac{1}{2}} + c\xi_0 - \frac{1}{2}c^2\xi_0^2\right]$

$= \frac{2\xi_0}{c} - \frac{1}{2}\xi_0^2, \quad \frac{a}{v_0}\left(\frac{2\xi_0}{2a}mv_0^2 - \frac{1}{2}\xi_0^2\right) = \xi_0 m v_0 - \frac{1}{2}\xi_0^2 \frac{a}{v_0} = \xi_0 m v_0 \left(1 - \frac{1}{2}\frac{a\xi_0}{mv_0^2}\right)$

$= \frac{3}{4}\xi_0 m v_0 \times 2\frac{v_0}{l} = \frac{3}{2}mv_0^2 \frac{\xi_0}{l} = \frac{1}{2}mv_0^2\, 3\frac{\xi_0}{l}\frac{a}{a} = \frac{3}{al}\left(\frac{1}{2}mv_0^2\right)^2$

$= \frac{3}{4}\frac{1}{2a}m^3v_0^3 \times 2\frac{v_0}{l} = \frac{3\left(\frac{1}{2}mv_0^2\right)}{al} \qquad a\xi_0 = \frac{1}{2}kT, \; a = \frac{\frac{1}{2}kT}{\xi_0} \cong \frac{2\times 10^{-14}}{2.10^{-8}} = 10^{-6}$



$$\mathcal{E} = \frac{1}{2}mv^2 + \int_0^\infty \frac{1}{\xi}dt$$

$$\frac{1}{\xi}d\xi = \frac{1}{\xi}\frac{d\xi}{dt}dt \quad v = \frac{1}{\xi} \quad F = -\frac{\partial v}{\partial \xi} = +\frac{1}{\xi^2} \quad m\ddot{\xi} = \frac{1}{\xi^2} \,|\, \dot{\xi}, \quad \frac{d\frac{1}{2}m\dot{\xi}^2}{dt} = -\frac{d\frac{1}{\xi}}{dt}$$

$$\frac{d\frac{1}{2}\dot{\xi}^2}{dt} = \frac{d\frac{1}{2}\dot{\xi}^2}{d\dot{\xi}}\ddot{\xi} = \dot{\xi}\ddot{\xi} \quad \frac{d\xi^{-1}}{dt} = \frac{d\xi^{-1}}{d\xi}\frac{d\xi}{dt} = -\xi^{-2}\dot{\xi} \quad \frac{1}{2}m\dot{\xi}^2 + \frac{1}{\xi} = \frac{1}{2}mv^2 = \mathcal{E}_0$$

$$\dot{\xi} = 0 \quad \frac{1}{\xi_0} = \frac{1}{2}mv^2, \ \dot{\xi} = v, \ \xi = \infty \quad v_1 = \int_0^t \frac{1}{\xi}dt = \int_{\xi_0}^\infty \frac{d\xi}{\xi\sqrt{a - \frac{b}{\xi}}} = \int_{\xi_0}^\infty \frac{d\xi}{\sqrt{a\xi^2 - b\xi}}$$

$$\dot{\xi}^2 = a - \frac{b}{\xi}, \quad \dot{\xi} = \sqrt{a - \frac{b}{\xi}} \quad d\xi = \sqrt{a - \frac{b}{\xi}}\,dt \quad \frac{1}{\xi} = \eta \quad \frac{d\eta}{d\xi} = -\frac{1}{\xi^2}, \ \frac{d\xi}{\xi} = -\xi d\eta = -\frac{1}{\eta}d\eta$$

$$\xi^2 - 2c\xi + c^2 = (\xi - c)^2 = \eta^2 \quad \xi - c = \pm\eta \quad d\xi = \pm d\eta \quad \int\frac{d\eta}{\sqrt{\eta^2 - c^2}}$$

$$\int_0^\infty \frac{a}{\xi^2}dt \quad \frac{1}{2}m\dot{\xi}^2 + \frac{a}{\xi^2} = \frac{1}{2}mv^2, \ \dot{\xi}^2 + \frac{2a}{m}\frac{1}{\xi^2} = v^2, \quad \dot{\xi} = \frac{d\xi}{dt} = \sqrt{v^2 - \frac{2a}{m}\frac{1}{\xi^2}}, \ dt = \frac{d\xi}{\sqrt{v^2 - \frac{2a}{m}\frac{1}{\xi^2}}}$$

$$\int_0^\infty \frac{a\,dt}{\xi^2} = \int_{\xi_0}^\infty \frac{a\,d\xi}{\xi^2\sqrt{v^2 - \frac{2a}{m}\frac{1}{\xi^2}}} = \frac{1}{v}\int_{\xi_0}^\infty \frac{a\,d\xi}{\xi\sqrt{\xi^2 - \frac{a}{\frac{1}{2}mv^2}}} = \frac{a}{v}\frac{1}{\sqrt{\frac{a}{\frac{1}{2}mv^2}}}\frac{1}{\cos\frac{\sqrt{\frac{a}{\frac{1}{2}mv^2}}}{x}}\Bigg|_{\xi_0}^\infty = \frac{a}{v}\sqrt{\frac{\frac{1}{2}mv^2}{a}}\left(1 - \frac{1}{\cos\sqrt{\frac{a\xi_0}{\frac{1}{2}mv^2}}}\right)$$

$$= \sqrt{\frac{ma}{2\cancel{x}}}\left(1 - \frac{1}{\cos 1}\right) = V_0 \quad \mathcal{E} = \frac{1}{2}mv^2 + \varepsilon_0 \quad \varepsilon_0 = \frac{v}{l}2V_0 \quad \mathcal{E} = \frac{1}{2}mv^2\left(1 + \frac{\xi_0}{l}\right)$$

$$\frac{\partial\mathcal{E}}{\partial l} = -\frac{v}{l^2}2V_0 \quad \frac{a}{\xi_0^2} = \frac{1}{2}mv^2 = \frac{1}{2}kT, \quad ma = \frac{1}{2}m^2v^2\xi_0^2 = \frac{1}{2}kT\,m\xi_0^2 = \mathcal{E}_0 m\xi_0^2$$

$$\sqrt{ma} = \sqrt{\tfrac{1}{2}kT}\,\xi_0\sqrt{m} \quad 2V_0 \simeq \sqrt{\frac{ma}{2}} = \frac{mv\xi_0}{2}$$

$$\varepsilon_0 = \frac{1}{2}mv^2\frac{\xi_0}{l}$$

$$\xi_0 = 10^{-8}\text{ cm} \quad \frac{a}{10^{-16}} \simeq \frac{1}{2}\times 1.4\times 10^{-16}\times 300 \simeq 2\times 10^{-14} \quad a \simeq 2\times 10^{-30} \quad m \simeq 10^{-22} \quad \sqrt{m} \simeq 10^{-11} \quad \xi_0\sqrt{m} = 10^{-19}$$

$$\mathcal{E} = \frac{1}{2}mv^2\left(1 + \frac{\xi_0}{\sqrt[3]{v}}\right)$$

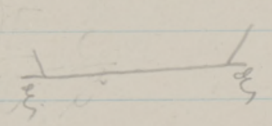
$$\overline{(p-\bar{p})^2} = kT\left[\frac{\partial \bar{p}}{\partial v} - \overline{\left(\frac{\partial p}{\partial v}\right)}\right] \qquad \bar{p} = \frac{nkT}{v} \qquad \frac{\partial \bar{p}}{\partial v} = -\frac{nkT}{v^2} = -\frac{\bar{p}^2}{nkT}$$

$$\frac{\overline{(p-\bar{p})^2}}{\bar{p}^2} = -\frac{1}{n}$$

$$S = k\ln v + c_0\ln T + S_0 \qquad \psi = E - TS = c_0 T - kT\ln v + c_0 T\ln T + S_0$$

$$\frac{\partial \psi}{\partial v} = -\frac{kT}{v} = -p$$

$$E = \frac{p^2}{2m} + a\xi$$



$$a\xi_0 = \frac{p^2}{2m}$$

$$\psi = -kT\ln\iint_{-\infty}^{+\infty} e^{-\frac{E}{kT}}\,dp\,dx \qquad \int_{-\infty}^{+\infty} e^{-\frac{p^2}{2mkT}}dp = (2mkT)^{\frac{1}{2}}\int_{-\infty}^{+\infty} e^{-x^2}dx = (2\pi mkT)^{\frac{1}{2}}$$

$$2\int_0^{\xi_0} e^{-\frac{a\xi}{kT}}d\xi = 2\frac{kT}{a}\int_0^{\frac{a\xi_0}{kT}} e^{-x}dx = 2\frac{kT}{a}\, e^{-x}\Big|^{0}_{\frac{p^2}{2mkT}} = 2\frac{kT}{a}\left(1-e^{-\frac{p^2}{2mkT}}\right)$$

$$\int dx = l + 2\frac{kT}{a}\left(1-e^{-\frac{p^2}{2mkT}}\right) = l\left[1+\frac{2kT}{al} - \frac{2kT}{al}e^{-\frac{p^2}{2mkT}}\right]$$

$$\iint e^{-\frac{E}{kT}}dp\,dx = \int_{-\infty}^{+\infty} dp\left[\int_0^{l} e^{-\frac{p^2}{2mkT}}dx + 2e^{-\frac{p^2}{2mkT}}\int_0^{\xi_0} e^{-\frac{a\xi}{kT}}d\xi\right]$$

$$= \int_{-\infty}^{+\infty} dp\; e^{-\frac{p^2}{2mkT}}\left[l + 2\frac{kT}{a}\left(1-e^{-\frac{p^2}{2mkT}}\right)\right]$$

$$= l\int_{-\infty}^{+\infty} e^{-\frac{p^2}{2mkT}}\left[1 + 2\frac{kT}{al} - \frac{2kT}{al}e^{-\frac{p^2}{2mkT}}\right]dp$$

$$= l\left[(2\pi mkT)^{\frac{1}{2}}\left(1+\frac{2kT}{al}\right) - \frac{2kT}{al}\int_{-\infty}^{+\infty} e^{-\frac{2p^2}{2mkT}}dp\right]$$

$$= l\left[(2\pi mkT)^{\frac{1}{2}}\left(1+\frac{2kT}{dl}\right) - \frac{2kT}{al}(\pi mkT)^{\frac{1}{2}}\right]$$

$$v = a\xi^n \quad \frac{\partial v}{\partial \xi} = an\xi^{n-1} \quad \frac{\partial^2 v}{\partial \xi^2} = an(n-1)\xi^{n-2}$$

$$\int_0^\infty e^{-\frac{v}{kT}} d\xi \quad \int_0^\infty \frac{\partial v}{\partial\xi} e^{-\frac{v}{kT}} d\xi = kT\int_0^\infty e^{-\frac{v}{kT}} d\frac{v}{kT} = kT e^{-\frac{v}{kT}}\Big|_\infty^0 = kT$$

$$\int_0^\infty \frac{\partial^2 v}{\partial\xi^2} e^{-\frac{v}{kT}} d\xi = \int_0^\infty \frac{\partial}{\partial\xi}\left(\frac{\partial v}{\partial \xi}\right) e^{-\frac{v}{kT}} d\xi = \int_0^\infty e^{-\frac{v}{kT}} d\left(\frac{\partial v}{\partial\xi}\right) = e^{-\frac{v}{kT}}\frac{\partial v}{\partial\xi}\Big|_0^\infty - \int_0^\infty \frac{\partial v}{\partial \xi} d\left(e^{-\frac{v}{kT}}\right)$$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \quad uv = \int u\,dv + \int v\,du \quad \int u\,dv = uv - \int v\,du$$

$$\frac{d e^{-\frac{v}{kT}}}{d\xi} = \frac{de^{-\frac{v}{kT}}}{d\frac{v}{kT}} \frac{d\frac{v}{kT}}{d\xi} = -e^{-\frac{v}{kT}}\frac{1}{kT}\frac{\partial v}{\partial \xi} \quad \int_0^\infty \frac{\partial^2 v}{\partial \xi^2} e^{-\frac{v}{kT}} d\xi = -\left(\frac{\partial v}{\partial\xi}\right)_0 + \frac{1}{kT}\int_0^\infty \left(\frac{\partial v}{\partial \xi}\right)^2 e^{-\frac{v}{kT}} d\xi$$

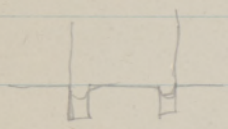
$$= \frac{1}{kT}\int_0^\infty \frac{\partial v}{\partial\xi} e^{-\frac{v}{kT}} dv = \int_0^\infty \frac{\partial v}{\partial \xi} e^{-\frac{v}{kT}} d\frac{v}{kT} = kT^{(1-\frac{1}{n})} a^{\frac{1}{n}} n \int_0^\infty x^{(1-\frac{1}{n})} e^{-x}dx$$

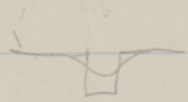
$$\frac{\partial v}{\partial \xi} = an\xi^{n-1} = \frac{nv}{\xi}, \quad \xi = \left(\frac{v}{a}\right)^{\frac{1}{n}}, \quad \frac{\partial v}{\partial \xi} = a^{\frac{1}{n}} n v^{(1-\frac{1}{n})} = kT^{(1-\frac{1}{n})} a^{\frac{1}{n}} n \left(\frac{v}{kT}\right)^{(1-\frac{1}{n})}$$

$$\int_0^\infty x e^{-x}dx = -\int_0^\infty x\, d(e^{-x}) = -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx = -xe^{-x} - e^{-x}\Big|_0^\infty = -(x+1)e^{-x}\Big|_0^\infty = 1$$

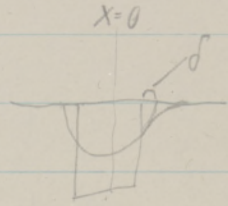
$$-c_v\,dT = p\,dv = \frac{RT}{v}dv,\quad -c_v\frac{dT}{T} = R\frac{dv}{v},\quad \frac{d\ln T}{d\ln v} = -\frac{R}{c_v} = -\frac{R}{\frac{3}{2}R} = -\frac{2}{3}$$

$$\ln T = \mathcal{C} - \frac{2}{3}\ln v = \mathcal{C} - \ln v^{\frac{2}{3}},\quad T v^{\frac{2}{3}} = e^{\mathcal{C}} = \text{konst} = T_1 v_1^{\frac{2}{3}},\quad \frac{T}{T_1} = \frac{v_1^{\frac{2}{3}}}{v^{\frac{2}{3}}}$$





$u_1 = A\cos\alpha$ für $x = -\frac{l}{2}$ und $x = +\frac{l}{2}$



$$\alpha = \frac{\pi}{2}\frac{x}{l} = \frac{\pi}{2}\frac{x}{l+\delta} = \frac{\pi}{2}\frac{x}{l}\frac{1}{1+\frac{\delta}{l}} = \frac{\pi}{2}\frac{x}{l}\left(1-\frac{\delta}{l}\right) = \frac{\pi}{2}\frac{x}{l} - \frac{\pi}{2}\frac{x}{l}\frac{\delta}{l}$$

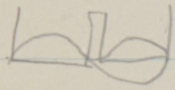
$$\cos\left(\frac{\pi}{2}-\varepsilon\right) = \cos-\left(\frac{\pi}{2}-\varepsilon\right) = \cos\frac{\pi}{2} - \varepsilon\cdot\sin\frac{\pi}{2} = \varepsilon \qquad 1-\frac{\delta}{l}$$

$$u = A\cos\frac{\pi}{2}\frac{x}{l} \quad x = \pm l,\ u=0 \quad x = l,\ u = \varepsilon = \cos\left(\frac{\pi}{2}-\varepsilon\right) = \cos\frac{\pi}{2}\frac{l}{l} = \cos\frac{\pi}{2}\frac{l-\delta}{l} = \cos\left(\frac{\pi}{2}-\frac{\pi}{2}\frac{\delta}{l}\right)$$

$$\varepsilon = \frac{\pi}{2}\frac{\delta}{l} \qquad \cos\vartheta = \frac{1}{2}\left(e^{i\vartheta}+e^{-i\vartheta}\right) = \frac{1}{2}\left(1+i\vartheta-\frac{1}{2}\vartheta^2+1-i\vartheta-\frac{1}{2}\vartheta^2\right) = 1-\frac{1}{2}\vartheta^2$$

$$u = A\cos\frac{\pi}{2}\frac{x}{l+\delta} = A\cos\frac{\pi}{2}\frac{x}{l}\left(1-\frac{\delta}{l}\right) = A\cos\frac{\pi}{2}\frac{x}{l} - \frac{\pi}{2}\frac{x}{l}\frac{\delta}{l}$$

daß Ausbruch nicht stattfindet, da sonst bei $T = 0$ sich Inhomogenität nach kurzer Zeit einstellen würde, was Energiezufuhr erfordert. Andererseits wie soll beim Abkühlen Inhomogenität zustandekommen, wenn Atome nichts von einander wissen? Schwierigkeit bei

 nicht so.