1)

$$\psi_1 = u_1(x) e^{-i\left(\frac{E_1}{h}t+\delta_1\right)} \quad \psi_2 = u_2(x) e^{-i\left(\frac{E_2}{h}t+\delta_2\right)}$$

$$\psi = u_1(x_a) e^{-i\left(\frac{E_1}{h}+\delta_1\right)} u_2(x_b) e^{-i\left(\frac{E_2}{h}t+\delta_2\right)} + u_1(x_b) e^{-i\left(\frac{E_1}{h}t+\delta_1\right)} u_2(x_a) e^{-i\left(\frac{E_2}{h}+\delta_2\right)}$$

$$|\psi|^2 = u_1(x_a)^2 \times u_2(x_b)^2 + u_1(x_b)^2 \times u_2^2(x_a) + 2u_1(x_a) \times u_1(x_b) \times u_2(x_a) \times u_2(x_b)$$

$$1 + e^{-\alpha} + e^{-2\alpha} + \cdots e^{-n\alpha} = S$$

$$e^{-\alpha} + e^{-2\alpha} + \cdots e^{-n\alpha} + e^{-(n+1)\alpha} = S e^{-\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$1 - e^{-(n+1)\alpha} = S(1-e^{-\alpha}), \quad S = \frac{1}{1-e^{-\alpha}} \ (n \to \infty)$$

$$\alpha e^{-\alpha} + 2\alpha e^{-2\alpha} + 3\alpha e^{-3\alpha} + \cdots = \alpha(e^{-\alpha} + 2e^{-2\alpha} + 3e^{-3\alpha} + \cdots) = -\alpha \frac{dS}{d\alpha}$$

$$-\frac{dS}{d\alpha} = e^{-\alpha} + 2e^{-2\alpha} + 3e^{-3\alpha} + \cdots$$

$$-\frac{dS}{d\alpha} = -\frac{d\frac{1}{1-e^{-\alpha}}}{d\alpha} = \frac{e^{-\alpha}}{(1-e^{-\alpha})^2} \quad \Bigg| \quad \bar{\mathcal{E}} = \frac{0e^{-0} + h\nu e^{-\frac{h\nu}{kT}} + 2h\nu e^{-2\frac{h\nu}{kT}} + \cdots}{1 + e^{-\frac{h\nu}{kT}} + e^{-2\frac{h\nu}{kT}} + \cdots} = kT \frac{\alpha e^{-\alpha} + 2\alpha e^{-2\alpha} + \cdots}{1 + e^{-\alpha} + e^{-2\alpha} + \cdots}$$

$$\frac{1}{kT}\bar{\mathcal{E}} = \frac{e^{-\alpha}\alpha}{(1-e^{-\alpha})^2}(1-e^{-\alpha}) = \frac{e^{-\alpha}}{1-e^{-\alpha}} = \frac{\alpha}{e^{\alpha}-1} \quad \mathcal{E} = \frac{kT\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}}-1} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

$$S = \int_0^T \frac{d\mathcal{E}}{T} = \frac{\mathcal{E}}{T}\Big|_0^T - \int_0^T \mathcal{E}\, d\frac{1}{T}, \quad \frac{1}{kT}\bar{\mathcal{E}} = -\frac{\alpha\frac{dS}{d\alpha}}{S} = -\alpha\frac{d\ln S}{d\alpha}, \quad \bar{\mathcal{E}} = -h\nu\frac{d\ln S}{d\alpha}$$

$$\int_0^T \bar{\mathcal{E}}\, d\frac{1}{T} = \frac{k}{h\nu}\int_0^T \bar{\mathcal{E}}\, d\alpha = -k\int_\infty^{\alpha_T} d\ln S = -k\ln S_T, \quad S = \frac{\mathcal{E}}{T} + k\ln\left(1 + e^{-\frac{h\nu}{kT}} + e^{-2\frac{h\nu}{kT}} + \cdots\right)$$

$$\frac{d\ln S}{d\alpha} = \frac{1}{S}\frac{dS}{d\alpha} = -\frac{e^{-\alpha}+2e^{-2\alpha}+\cdots}{1+e^{-\alpha}+e^{-2\alpha}+\cdots}, \quad \bar{\mathcal{E}} = -h\nu\frac{d\ln S}{d\alpha} \qquad P = \mathcal{E} - TS = -kT\ln(\quad)$$

$$S = k\left[\frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}}-1} - \ln\left(1-e^{-\frac{h\nu}{kT}}\right)\right], \quad \frac{h\nu}{kT} \gg 1, \quad S = k\left[\frac{h\nu}{kT}e^{-\frac{h\nu}{kT}} + e^{-\frac{h\nu}{kT}}\right] = k\, e^{-\frac{h\nu}{kT}}\left(\frac{h\nu}{kT}+1\right)$$

$$S = k\left[-\frac{h\nu}{kT}\frac{d\ln\Sigma}{d\alpha} + \ln\Sigma\right] \quad d\ln\alpha = \frac{d\alpha}{\alpha} \quad S = k\left[-\frac{d\ln\Sigma}{d\ln\alpha} + \ln\Sigma\right]$$

$$S = k\left[-\frac{d\ln\Sigma}{d\ln\alpha} + \ln\Sigma\right] \quad \alpha = \frac{h\nu}{kT} \quad \Sigma = 1 + e^{-\alpha} + e^{-2\alpha} + \cdots = \frac{1}{1-e^{-\alpha}}$$

$$\frac{d\ln\Sigma}{d\ln\alpha} = \frac{\alpha}{\Sigma}\frac{d\Sigma}{d\alpha} = \alpha(1-e^{-\alpha})\cdot -\frac{e^{-\alpha}}{(1-e^{-\alpha})^2} = -\frac{\alpha e^{-\alpha}}{1-e^{-\alpha}} = -\frac{\alpha}{e^{\alpha}-1}$$

$$\ln\Sigma = -\ln(1-e^{-\alpha}) \quad \frac{d\ln\Sigma}{d\ln\alpha} = \alpha\frac{d\ln\Sigma}{d\alpha} = -\alpha\frac{d\ln(1-e^{-\alpha})}{d\alpha} = -\alpha\frac{e^{-\alpha}}{1-e^{-\alpha}}$$

$$S = k\left[\frac{\alpha}{e^{\alpha}-1} - \ln(1-e^{-\alpha})\right] = k\left[\frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}}-1} - \ln\left(1-e^{-\frac{h\nu}{kT}}\right)\right]$$

$$T\to\infty,\ \alpha\to 0,\ S\to k\left[1-\ln\frac{h\nu}{kT}\right]$$

$$T\to 0,\ \alpha\to\infty,\ S\to k\left[\alpha e^{-\alpha}+e^{-\alpha}\right] = k e^{-\frac{h\nu}{kT}}\left(\frac{h\nu}{kT}+1\right)$$

$$\bar{\mathcal{E}} = \frac{\varepsilon_1 e^{-\frac{\varepsilon_1}{kT}} + \varepsilon_2 e^{-\frac{\varepsilon_2}{kT}} + \cdots}{1+e^{-\frac{\varepsilon_1}{kT}}+e^{-\frac{\varepsilon_2}{kT}}+\cdots} = kT^2\frac{d\ln\Sigma}{dT},\ \Sigma = 1+e^{-\frac{\varepsilon_1}{kT}}+e^{-\frac{\varepsilon_2}{kT}}+\cdots$$

$$S = \int_0^T \frac{d\bar{\mathcal{E}}}{T} = \frac{\bar{\mathcal{E}}}{T}\Big|_0^T - \int_0^T \bar{\mathcal{E}}\, d\frac{1}{T} = \frac{\bar{\mathcal{E}}}{T} + \int_0^T \frac{\bar{\mathcal{E}}}{T^2}dT = \frac{\bar{\mathcal{E}}}{T} + \int_0^T k\, d\ln\Sigma = \frac{\bar{\mathcal{E}}}{T} + k\ln\Sigma_T$$

$$S = \left[kT\frac{d\ln\Sigma}{dT} + k\ln\Sigma\right] = k\frac{d(T\ln\Sigma)}{dT}$$

$$\frac{d(T\ln\Sigma)}{dT} = T\frac{d\ln\Sigma}{dT} + \ln\Sigma$$

$$\xi_0 = \frac{1}{\Sigma},\ \xi_1 = \frac{e^{-\frac{\varepsilon_1}{kT}}}{\Sigma},\ \xi_2 = \frac{e^{-\frac{\varepsilon_2}{kT}}}{\Sigma}, \quad \sum\xi_i = 1,\ \sum\xi_i\ln\xi_i = \sum_i\xi_i\left[-\frac{\varepsilon_i}{kT} - \ln\Sigma\right] = -\frac{\sum\varepsilon_i\xi_i}{kT} - \ln\Sigma = -\frac{\bar{\mathcal{E}}}{kT} - \ln\Sigma$$

$$\bar{\mathcal{E}} = \frac{\varepsilon_1 e^{-\frac{\varepsilon_1}{kT}}}{\Sigma} + \frac{\varepsilon_2 e^{-\frac{\varepsilon_2}{kT}}}{\Sigma} = \sum\varepsilon_i\xi_i \quad (\varepsilon_0 = 0) \qquad S = -k\sum\xi_i\ln\xi_i$$

3)

$$\lambda = \frac{2l}{n} = \frac{h}{p}, \quad p = n\frac{h}{2l}, \quad \varepsilon = \frac{p^2}{2m} = n^2\frac{h^2}{8ml^2} = n^2\beta$$

$$\alpha = \frac{\beta}{kT} \qquad \frac{n^2\beta}{kT} = n^2\alpha$$

$$S = \frac{\bar{\varepsilon}}{T} + k\ln\Sigma, \quad T \to \infty, \quad \Sigma = 1 + e^{-\alpha} + e^{-4\alpha} + \cdots e^{-n^2\alpha} \qquad \alpha = \frac{h^2}{8ml^2kT}$$

$$\Sigma \to \int_0^\infty e^{-n^2\alpha}dn = \frac{1}{\sqrt{\alpha}}\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} \qquad \frac{1}{\sqrt{\alpha}} = \frac{2\sqrt{2mkT}\,l}{h}$$

$T \to \infty$

$$S = \tfrac{1}{2}k + k\ln\frac{\sqrt{\pi}}{2\sqrt{\alpha}} = \frac{1}{2}k + k\ln\frac{l}{h}\sqrt{2\pi mkT} = k\ln l + \tfrac{1}{2}k\ln T + \tfrac{1}{2}k + k\ln\frac{(2\pi mk)^{\frac{1}{2}}}{h}$$

$$S_1 = k\ln l + \tfrac{1}{2}k\ln T + \tfrac{1}{2}k + k\ln\frac{(2\pi Mk)^{\frac{1}{2}}}{h}$$

$$S_1 = k(N-1) - k(N-1)\ln\frac{h}{kT} - k\ln\prod_e^{1,N-1}\nu^e$$

$$(x-x_1)(x-x_2)(x-x_3) = 0 = x^3 - (x_1+x_2+x_3)x^2 + (x_1x_2 + x_1x_3 + x_2x_3)x - x_1x_2x_3 = 0$$

$$\begin{aligned} m\ddot{\xi}_1 &= -\alpha\xi_1 + \alpha\xi_2 & \xi_1 &= u_1e^{i2\pi\nu t} & \ddot{\xi} &= -(2\pi\nu)^2\xi = +x\xi \\ m\ddot{\xi}_2 &= \alpha\xi_1 - 2\alpha\xi_2 + \alpha\xi_3 & \xi_2 &= u_2e^{i2\pi\nu t} \\ m\ddot{\xi}_3 &= \alpha\xi_2 - \alpha\xi_3 & \xi_3 &= u_3e^{i2\pi\nu t} \end{aligned}$$

$$\begin{aligned} -mx\xi_1 &= -\alpha\xi_1 + \alpha\xi_2 & \xi_1\left(-\frac{\alpha}{m} + x\right) + \frac{\alpha}{m}\xi_2 &= 0 \\ -mx\xi_2 &= \alpha\xi_1 - 2\alpha\xi_2 + \alpha\xi_3 & \xi_1\frac{\alpha}{m} + \left(-2\frac{\alpha}{m} + x\right)\xi_2 + \frac{\alpha}{m}\xi_3 &= 0 \qquad \frac{\alpha}{m} = a \\ -mx\xi_3 &= \alpha\xi_2 - \alpha\xi_3 & \xi_2\frac{\alpha}{m} + \left(-\frac{\alpha}{m} + x\right)\xi_3 &= 0 \end{aligned}$$

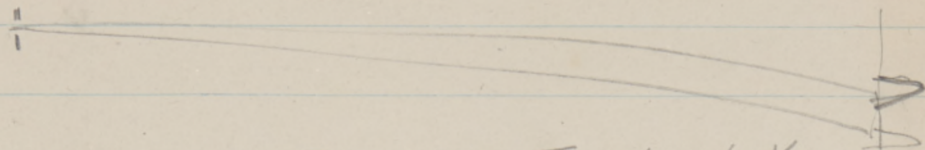
$$\begin{vmatrix} -a+x & a & 0 \\ a & -2a+x & a \\ 0 & a & -a+x \end{vmatrix} = 0 = (x-a)\begin{vmatrix} x-2a & a \\ a & x-a \end{vmatrix} - a\begin{vmatrix} a & a \\ 0 & x-a \end{vmatrix}$$

$$= (x-a)\left[(x-2a)(x-a) - a^2\right] - a^2(x-a)$$

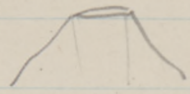
$$= (x-a)(x^2 - 3ax + a^2) - a^2x + a^3 = 0$$

$\sqrt{2}$

$$K' = \frac{p'^2}{p} = P\frac{4\alpha^2}{(1+\alpha)^2}\frac{1+\alpha}{1-\alpha} = P\frac{4\alpha^2}{1-\alpha^2}$$



0.01 cm^2



					x
T	t	$\log K_p$	$-\frac{1}{2}\log K_p$	$P=1$ atm	10^{-10} atm
$T=400$	127	14.60	$0.7-8$	5×10^{-8}	5×10^{-8}
673	400	6.41	$0.8-4$	6×10^{-4}	$1-1.3\times10^{-4}$

0.005 cm^2

$$K_p = 10^{6.41} \quad P = 10^{-10} \quad K_pP = 10^{-3.59} = 10^{0.41}\times10^{-4} = 2.6\times10^{-4}$$

$2\ 10^{-3}$ cm^2

$\cancel{\pi\, 200^2}$

$4\ 10^{2}$

$\sqrt{2}$

$K_pP \ll 1$, $\cancel{\frac{1}{x^2} = 1+K_pP = \frac{1}{x}}$

$x = 1-\varepsilon$, $x^2 = 1-2\varepsilon$, $\frac{1-x^2}{x^2} = 2\varepsilon = K_pP$, $\varepsilon = 1-x = \frac{1}{2}K_pP$

$$\frac{1-x^2}{x^2} = \frac{1}{x^2} - 1 = K_pP$$

$\frac{1}{x^2} = K_pP + 1$, $-2\log x = \log K_pP + \log\left(1+\frac{1}{K_pP}\right)$

$$\log x = -\frac{1}{2}\log K_p - \frac{1}{2}\log P - \frac{1}{2}\log\left(1+\frac{1}{K_pP}\right)$$

$P = P_{J_2} + P_J \qquad \frac{P_{J_2}}{P_J} = \frac{1-x}{2x}$

$P_0(1-x) + P_0 2x = P = P_0(1+x) \quad P_{J_2} = P_0(1-x) = P\frac{1-x}{1+x} \quad P_J = P_0 2x = P\frac{2x}{1+x} \quad \frac{P_{J_2}}{P_J^2} = \frac{1}{P}\frac{1-x}{1+x}\frac{(1+x)^2}{4x^2} = \frac{1-x^2}{4x^2}$

$\log K_p = \frac{7762}{T} - 1.75\log T + 4.16\times10^{-4}T - 0.422 \qquad K_p = \frac{1-x^2}{x^2P}$

$x \ll 1 \quad x^2 = \frac{1}{K_pP}$

$T = 400 \quad \log K_p = 19.40_5 - 1.75\times2.602 + 0.166 \quad \begin{matrix}-4.55\\ -0.422\\ \hline -4.97\end{matrix}$

$\begin{matrix}0.166\\ \hline 19.57\\ -4.97\\ \hline 14.60\end{matrix}$

$2\log x = -\log K_p \quad (P=1)$

$\log x = -7.3 = 0.7-8$, $x = 5\times10^{-8} = 5\times10^{-6}\%$ $\quad P = 10^{-10}$

$\log x = -\frac{1}{2}\log K_p - \frac{1}{2}\log P$

$T = 673 \quad \log K_p = \overset{28}{11.50} - 1.75\times\overset{2.721}{2.828} + 0.28_0 \quad \begin{matrix}-4.95\\ -0.422\\ \hline -5.37\end{matrix}$

$\begin{matrix}11.78\\ -5.37\\ \hline 6.41\end{matrix} \qquad 4.95$

$\log x = -3.20 = 0.80-4$, $x = 0.0006 = 0.06\%$

$$\psi = c_1 e^{-i\alpha_1} e^{-i\frac{W_1}{h}t} u_1 + c_2 e^{-i\alpha_2} e^{-i\frac{W_2}{h}t} u_2$$

$$\int \psi^{\times}\psi\,dx = \int\left(c_1 e^{+i\alpha_1} e^{+i\frac{W_1}{h}t} u_1^{\times} + c_2 e^{+i\alpha_2} e^{+i\frac{W_2}{h}t} u_2^{\times}\right)\left(c_1 e^{-i\alpha_1} e^{-i\frac{W_1}{h}t} u_1 + c_2 e^{-i\alpha_2} e^{-i\frac{W_2}{h}t} u_2\right) dx$$
$$= c_1^2 + c_2^2 + 0 = 1$$

$$c_1' = c_1 e^{-i\alpha_1} e^{-i\frac{W_1}{h}t} \quad P_{11} = c_1^{\times} c_1 = c_1^2, \; P_{22} = c_2^{\times} c_2 = c_2^2, \; P_{12} = c_2^{\times} c_1 = c_1 c_2 e^{-i(\alpha_1-\alpha_2)} e^{-i\frac{W_1-W_2}{h}t} = c_1 c_2 e^{-i\alpha} e^{-i\nu t}$$
$$P_{21} = c_1^{\times} c_2 = c_1 c_2 e^{+i(\alpha_1-\alpha_2)} e^{+i\frac{W_1-W_2}{h}t} = c_1 c_2 e^{+i\alpha} e^{+i\nu t}$$

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} c_1^2 & c_1 c_2 e^{-i(\alpha+\nu t)} \\ c_1 c_2 e^{+i(\alpha+\nu t)} & c_2^2 \end{pmatrix} \qquad \begin{vmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{vmatrix} = P_{11}P_{22} - P_{21}P_{12} = c_1^2 c_2^2 - c_1^2 c_2^2 = 0$$

$$\ln\frac{v_1+h}{v_2+h} = \ln v_1\left(1+\frac{h}{v_1}\right) - \ln v_2\left(1+\frac{h}{v_2}\right) = \ln v_1 + \frac{h}{v_1} - \ln v_2 - \frac{h}{v_2} = \ln\frac{v_1}{v_2} + h\left(\frac{1}{v_1} - \frac{1}{v_2}\right)$$

$\frac{1}{v_1} - \frac{1}{v_2} = z, \; v_2 - v_1 = z v_1 v_2, \; z = \frac{v_1 v_2}{v_2 - v_1} \rightarrow \infty$ für $v_2 = v_1$ zugleich mit $\ln\frac{v_1}{v_2} \rightarrow 0$

$$\ln\frac{v_1+v_2+h}{v_2+h} = \ln\frac{v_1+v_2}{v_2} + h\left(\frac{1}{v_1+v_2} - \frac{1}{v_2}\right) = \ln\frac{v_1+v_2}{v_2} - h\frac{v_1}{(v_1+v_2)v_2} = \ln\frac{v_1+v_2}{v_2} - h\frac{v_1}{v_2^2}$$ für $v_1 \ll v_2$

$$\ln\left(1+\frac{v_1}{v_2}\right) = \frac{v_1}{v_2}\left(1 - \frac{h}{v_2}\right)$$

$$s_\alpha = \frac{M}{4RT}\frac{d\mathcal{H}}{ds}l^2 = \frac{5585}{4\times 8.314\times 10^7}\frac{d\mathcal{H}}{ds}\frac{l^2}{T} = 1.67_5\times 10^{-5}\frac{d\mathcal{H}}{ds}\frac{l^2}{T}\,\mathrm{cm} = 1.67_5\times 10^{-5}\times 5\times 10^4\frac{50}{250} = 1.67_5\ \mathrm{mm}$$

$$33.26$$

$$dJ = \frac{dn}{2s} = \frac{n_0 2 e^{-\frac{v^2}{\alpha^2}}\frac{v^3}{\alpha^3}d\frac{v}{\alpha}}{2s_\alpha\frac{\alpha^2}{v^2}} = \frac{n_0 e^{-\frac{s_\alpha}{s}}\left(\frac{s_\alpha}{s}\right)^2 d\frac{s_\alpha}{s}}{2s_\alpha} = \frac{1}{2}J_0\frac{b}{s_\alpha}e^{-x}x^2dx,\quad x = \frac{s_\alpha}{s}$$

$$\left(1+\frac{1}{2}\frac{s_\alpha}{s}\right)^2 + \left(\frac{1}{2}\frac{s_\alpha}{s}\right)^2$$

$$J = \int_{s=\infty}^{s=s} dJ = \int_0^{\frac{s_\alpha}{s}}\frac{1}{2}J_0\frac{b}{s_\alpha}x^2e^{-x}dx = \underline{\underline{J_0\frac{b}{s_\alpha}\left[1-\left(1+\frac{s_\alpha}{s}+\frac{1}{2}\frac{s_\alpha^2}{s^2}\right)e^{-\frac{s_\alpha}{s}}\right]}}$$

$$\int_0^a x^2e^{-x}dx = \int_a^0 x^2 de^{-x} = x^2e^{-x}\Big|_a^0 - 2\int_a^0 xe^{-x}dx = -a^2e^{-a} + 2\int_a^0 x\,de^{-x} = -a^2e^{-a} + 2\left[xe^{-x}+e^{-x}\right]_a^0$$

$$= -a^2e^{-a} + 2\left[1 - ae^{-a} - e^{-a}\right] = 2 - 2e^{-a} - 2ae^{-a} - a^2e^{-a} = 2\left[1-\left(1+a+\tfrac{1}{2}a^2\right)e^{-a}\right]$$

$s=0:\ J = J_0\frac{b}{s_\alpha}$

$\frac{s}{s_\alpha} = 1:\ [\,] = 1 - 2\frac{1}{2}\times 0.368 = 1 - 0.920 = 0.080$

$\frac{s}{s_\alpha} = \frac{1}{2}:\ \frac{s_\alpha}{s} = 2,\ [\,] = 1 - 5\times 0.1353_4 = 1 - 0.6767 = 0.3233$

$\frac{s}{s_\alpha} = \frac{1}{4}:\ \frac{s_\alpha}{s} = 4,\ [\,] = 1 - 13\times 0.01832 = 1 - 0.238 = 0.762$ (183 5496)

$\frac{s}{s_\alpha} = \frac{1}{6}:\ \frac{s_\alpha}{s} = 6,\ [\,] = 1 - 25\times 0.00248 = 1 - 0.062 = 0.938$

$\frac{s}{s_\alpha} = \frac{1}{10}:\ \frac{s_\alpha}{s} = 10,\ [\,] = 1 - 61\times 0.0000458 = 1 - 0.0028 = 0.997_2$

$\frac{s}{s_\alpha} = \frac{1}{5}:\ \frac{s_\alpha}{s} = 5,\ [\,] = 1 - 18\frac{1}{2}\times 0.00674 = 1 - 0.124_7 = 0.875_3$

$\frac{s}{s_\alpha} = \frac{2}{3}:\ \frac{s_\alpha}{s} = 1.5,\ [\,] = 1 - 3.62_5\times 0.223 = 1 - 0.809 = 0.191$

$\frac{s}{s_\alpha} = \frac{4}{5}:\ \frac{s_\alpha}{s} = 1.25,\ [\,] = 1 - 3.03\frac{1}{8}\times 0.286_5 = 1 - 0.868 = 0.132$

$\frac{s}{s_\alpha} = 2:\ \frac{s_\alpha}{s} = 0.5,\ [\,] = 1 - 1.625\times 0.6065_3 = 1 - 0.9856 = 0.0144$ (1516 36392)

$$\frac{dJ}{ds} \sim \frac{d[\,]}{ds} = \frac{d\left[\frac{s_\alpha}{s}=x\right]}{dx}\frac{dx}{ds} = \frac{d\left[1-\left(1+x+\frac{1}{2}x^2\right)e^{-x}\right]}{dx}\times -\frac{s_\alpha}{s^2} = \cancel{(1+x)e^{-x}} - \cancel{\left(1+x+\tfrac{1}{2}x^2\right)}e^{-x} - \frac{1}{2}x^2e^{-x}\times\frac{x^2}{s_\alpha} = -\frac{1}{2s_\alpha}\left(\frac{s_\alpha}{s}\right)^4e^{-\frac{s_\alpha}{s}}$$

$$s=0\ \frac{dJ}{ds} = 0,\quad s=\infty\ \frac{dJ}{ds} = \infty \qquad \frac{d\frac{dJ}{ds}}{ds} = -\frac{1}{2s_\alpha}\frac{dx^4e^{-x}}{dx}\times -\frac{x^2}{s_\alpha} = \frac{x^2}{2s_\alpha^2}\left(4x^3e^{-x} - x^4e^{-x}\right) = 0,\ x_m = 4 = \frac{s_\alpha}{s_{\left(\frac{dJ}{ds}\right)_m}}$$

$$\underline{\underline{s_{\left(\frac{dJ}{ds}\right)_m} = \frac{1}{4}s_\alpha}}$$

Kontrolle: $\int_0^\infty J\,ds = J_0 b\int_0^\infty\left[1-\left(1+\frac{s_\alpha}{s}+\frac{1}{2}\frac{s_\alpha^2}{s^2}\right)e^{-\frac{s_\alpha}{s}}\right]d\frac{s}{s_\alpha} = J_0 b\int_\infty^0\left[1-\left(1+x+\frac{1}{2}x^2\right)e^{-x}\right]d\frac{1}{x} = J_0 b\times\frac{1}{2}$

$$\int_\infty^0[\,]\,d\frac{1}{x} = [\,]\cdot\frac{1}{x}\Big|_\infty^0 - \int_\infty^0\frac{1}{x}d[\,],\quad \lim_{x\to 0}\frac{[\,]}{x} = \frac{1-\left(1+x+\frac{1}{2}x^2\right)\left(1-x+\frac{1}{2}x^2-\frac{1}{6}x^3\right)}{x} = \cancel{\frac{1-1+x-\frac{1}{2}x^2-x+x^2-\frac{1}{2}x^3+\frac{1}{2}x^2-\frac{1}{2}x^3+\frac{1}{6}x^3}{x}} = \frac{1}{6}x^2 \to 0$$

$$d[\,] = \frac{d[\,]}{dx}dx = \left[-(1+x)e^{-x} + \left(1+x+\tfrac{1}{2}x^2\right)e^{-x}\right]dx = \frac{1}{2}x^2e^{-x}dx,\quad \int_\infty^0 xe^{-x}dx = \int_0^\infty x\,de^{-x} = xe^{-x}\Big|_0^\infty - \int_0^\infty e^{-x}dx = -1$$

$$\int_\infty^0[\,]\,d\frac{1}{x} = 0 - \int_\infty^0\frac{1}{2}xe^{-x}dx = \frac{1}{2}$$

Journal of Physics, USSR, 11 (№ 1) 1947 p. 33-43, B. Davydov
Quantum mechanics and thermodynamic irreversibility.

$$W_E = \frac{1}{2}\lambda^2\left[\frac{k(k+1)-3p^2}{k(k+1)(2k-1)(2k+3)}\right]\frac{\hbar^2}{2I} \qquad \lambda = \frac{\mu E 2I}{\hbar^2},\ \frac{d\lambda}{dE} = \frac{\lambda}{E} = \frac{\mu}{\left(\frac{\hbar^2}{2I}\right)}$$

$$-\mu_E\frac{dW_E}{dE} = \frac{dW_E}{d\lambda}\frac{d\lambda}{dE} = \lambda[\ ]\frac{\mu 2I}{\hbar^2} = \frac{\lambda^2}{E}[\ ] = E\left(\frac{\lambda}{E}\right)^2[\ ] = \lambda\frac{\lambda}{E}[\ ]\frac{\hbar^2}{2I} = \lambda[\ ]\mu$$

$$\frac{\mu_e}{\mu} = -\lambda[\ ] \qquad \begin{matrix} k=2 \\ p=\pm 1\end{matrix} \quad [\ ] = \frac{2\times3-3}{2\times3\times3\times7} = \frac{1}{42}$$

$$I \simeq 2\times10^{-38}\ g\,cm^2,\ \mu \simeq 7\times10^{-18}\ e.s.u$$

$$\hbar^2 = 1.11\times10^{-54} \simeq 1.10^{-54}\ \frac{\hbar^2}{2I} = \frac{1.11\times10^{-54}}{3.8\ 10^{-38}} = 2.9\times10^{-17} \quad T = 850°K,\ k = 1.38\times10^{-16},\ kT = 1.17\times10^{-13},\ \frac{\left(\frac{\hbar^2}{2I}\right)}{kT} = \frac{2.9\ 10^{-17}}{1.17\times10^{-13}} = 2.5\times10^{-4}$$

$$\frac{\lambda}{E} = \frac{\mu}{\left(\frac{\hbar^2}{2I}\right)} = \frac{7\times10^{-18}}{2.8\times10^{-14}} = 2.5\times10^{-1},\ \lambda = 2.5\times10^{-1}\times E,\ \lambda = 1 \text{ for } E = 4\ e.s.u = 120\ \frac{V}{cm} \qquad \sigma_0 = \frac{0.21}{T} = \frac{2100}{850}10^{-4} = 2.47\times10^{-4}$$

$$\lambda = 5 = \frac{\mu}{\frac{\hbar^2}{2I}}E = 2.5\times10^{-1}E,\ E = 20\ esu = 6000\frac{V}{cm}$$

$$\lambda = \frac{2.5\times10^{-1}}{300}E_{\frac{V}{cm}} = 8.3\times10^{-4}E_{\frac{V}{cm}}$$

$$W = \frac{\hbar^2}{2I}\left[k(k+1) + \frac{1}{2}\lambda^2\left\{\frac{k(k+1)-3p^2}{k(k+1)(2k-1)(2k+3)}\right\}\right],\ \varepsilon = \frac{W}{\left(\frac{\hbar^2}{2I}\right)} \qquad k=0,\ p=0$$

$$\varepsilon = -\frac{1}{6}\lambda^2$$

$$k=1,\ p=0,\ \varepsilon = 2+\frac{1}{2}\lambda^2\left\{\frac{1.2}{1.2.1.5}\right\} = 2+\frac{1}{10}\lambda^2$$

$$k=1,\ p=\pm1,\ \varepsilon = 2+\frac{1}{2}\lambda^2\left\{\frac{1.2-3}{1.2.1.5}\right\} = 2-\frac{1}{20}\lambda^2$$

$$k=2,\ p=0,\ \varepsilon = 6+\frac{1}{2}\lambda^2\left\{\frac{2.3}{2.3.3.7}\right\} = 6+\frac{1}{42}\lambda^2$$

$$k=2,\ p=\pm1,\ \varepsilon = 6+\frac{1}{2}\lambda^2\left\{\frac{2.3-3}{2.3.3.7}\right\} = 6+\frac{1}{84}\lambda^2$$

$$k=2,\ p=\pm2,\ \varepsilon = 6+\frac{1}{2}\lambda^2\left\{\frac{2.3-3.4}{2.3.3.7}\right\} = 6-\frac{1}{42}\lambda^2$$

$$n_k = \frac{(2k+1)e^{-\frac{E_k}{kT}}}{\sum(2k+1)e^{-\frac{E_i}{kT}}},\ \varepsilon_k = \frac{\hbar^2}{2I}k(k+1) = \frac{h^2}{8\pi^2 I}k(k+1),\ \sigma = \frac{\hbar^2}{2IkT},\ n_k = \frac{(2k+1)e^{-\sigma k(k+1)}}{\sum(2k+1)e^{-\sigma k(k+1)}}$$

$$\sum(2k+1)e^{-\sigma k(k+1)} \cong \frac{1}{\sigma}\int_0^\infty e^{-x}dx = \frac{1}{\sigma},\ n_k \simeq \sigma(2k+1)\times1 = \frac{\hbar^2(2k+1)}{2IkT}\ (\ll 1)$$

$$\left(\sim\int 2k\,e^{-\sigma k^2}dk\right)$$

$$-\sum a_l \left(\kappa \frac{\partial^2 \psi_l^0}{\partial \varphi^2} + W_k \psi_l^0\right) = (W_k' - v)\, \psi_k^0$$

$$-\sum a_l (-W_l^0 \psi_l^0 + W_k^0 \psi_l^0) = \sum a_l (W_l^0 - W_k^0)\, \psi_l^0 \,\Big|\, \int \psi_k^{0\times} d\tau = 0$$

$$\int \psi_k^{0\times} (W_k' - v)\, \psi_k^0\, d\tau = 0, \quad W_k' = \int \psi_k^{0\times}\, v\, \psi_k^0\, d\tau = 0$$

$$-\kappa \frac{d^2\psi}{d\varphi^2} + v\psi = W\psi, \quad -\kappa \frac{d^2\psi_k^0}{d\varphi^2} = W_k \psi_k^0, \quad -\kappa \frac{d^2\psi_l}{d\varphi^2} + v\psi_l = W_l \psi_l$$

$$\psi_l = \sum_k a_{lk} \psi_k^0, \quad -\kappa \sum_k a_{lk} \frac{d^2 \psi_k^0}{d\varphi^2} + \sum_k a_{lk}\, v\, \psi_k^0 = W_l \sum_k a_{lk} \psi_k^0 \,\Big|\, \int \psi_m^{\times} d\tau$$

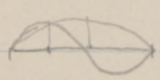
$$-\kappa \sum_k a_{lk} W_k^0 \psi_k^0 - \sum_k a_{lk} W_l \psi_k^0 + \sum_k a_{lk}\, v\, \psi_k^0 = 0 \,\Big|\, \int \psi_m^0\, d\tau$$

$$a_{lm} W_m^0 - a_{lm} W_m + \sum_k a_{lk} v_{km} = 0, \quad \sum_k [v_{km} - W_m \delta$$

$$u_n = \sum_m S_{mn} u_m^0, \quad H u_n = \mathcal{E}_n u_n \,\Big|\, \int u_k^{0\times} d\tau \qquad H u_n = -\kappa \frac{d^2 u_n}{d\varphi^2} + V u_n = E_n u_n$$

$$\sum_m S_{mn} \int u_k^{0\times} H u_m^0\, d\tau = \sum_m \mathcal{E}_n S_{mn} \int u_k^{0\times} u_m^0\, d\tau = \mathcal{E}_n S_{mn} \delta_{km}$$

$$\sum_m S_{mn} (H_{km} - \mathcal{E}_n \delta_{km}) = 0$$



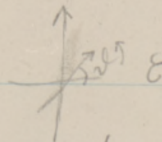
$$u_1 = \sqrt{\frac{2}{l}} \sin \pi \frac{x}{l} \quad u_2 = \sqrt{\frac{2}{l}} \sin 2\pi \frac{x}{l} \quad \int_0^l u_1^2\, dx = \frac{2}{l} \int_0^l \sin^2 \frac{\pi}{l} x\, dx = \frac{2}{\pi} \int_0^x \sin^2 y\, dy = 1 \qquad y = \pi \frac{x}{l}$$

$$u = u_1^a u_2^b - u_1^b u_2^a = \frac{2}{l}\left[\sin \pi \frac{x_a}{l} \sin 2\pi \frac{x_b}{l} - \sin \pi \frac{x_b}{l} \sin 2\pi \frac{x_a}{l}\right]$$

$$x_a = \frac{l}{2} \quad x_b = \frac{l}{4} \quad \sin \pi \frac{x_a}{l} \sin 2\pi \frac{x_b}{l} = \sin \frac{\pi}{2} \sin \frac{\pi}{2} = 1$$

$$x_a = \frac{l}{4} \quad x_b = \frac{l}{2} \quad \sin \pi \frac{x_a}{l} \sin 2\pi \frac{x_a}{l} = \sin \frac{\pi}{4} \sin \pi = 0$$

$$\mu = -\frac{\mu_0^2 E}{2I\omega_0^2}$$

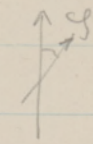


$$\varepsilon = -\mu E \sin\vartheta$$

$$\bar{\mu} = \frac{\int_0^{2\pi} \mu \sin\vartheta \, e^{\frac{\mu E \sin\vartheta}{kT}} d\vartheta}{\int_0^{2\pi} e^{\frac{\mu E \sin\vartheta}{kT}} d\vartheta} \cong \frac{\int_0^{2\pi} \mu \sin\vartheta \, d\vartheta + \int_0^{2\pi} \frac{\mu^2 E}{kT} \sin^2\vartheta \, d\vartheta}{\int_0^{2\pi} \left(1 + \frac{\mu E}{kT}\sin\vartheta\right) d\vartheta} = \frac{\mu^2 E}{kT} \frac{\pi}{2\pi} = \frac{1}{2}\frac{\mu^2 E}{kT}$$

$$\mu = \frac{dW}{dE} = \frac{8\pi^2 I \mu^2 E}{h^2}, \quad 2kT = \frac{h^2}{8\pi^2 I}, \quad kT = \frac{\hbar^2}{I} = I\omega^2 = \frac{(I\omega)^2}{I}$$

$$m^2 - (m \pm 1)^2 = \mp 2m - 1 \qquad -\mu E \cos\varphi$$



$$\frac{1}{2m-1} + \frac{1}{2m+1} = \frac{2}{4m^2-1}$$

$$\frac{d^2\psi}{d\varphi^2} + \frac{2I}{\hbar^2} W\psi = 0 \qquad \psi = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \quad \frac{d^2\psi}{d\varphi^2} = -m^2\psi$$

$$-m^2 + \frac{2I}{\hbar^2} W_m = 0, \quad W_m = m^2 \frac{\hbar^2}{2I}$$

$$\frac{d^2\psi}{d\varphi^2} + \frac{2I}{\hbar^2}(W + \mu E \cos\varphi)\psi = 0 \qquad \mu E \ll W$$

$$\psi_k = \psi_k^0 + \lambda \psi_k' + \lambda^2 \psi_k'' + \cdots \qquad W = W_k^0 + \lambda W_k' + \lambda^2 W_k'' + \cdots$$

$$-\frac{\hbar^2}{2I}\frac{d^2\psi}{\partial\varphi^2} + \overset{v}{\mu E \cos\varphi}\cdot\psi - W\psi = 0, \quad \frac{\hbar^2}{2I} = \kappa$$

$$-\kappa \frac{\partial^2 \psi_k^0}{\partial\varphi^2} - W_k^0 \psi_k^0 = 0$$

$$-\kappa \frac{\partial^2 \psi_k'}{\partial\varphi^2} + v\psi_k^0 - W_k^0 \psi_k' - W_k' \psi_k^0 = 0 \qquad \psi' = \sum a_l \psi_l^0$$

$$-\kappa \sum a_l \frac{\partial^2 \psi_l^0}{\partial\varphi^2} + v\psi_k^0 - W_k^0 a_l \psi_l^0 - W_k' \psi_k^0 = 0$$

$$I\ddot{\psi} = \mu_e X \cos\psi \,/\, \dot\psi \quad \frac{d(\dot\psi)^2}{dt} = 2\dot\psi\ddot\psi = 2\dot\psi\frac{d\dot\psi}{dt} \quad d\dot\psi^2 = 2\dot\psi\frac{d\dot\psi}{dt}dt$$

$$\dot\psi\frac{d\dot\psi}{dt} = \frac{\mu_e X}{I}\cos\psi\frac{d\psi}{dt}, \quad d\left(\frac{1}{2}\dot\psi^2\right) = \frac{\mu_e X}{I}d(\sin\psi), \quad \frac{1}{2}\dot\psi^2 = \frac{\mu_e X}{I}\sin\psi + C$$

$$\dot\psi^2 = \frac{2\mu_e X}{I}\sin\psi + \omega_0^2 = \left(1+\frac{2\mu_e X\sin\psi}{\omega_0^2 I}\right)\omega_0^2, \quad \dot\psi = \omega_0\left(1+\frac{2\mu_e X}{I\omega_0^2}\sin\psi\right)^{\frac{1}{2}}, \quad \frac{d\psi}{dt} = \dot\psi$$

$$dt = \frac{d\psi}{\dot\psi}$$

$$\bar\mu_1 = \frac{\int_0 \mu_e \sin\psi\, dt}{\int_0 dt} = \frac{\int_0^{2\pi}\mu_e\frac{\sin\psi}{\dot\psi}d\psi}{\int_0^{2\pi}\frac{d\psi}{\dot\psi}} = \mu_e\frac{\int_0^{2\pi}\frac{\sin\psi}{\sqrt{1+\varepsilon\sin\psi}}d\psi}{\int_0^{2\pi}\frac{1}{\sqrt{1+\varepsilon\sin\psi}}d\psi}$$

$$\frac{\sin\psi}{\sqrt{1+\varepsilon\sin\psi}} = \sin\psi\left(1-\frac{1}{2}\varepsilon\sin\psi\right) = \sin\psi - \frac{1}{2}\varepsilon\sin^2\psi$$

$$\bar\mu_1 = \mu_e\frac{0-\frac{1}{2}\varepsilon\pi}{2\pi} = -\mu_e\frac{\varepsilon}{4} = -\mu_e\frac{2\mu_e X}{4I\omega_0^2} = -\frac{\mu_e^2 X}{2I\omega_0^2}$$

$$\bar\mu = +\frac{\mu_e^2 X}{2I\omega_0^2}(3\cos^2\vartheta - 1) = \frac{\mu_e^2 X}{2\frac{(I\omega_0)^2}{I}}(3\cos^2\vartheta - 1)$$

$$\bar\mu = \frac{\mu_e^2 X}{2\frac{\hbar^2}{I}}\frac{1}{(J-\frac{1}{2})(J+\frac{3}{2})}\left[3\frac{M^2}{J(J+1)} - 1\right] \quad J \gg 1, \quad \bar\mu = \frac{\mu_e^2 X}{2\frac{(J\hbar)^2}{I}}\left[3\left(\frac{M}{J}\right)^2 - 1\right]$$

$$(\alpha\beta)_{pq} = \sum_r \alpha_{pr}\beta_{rq}$$

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \qquad (\alpha\beta) = \begin{pmatrix} \alpha_{11}\beta_{11} + \alpha_{12}\beta_{21}, & \alpha_{11}\beta_{12} + \alpha_{12}\beta_{22} \\ \alpha_{21}\beta_{11} + \alpha_{22}\beta_{21}, & \alpha_{21}\beta_{12} + \alpha_{22}\beta_{22} \end{pmatrix}$$

$$\overset{\alpha}{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}} \overset{\beta}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \qquad (\alpha\beta) = \begin{pmatrix} 0+0, & 0+0 \\ 0+0, & 1+0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(\beta\alpha) = \begin{pmatrix} 0+1, & 0+0 \\ 0+0, & 0+0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\overset{\beta}{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}} \overset{\alpha}{\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}} \qquad (\beta\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{iw_a}(n_a+1)e^{-iw_a} = e^{iw_a} n_a e^{-iw_a} + \mp 1 = n_a + \mp 1$$

$$x^3 - 3ax^2 + a^2x - ax^2 + 3a^2x - a^3 - a^2x + a^3 = 0$$

$$x^3 - 4ax^2 + 3a^2x = 0, \quad x_1 = 0, \quad x^2 - 4ax + 3a^2 = 0$$

$$x^2 - 4ax + 4a^2 = a^2 = (x-2a)^2, \quad x - 2a = \pm a, \quad x = 2a \pm a, \quad x_2 = 3a, \quad x_3 = a$$

$$x_2 \cdot x_3 = 3a^2$$

$$\cancel{2\pi \ln 2a^{\frac{1}{2}} + 2\int_0^{\pi} \ln \sin x \, dx = 2\pi \ln 2a^{\frac{1}{2}} - 2\pi \ln 2 = 2\pi \left[\ln 2a^{\frac{1}{2}} - \ln 2^{\frac{1}{2}}\right] = 2\pi \ln (2a)^{\frac{1}{2}}}$$

$$\prod_e^{1,N-1} (2\pi\nu_e)^2 = N\left(\frac{\alpha}{m}\right)^{N-1}, \quad \left(\frac{\alpha}{m}\right)^{\frac{1}{2}} = 2\pi\nu_0, \quad \prod_e^{1,N-1} (2\pi\nu_e)^2 = N\left(4\pi^2\nu_0^2\right)^{N-1}, \quad \prod_e^{1,N-1} \nu_e = N^{\frac{1}{2}} \nu_0^{N-1}$$

$$S_f = k(N-1) - k(N-1)\ln\frac{h}{kT} - k\ln N^{\frac{1}{2}}\nu_0^{N-1} = k(N-1) - k(N-1)\ln\frac{h\nu_0}{kT} - \frac{1}{2}k\ln N$$

$$= k(N-1)\left(1 - \ln\frac{h\nu_0}{kT}\right) - \frac{1}{2}k\ln N$$

$$S_q = k\ln l + \frac{1}{2}k\ln T + \frac{1}{2}k + k\ln\frac{(2\pi m k)^{\frac{1}{2}}}{h} + \frac{1}{2}k\ln N$$

$$S_f + S_q = k\ln l + \frac{1}{2}k\ln T + \frac{1}{2}k + k\ln\frac{(2\pi m k)^{\frac{1}{2}}}{h} + k(N-1)\left(1 - \ln\frac{h\nu_0}{kT}\right)$$

$$S_{f_1} + S_{q_1} = k\ln l + \frac{1}{2}k\ln T + S^0_{q_1} + k(N_1 - 1)\left(1 - \ln\frac{h\nu_0}{kT}\right) - \frac{1}{2}k\ln N_1$$

$$S_{f_2} + S_{q_2} = k\ln l + \frac{1}{2}k\ln T + S^0_{q_2} + k(N_2 - 1)\left(1 - \ln\frac{h\nu_0}{kT}\right) - \frac{1}{2}k\ln N_2$$

$$S_f + S_q = 2k\ln l + k\ln T + \left(S^0_{q_1} + S^0_{q_2}\right) + k(N_1 + N_2 - 1)\left(1 - \ln\frac{h\nu_0}{kT}\right) - k\left(1 - \frac{h\nu_0}{kT}\right) - \frac{1}{2}k\ln N_1 N_2$$

$$- k(N_1 + N_2 - 1)\left(1 - \ln\frac{h\nu_0}{kT}\right) + \frac{1}{2}k\ln N_1 + N_2$$

$$S_g = nk\ln l + \tfrac{1}{2}nk\ln T + \tfrac{3}{2}nk + nk\ln\frac{(2\pi mk)^{\frac{1}{2}}}{h}$$
$$= nk\ln l + \tfrac{1}{2}nk\ln\frac{(2\pi mkT)}{h^2} + \tfrac{3}{2}nk$$

$$k\ln n! = k(n\ln n - n) = nk\ln n - nk$$

$$nS_g = nk\ln\frac{l}{n} + nk\ln\frac{(2\pi mkT)^{\frac{1}{2}}}{h} + \tfrac{1}{2}nk$$ gerade in einzelne box $\left(\frac{l}{n}\right)$

alle verschieden: Zwischenwände weggenommen, Entropiezunahme für jedes Atom:

$$\Delta S = +k\ln\frac{l}{\frac{l}{n}} = k\ln n, \text{ für } n \text{ Atome } nk\ln n$$

$$S_g = nk\ln\frac{l}{n} + nk\ln\frac{(2\pi mkT)^{\frac{1}{2}}}{h} + \tfrac{1}{2}nk + nk\ln n$$

alle gleich: $k(-\ln n!) = (-n\ln n + n)k$

$$S_g = nk\ln\frac{l}{n} + nk\ln\frac{(2\pi mkT)^{\frac{1}{2}}}{h} + \tfrac{3}{2}nk$$

Gesamtmasse: $M = N\mu$

Geteilt in Z gleiche Mol., $n = \frac{N}{Z}$, $m = n\mu$

$$S_g^{gl} = nk\ln\frac{l}{n} + nk\ln\frac{(2\pi mkT)^{\frac{1}{2}}}{h} + \tfrac{3}{2}nk$$

Geteilt in Z Mol., alle ungleich: $n = \frac{N}{Z}$, $m_1 = Z_1\mu$, … $m_i = Z_i\mu$ $\quad \sum Z_i = N$

$$S_g^{ungl} = nk\ln\frac{l}{n} + nk\ln\frac{(2\pi\tilde{m}kT)^{\frac{1}{2}}}{h} + \tfrac{1}{2}kn + nk\ln n, \quad \tilde{m}^n = \prod_i^{n} m_i$$

$$S_g^{ungl} - S_g^{gl} = nk\ln\left(\frac{\tilde{m}}{m}\right)^{\frac{1}{2}} + nk\ln n - nk = nk\ln\left(\frac{\tilde{m}}{m}\right)^{\frac{1}{2}} + k\ln n!$$

$1+2+3+4+5 = 15$, $3\times 5 = 15$, $x^5 = 1.2.3.4.5 = 120$, $5\log x = \log 120 = 2.07918 : 5 = \overset{\log x}{0.41554}$ $\quad x = 2.6034$

$\frac{\tilde{m}}{m} = \frac{2.6034}{3} = 0.8678$

$1+2+3+4+5+6+7 = 28$, $4\times 7 = 28$, $x^7 = 1.2.3.4.5.6.7 = \overset{720\cdot 7}{5040}$, $7\log x = \log 5040 = 3.70243 : 7 = 0.52892$, $x = 3.3800$

$\frac{\tilde{m}}{m} = \frac{3.3800}{4} = 0.8450$

$n \quad 1+2+3+\ldots+n = (n+1) + (n-1+2) + (n-2+3) + \ldots = N = \frac{n-1}{2}(n+1) + \frac{n+1}{2} = (n-1)\frac{n+1}{2} + \frac{n+1}{2} = n\frac{(n+1)}{2}$

$n \gg 1$, $n^2 = 2N$ $\quad n = \sqrt{2N}$ $\quad N = 15$ $\quad n\frac{n+1}{2} = 5\,\frac{5+1}{2} = 15$, $n\frac{n+1}{2} = 7\,\frac{7+1}{2} = 28$

$1.2.3.\ldots n = n!$ $\quad x^n = n!$ $\quad x = \sqrt[n]{n!}$ $\quad \frac{\tilde{m}}{m} = \frac{\sqrt[n]{n!}}{\frac{n+1}{2}}$ $\quad n\ln\frac{\tilde{m}}{m} = n\ln\frac{2\sqrt[n]{n!}}{n+1} = \ln\frac{2^n n!}{(n+1)^n} = n\ln 2 + \ln n! - n\ln(n+1)$

6)

$$\begin{array}{r} 1{,}00000 \\ -\underline{0{,}69315} \\ 0{,}30685 \end{array}$$

$$n\ln 2 + \ln n! - n\ln(n+1) = n\ln 2 + n\ln n - n - n\ln(n+1) = n(\ln 2 - 1) - 1 = -(n\,0.30685 + 1)$$

$$-n\ln\frac{n+1}{n} = n\ln\left(1+\frac{1}{n}\right) = -n\frac{1}{n} = -1 \quad (n \gg 1)$$

$n=7 \quad \frac{\tilde{m}}{m} = 0.8450 \quad \ln 0.845 = 9.83157-10 = -0.16843 \cdot 7 = -1.17901$ $\quad 0.30685 \cdot 7 \quad 2.14795 \quad -3.148$ $\quad$ statt -1.179

$$\begin{array}{rr} 1198 & 565 \\ 1183 & 592 \\ 592 & 1157 \end{array}$$

$$n\ln\frac{\tilde{m}}{m} = -n\cdot 0.30685 \quad (n \gg 1) \quad \ln\frac{\tilde{m}}{m} = -0.30685 = 9{,}69315 - 10$$

$$\frac{\tilde{m}}{m} \cong 0.736 \quad (n \gg 1)$$

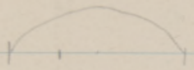
$$n\ln\left(\frac{\tilde{m}}{m}\right)^{\frac{1}{2}} \cong -\frac{1}{2}(n\cdot 0.30685 + 1) \quad (n \gg 1)$$

$$\cong -n\cdot 0.1534 \qquad n\ln\left(\frac{\tilde{m}}{m}\right)^{\frac{1}{2}} + \ln n! = n(\ln n - 1.1534)$$

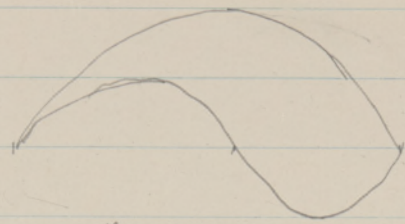
$$\ln n! = n\ln n - n = n(\ln n - 1)$$

$$M = \log e = 0.4343$$

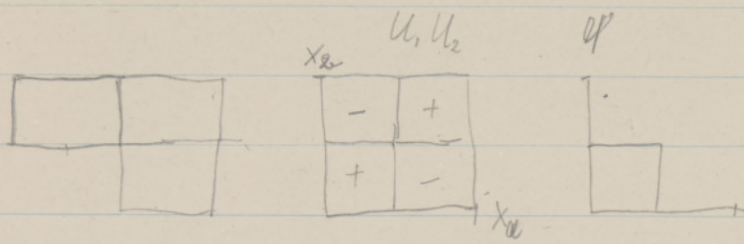
$$\log z = \log e^{\ln z} = \ln z \cdot \log e, \quad \ln z = \frac{1}{\log e}\log z = 2.3026 \times \log z \quad \frac{1}{M} = \ln 10 = 2.3026$$



$\psi_1 =$



$$\psi = u_1(x_a)\,u_2(x_b) + u_1(x_b)\,u_2(x_a)$$



$$\psi_1 = A_1 e^{-i\left(\frac{E_1}{\hbar}t+\delta_1\right)} u_1 \qquad \psi_2 = A_2 e^{-i\left(\frac{E_2}{\hbar}t+\delta_2\right)} u_2 \qquad A_1^2\int_0^l |u_1^2|\,dx = 1 \qquad A_2^2\int_0^l |u_2^2|\,dx = 1$$

$$\psi = \psi_1(x_a)\,\psi_2(x_b) + \psi_1(x_b)\,\psi_2(x_a) = A_1A_2\, e^{-i\left(\frac{E_1+E_2}{\hbar}t+\delta_1+\delta_2\right)} u_1(x_a)\,u_2(x_b)$$
$$+ A_1A_2\, e^{-i\left(\frac{E_1+E_2}{\hbar}t+\delta_1+\delta_2\right)} u_1(x_b)\,u_2(x_a)$$

$$\psi = A_1A_2\, e^{-i\left(\frac{E_1+E_2}{\hbar}t+\delta_1+\delta_2\right)} \left[u_1(x_a)\,u_2(x_b) + u_1(x_b)\,u_2(x_a)\right]$$

$$\psi\psi^* = A_1^2A_2^2\left[|u_1(x_a)|^2\,|u_2(x_b)|^2 + |u_1(x_b)|^2\,|u_2(x_a)|^2 + u_1^*(x_a)\,u_2(x_a)\,u_1(x_b)\,u_2^*(x_b) + u_1(x_a)\,u_2^*(x_a)\,u_1^*(x_b)\,u_2(x_b)\right]$$

$$\int_0^l \psi\psi^*\,dx = 2$$

$$\frac{4\times 8\times 4}{5^7}\ \frac{7}{3} \qquad \frac{32\times 28}{125\times 125\times 15} = \frac{3.2\times 2.8\times 10^2}{1.25\times 1.25\times 1.5\times 10^5} = 3.8\times 10^{-3} = 0.0038$$

$$\frac{4.8\times 10^2}{21^{23}}\ \frac{23}{19} = \frac{3840}{21^{23}}$$

$\lg 3840 = 3.5844$ $\quad 30 \quad -27$

-30.4111

$0.1466-27$

$\log 21 = 1.32222\times 23$

3.96666
26.4444
30.4111

1.5×10^{-27}

$$\frac{0.75\times 10^{-16}}{1.5\times 10^{-27}} = 0.5\times 10^{11}$$

I/

$p = m\dot{x} = \frac{h}{\lambda}$, $l_n = \frac{2l}{n}$, $E_n = \frac{p_n^2}{2m} = \frac{h^2}{2m\,4l^2}n^2 = n^2\frac{h^2}{8ml^2}$, $u_n = \sin\frac{2\pi x}{\lambda_n}$, $\psi = \sum_n c_n e^{-\frac{i}{\hbar}E_n t} u_n = \sum_n c_n \psi_n$

$c_n = a_n e^{-i\delta_n}$

$P_{mn} = c_n^* c_m$ $\quad \bar{E} = \sum c_n^* c_n E_n = \sum P_{nn}E_n = \sum_n (PE)_{n,n}$

$\bar q = \int q\psi^*\psi\, dq = \sum_{n,m} c_n^* c_m \int q \psi_n^* \psi_m\, dq = \sum_{n,m} c_n^* c_m q_{nm} = \sum_{n,m} q_{nm} P_{mn} = \sum_n (qP)_{n,n}$

$\bar F = \int \psi^*(F\psi)\, dq = \sum_n (FP)_{nn}$

$u_1 = \sqrt{\frac{2}{l}}\sin\frac{2\pi}{2l}x$ $\quad u_2 = \sqrt{\frac2l}\sin\frac{2\pi}{l}x$ $\quad E_1 = \frac{h^2}{8ml^2}$ $\quad E_2 = 4E_1$ $\quad \frac{E_1}{h}=\nu_1$ $\quad \frac{E_2}{h} = 4\frac{E_1}{h} = 4\nu_1 = \nu_2$

$\psi_1 = e^{-i\nu_1 t}\sin\frac{\pi}{l}x$

$q_{11} = \frac2l\int_0^l x\sin^2\frac{\pi}{l}x\,dx = \frac{l^2}{\pi^2}\frac{2}{l}\int_0^\pi y\sin^2 y\,dy \quad (y=\frac{\pi}{l}x) = \frac{2}{l}\frac{l^2}{\pi^2}\frac{\pi^2}{4} = \frac{l}{2}$

$n^2\int x^m\sin^n x\,dx = x^{m-1}\sin^{n-1}x(m\sin x - nx\cos x) + n(n-1)\int x^m \sin^{n-2}x\,dx - m(m-1)\int x^{m-2}\sin^n x\,dx$

$4\int_0^\pi y\sin^2 y\,dy = \sin y(\sin y - 2y\cos y) + 2\int_0^\pi y\,dy - 0\Big|_0^\pi = y^2\Big|_0^\pi = \pi^2$, $\int_0^\pi y\sin^2 y\,dy = \frac{\pi^2}{4}$

$\int_0^l u_n^2\, dx = 1 = C^2\int_0^l \sin^2 n\frac{\pi}{l}x\,dx = \frac{l}{\pi n}C^2\int_0^{n\pi}\sin^2 y\,dy \quad (y = n\frac{\pi}{l}x) = \frac{l}{n\pi}C^2\frac{n\pi}{2} = C^2\frac{l}{2} = 1$, $C^2 = \frac2l$, $C = \sqrt{\frac2l}$

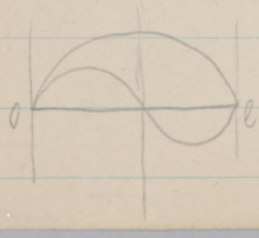
$q_{12} = \frac2l\int_0^l x\sin\frac{\pi}{l}x\sin 2\frac{\pi}{l}x\,dx = \frac2l\frac{l^2}{\pi^2}\int_0^\pi y\sin y\sin 2y\,dy \quad (y=\frac{\pi}{l}x) = -\frac2l\frac{l^2}{\pi^2}\frac89 = -\frac{16}{9\pi^2}l$

$\int_0^\pi y\sin y\sin 2y\,dy = \int_0^\pi y\,2\sin^2 y\cos y\,dy = \int_0^\pi 2y\sin^2 y\,d\sin y = \int_0^\pi \frac23 y\,d\sin^3 y = \cancel{\frac23 y\sin^3 y\Big|_0^\pi} - \int_0^\pi\frac23\sin^3 y\,dy$

$\int_0^\pi \sin^3 y\,dy = \int_\pi^0 \sin^2 y\,d\cos y = \int_\pi^0(1-\cos^2 y)\,d\cos y = \cos y - \frac13\cos^3 y\Big|_\pi^0 = 2-\frac13-\frac13 = \frac43$, $\int_0^\pi y\sin y\sin 2y\,dy = -\frac89$

$q_{21} = \frac2l\int_0^l x\sin 2\frac{\pi}{l}x\sin\frac{\pi}{l}x\,dx = q_{12}$

$q_{22} = \frac2l\int_0^l x\sin^2 2\frac{\pi}{l}x\,dx = \frac2l\left(\frac{l}{2\pi}\right)^2\int_0^{2\pi} y\sin^2 y\,dy \quad (y = 2\pi\frac{x}{l}) = \frac2l\frac{l^2}{(2\pi)^2}\frac{(2\pi)^2}{4} = \frac{l}{2}$



$\int_0^{\pi/2} y\,d\sin^3 y = y\sin^3 y\Big|_0^{\pi/2} - \int_0^{\pi/2}\sin^3 y\,dy = \frac\pi2 - \int_0^{\pi/2}\sin^3 y\,dy$

$\int_{\pi/2}^\pi y\,d\sin^3 y = y\sin^3 y\Big|_{\pi/2}^\pi - \int_{\pi/2}^\pi \sin^3 y\,dy = -\frac\pi2$

II)

$$\psi = \sqrt{\tfrac{2}{l}}\; e^{-i(\nu_1 t+\delta_1)} \sin\pi\frac{x}{l} + \sqrt{\tfrac{2}{l}}\; e^{-i(\nu_2 t+\delta_2)} \sin 2\pi\frac{x}{l}$$

$$\int_0^l \psi\psi^* dx = \frac{2}{l}\int_0^l \left[e^{-i(\nu_1 t+\delta_1)}\sin\pi\frac{x}{l} + e^{-i(\nu_2 t+\delta_2)}\sin 2\pi\frac{x}{l}\right]\left[e^{+\cdots}\sin\pi\frac{x}{l} + e^{+\cdots}\sin 2\pi\frac{x}{l}\right]dx$$

$$= \frac{2}{l}\left[\int_0^l \sin^2\pi\frac{x}{l}\,dx + \int_0^l \sin^2 2\pi\frac{x}{l}\,dx + \left(e^{-i[(\nu_1-\nu_2)t+\delta_1-\delta_2]} + e^{-i[(\nu_2-\nu_1)t+\delta_2-\delta_1]}\right)\int_0^l \sin\pi\frac{x}{l}\sin 2\pi\frac{x}{l}\,dx\right]$$

$$= \frac{2}{l}\left[\frac{l}{2} + \frac{l}{2} + 0\right] = 2$$

$$\int_0^l x\psi\psi^* dx = \frac{1}{l}\left[\int_0^l x\sin^2\pi\frac{x}{l}\,dx + \int_0^l x\sin^2 2\pi\frac{x}{l}\,dx + \left(e^{-i[(\nu_1-\nu_2)t+\delta_1-\delta_2]} + e^{+i[\;\;]}\right)\int_0^l x\sin\pi\frac{x}{l}\sin 2\pi\frac{x}{l}\,dx\right]$$

$$= \frac{1}{2}q_{11} + \frac{1}{2}q_{22} + \frac{1}{2}q_{12}\,2\cos[(\nu_1-\nu_2)t + \delta_1-\delta_2]$$

$$P_{nm} = C_n^* C_m \quad C_1 = e^{-i(\nu_1 t+\delta_1)} \quad C_2 = e^{-i(\nu_2 t+\delta_2)}$$

$$P_{11} = C_1^*C_1 = 1,\quad P_{22} = C_2^*C_2 = 1,\quad P_{12} = C_2^*C_1 = e^{-i[(\nu_1-\nu_2)t+(\delta_1-\delta_2)]},\quad P_{21} = C_1^*C_2 = e^{+i[(\nu_1-\nu_2)t+(\delta_1-\delta_2)]}$$

$$\begin{vmatrix} P_{11}-\lambda & P_{12} \\ P_{21} & P_{22}-\lambda \end{vmatrix} = 0 = (P_{11}-\lambda)(P_{22}-\lambda) - P_{12}P_{21},\quad P_{11}P_{22} - \lambda(P_{11}+P_{22}) + \lambda^2 = P_{12}P_{21}$$

$$\lambda^2 - \lambda(P_{11}+P_{22}) + \frac{1}{4}(P_{11}+P_{22})^2 = \frac{1}{4}(P_{11}+P_{22})^2 - P_{11}P_{22} + P_{12}P_{21}$$

$$\lambda = \frac{1}{2}(P_{11}+P_{22}) \pm \sqrt{\frac{1}{4}(P_{11}+P_{22})^2 - P_{11}P_{22} + P_{12}P_{21}}$$

$$= \frac{1}{2}\,2 \pm \sqrt{\frac{1}{4}4 - 1 + 1} = 1 \pm \sqrt{1} = 2 \text{ oder } 0$$

III)

$$f_{ww}(R) = \alpha f_{ww}'(R) + \beta f_{ww}''(R), \quad \alpha > 0, \ \beta > 0, \ \alpha + \beta = 1$$

$$\psi_e = \frac{1}{\sqrt{2}}\left(e^{-i(\nu_1 t + \delta_1)} u_1 + e^{-i(\nu_2 t + \delta_2)} u_2\right) \quad u_1, u_2 \text{ orthonormal}$$

$$\int_0 \psi^* \psi \, dx = 1 = \frac{1}{2}\int \left(e^{+\nu_1 \delta_1} u_1^* + e^{+\nu_2 \delta_2} u_2^*\right)\left(e^{-\nu_1 \delta_1} u_1 + e^{-\nu_2 \delta_2} u_2\right) dx = \frac{1}{2}(1 + 1 + 0 + 0) = 1$$

$$\psi' = e^{-i(\nu_1 t + \delta_1)} u_1 \quad \int_0 \psi'^* \psi' \, dx = \int_0 u_1^* u_1 \, dx = 1 \qquad u_1^* = u_1 \quad u_2^* = u_2$$

$$\psi'' = e^{-i(\nu_2 t + \delta_2)} u_2$$

$$\bar{x} = \frac{1}{2}\int_0 \left(e^{-i(\nu_1 t + \delta_1)} u_1 + e^{-i(\nu_2 t + \delta_2)} u_2\right) x \left(e^{+} u_1 + e^{+} u_2\right) dx$$

$$= \frac{1}{2}\left[\int_0 u_1 x u_1 \, dx + \int_0 u_2 x u_2 \, dx + \left(e^{-i[(\nu_1 - \nu_2)t + \delta_1 - \delta_2]} + e^{+i[(\nu_1 - \nu_2)t + \delta_1 - \delta_2]}\right)\int_0 u_1 x u_2 \, dx\right]$$

$$= \frac{1}{2}\left\{q_{11} + q_{22} + 2 q_{12} \cos[(\nu_1 - \nu_2)t + \delta_1 - \delta_2]\right\}$$

$$\bar{x}' = q_{11} \quad \bar{x}'' = q_{22} \qquad \alpha q_{11} + \beta q_{22} \neq \frac{1}{2}\left\{q_{11} + q_{22} + 2 q_{12} \cos[(\nu_1 - \nu_2)t + \delta_1 - \delta_2]\right\}$$

$$P_{mn} = c_n^* c_m \qquad \lambda = \frac{1}{2}(P_{11} + P_{22}) \pm \sqrt{\left[\frac{1}{2}(P_{11} + P_{22})\right]^2 - (P_{11} P_{22} - P_{12} P_{21})}$$

$$c_1 = \frac{1}{\sqrt{2}} e^{-i(\nu_1 t + \delta_1)} \quad c_2 = \frac{1}{\sqrt{2}} e^{-i(\nu_2 t + \delta_2)}, \quad P_{11} = \frac{1}{2}, \ P_{22} = \frac{1}{2}, \ P_{21} = P_{12}^* = \frac{1}{2} e^{-i[(\nu_1 - \nu_2)t + \delta_1 - \delta_2]}$$

$$\lambda = \frac{1}{2} \cdot 1 \pm \sqrt{\frac{1}{4} - 0} = \frac{1}{2} \pm \frac{1}{2} = \begin{matrix} 1 \\ 0 \end{matrix} \qquad P_{11} + P_{22} = 1, \ P_{11} P_{22} = \frac{1}{4}, \ P_{12} P_{21} = \frac{1}{4}$$

$$c_1 = \alpha \quad c_2 = \beta, \quad P_{11} = \alpha^2, \ P_{22} = \beta^2, \ P_{12} = P_{21} = \alpha\beta, \ P_{11} + P_{22} = \alpha^2 + \beta^2, \ P_{11} P_{22} = \alpha^2 \beta^2, \ P_{12} P_{21} = \alpha^2 \beta^2$$

$$\psi_e = \alpha \psi' + \beta \psi'' \quad \lambda = \frac{1}{2}(P_{11} + P_{22}) \pm \frac{1}{2}(P_{11} + P_{22}) = \begin{matrix} P_{11} + P_{22} = \alpha^2 + \beta^2 \\ 0 \end{matrix}$$

$\int_0 \psi^* \psi \, dx = \alpha^2 + \beta^2 = 1$ unvereinbar mit $\alpha + \beta = 1$ $(2\alpha\beta = 0!)$

IV

$$\left[(p_1 - p_2)2l = h \quad \mathcal{E} = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 2m} = \frac{h^2}{8ml}n^2 \quad p = \frac{h}{2l}n, \; p_1 - p_2 = \frac{h}{2l}(n_1 - n_2)\right]$$

$$\bar{q} = \sum (qP)_{n,n} \quad \begin{matrix} q_{11} & q_{12} \\ q_{21} & q_{22} \\ \frac{1}{2}l & -\frac{16}{9\pi}l \\ -\frac{16}{9\pi}l & \frac{1}{2}l \end{matrix} \quad \begin{matrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ \frac{1}{2} & \frac{1}{2}e^{-i[\,]} \\ \frac{1}{2}e^{+i[\,]} & \frac{1}{2} \end{matrix} \quad \begin{matrix} (qP)_{11} = q_{11}c_{11} + q_{12}c_{21} \\ (qP)_{22} = q_{21}c_{12} + q_{22}c_{22} \end{matrix}$$

$$\bar{q} = \frac{1}{4}l + \frac{1}{4}l - \frac{16}{9\pi}l\,\frac{1}{2}\left\{e^{i[\,]} + e^{-i[\,]}\right\} = \frac{1}{2}l - \frac{16}{9\pi}l\cos\left[(\nu_1 - \nu_2)t + (\delta_1 - \delta_2)\right]$$

$$\Delta S = \int_{T_0}^{T} \mathcal{C}\frac{dT}{T} - \frac{\mathcal{C}(T-T_0)}{T_0} = \mathcal{C}\ln\frac{T}{T_0} - \frac{\mathcal{C}(T-T_0)}{T_0} = -\frac{1}{2}\frac{\mathcal{C}(T-T_0)^2}{T_0^2} = -\frac{1}{2}\mathcal{C}\left(\frac{T}{T_0} - 1\right)^2$$

$$\ln\frac{T}{T_0} = \ln\frac{T_0 + T - T_0}{T_0} = \ln\left(1 + \frac{T-T_0}{T_0}\right) \simeq \frac{T-T_0}{T_0} - \frac{1}{2}\left(\frac{T-T_0}{T_0}\right)^2$$

$$\frac{df(x_0+x)}{dx} = f(x_0) + x\left|\frac{df(x_0+x)}{dx}\right|_0 + \frac{1}{2}x^2\left|\frac{d^2 f(x_0+x)}{dx^2}\right|$$

$$\frac{d\ln(1+x)}{dx} = \ln 1 + x\left|\frac{d\ln(1+x)}{dx}\right|_{x=0} + \frac{1}{2}x^2\left|\frac{d^2\ln(1+x)}{dx^2}\right|_0$$

$$\frac{1}{1+x} \to 1 \qquad -\frac{1}{(1+x)^2} \to -1$$

$$\Delta S = \Delta E \qquad S(x) = S_0(x_0) + (x - x_0)\left|\frac{dS}{dx}\right|_{x_0} + \frac{1}{2}(x-x_0)^2\left|\frac{d^2S}{dx^2}\right|_{x_0}$$

$$S = w_1 \ln w_1 + w_2 \ln w_2 \qquad w_1 = 1 - x \quad w_2 = x$$

$$\Delta S = -\frac{E}{T_0} \quad w \qquad S_m = k\ln 2 \quad \left(\sum w_i \ln w_i\,!\right)$$

V

$\mathcal{E}_1 \quad \mathcal{E}_2 \qquad w_1 = \mathcal{C} e^{-\frac{\mathcal{E}_1}{kT}} \quad w_2 = \mathcal{C} e^{-\frac{\mathcal{E}_2}{kT}} \qquad \ln w_1 = -\frac{\mathcal{E}_1}{kT} + \ln \mathcal{C}$

$w_1 + w_2 = 1 = \mathcal{C}\left(e^{-\frac{\mathcal{E}_1}{kT}} + e^{-\frac{\mathcal{E}_2}{kT}}\right)$

$k\, n_1 \qquad n_1 \quad n_2 \quad p_1 = \frac{n_1}{V} kT, \; p_2 = \frac{n_2}{V} kT$

$k(w_1 \ln w_1 + w_2 \ln w_2) = w_1 S_1 + w_2 S_2$

$k \ln w_1 = S_1 = k_x - \frac{\mathcal{E}_1}{kT} \quad S_1 = -\frac{\mathcal{E}_1}{T}$

$\frac{S_1}{k} + \ln w_1 = \frac{S_2}{k} + \ln w_2, \quad \frac{S_1}{k} - \frac{\mathcal{E}_1}{kT} = \frac{S_2}{k} - \frac{\mathcal{E}_2}{kT}$

S muß unabhängig von W sein, deshalb muß
$k(w_1 \ln w_1 + w_2 \ln w_2) + w_1 S_1 + w_2 S_2 \geqq 0$ sein auch für die Werte von w_1 und w_2,
für die der Ausdruck ein Minimum wird. $(w_1 + w_2 = 1)$

$\delta\{w_1(k \ln w_1 + S_1) + w_2(k \ln w_2 + S_2)\} = 0, \quad \delta w_1 + \delta w_2 = 0$

$\delta w_1 (k \ln w_1 + S_1) + w_1\Big(k\, \underbrace{\delta \ln w_1}_{\frac{\delta w_1}{w_1}} + 0\Big) + \delta w_2 (k \ln w_2 + S_2) + w_2\Big(k\, \underbrace{\delta \ln w_2}_{\frac{\delta w_2}{w_2}} + 0\Big) = 0$

$\delta w_2 = -\delta w_1$

$\delta w_1 (k \ln w_1 + S_1 + k) - \delta w_1 (k \ln w_2 + S_2 + k) = 0$

$k \ln w_1 + S_1 = k \ln w_2 + S_2 = \lambda k$

$\ln w_1 = \lambda - \frac{S_1}{k}, \quad w_1 = e^{\lambda} e^{-\frac{S_1}{k}}, \quad w_2 = e^{\lambda} e^{-\frac{S_2}{k}}$

$e^{\lambda} e^{-\frac{S_1}{k}}(k\lambda - S_1 + S_1) + e^{\lambda} e^{-\frac{S_2}{k}}(k\lambda - S_2 + S_2) \geqq 0$

$\lambda e^{\lambda}\left(e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}}\right) \geqq 0, \quad \lambda \geqq 0, \quad w_1 + w_2 = 1, \quad e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} = e^{-\lambda} \leqq 1$

$S_1 = \frac{\mathcal{E}_1}{T} \quad S_2 = \frac{\mathcal{E}_2}{T}$

VI

$$k(w_1 \ln w_1 + w_2 \ln w_2 + w_3 \ln w_3) + w_1 S_1 + w_2 S_2 + w_3 S_3 \geq 0, \quad w_1 + w_2 + w_3 = 0$$

$$-\lambda'(w_1 + w_2 + w_3)$$

$$\delta\{w_1(k \ln w_1 + S_1 - \lambda') + \cdot + \cdot \quad \} = 0$$

$$\delta w_1 (k \ln w_1 + S_1 - \lambda') + \cancel{w_1}\, k \frac{\delta w_1}{\cancel{w_1}} = 0$$

$$k \ln w_1 + S_1 - \lambda' + k = 0 \qquad \ln w_1 = \frac{\lambda'}{k} - 1 - \frac{S_1}{k} \qquad w_1 = e^{\frac{\lambda'}{k} - 1} e^{-\frac{S_1}{k}}$$

$$e^{\frac{\lambda'}{k} - 1}\left(e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} + e^{-\frac{S_n}{k}}\right) = 1$$

$$e^{\frac{\lambda'}{k} - 1} e^{-\frac{S_1}{k}} (\cancel{\lambda'} - k \cancel{- S_1} \cancel{+ S_1} \cancel{- \lambda'}) = -k e^{\frac{\lambda'}{k} - 1} e^{-\frac{S_1}{k}}$$

$S_i \overset{?}{=} k \ln w_i$, Mindestaufwand von Entropie, um festzustellen, ob System im i-ten Zustand. S_i muß unabhängig von w_i sein?

$$\lambda = \frac{\lambda'}{k} - 1 \qquad \sum_i^{1,n} w_i \ln w_i \text{ Ext. mit } \sum_i^{1,n} w_i = 1$$

$$\delta(w_i \ln w_i) = \frac{d(w_i \ln w_i)}{d w_i} \delta w_i = (\ln w_i + 1)\, \delta w_i,$$

$$\ln w_i + \lambda + 1 = 0, \quad w_i = e^{-(\lambda+1)} \quad \sum_i^{1,n} e^{-(\lambda+1)} = 1 = n e^{-(\lambda+1)} = w_i \times n, \quad w_i = \frac{1}{n}$$

Unabhängigkeit von S_i von w_i erweist dadurch, daß auch für die Messung größter Entropie, für die $w_i = \frac{1}{n}$ ist, $k \sum w_i \ln w_i + \sum w_i S_i \geq 0$ ist. Daraus folgt $S_i \geq k \ln \frac{1}{n}$ für die bei der Messung erzeugte Entropie.

$$S_1 = k \ln \frac{v}{v_1} \quad S_2 = k \ln \frac{v}{v_2} \quad S_1 = k \ln \frac{1}{w_1} \quad S_2 = k \ln \frac{1}{w_2}$$

$$e^{-\frac{S}{k}} + e^{-\frac{S_2}{k}} = e^{-\ln \frac{1}{w_1}} + e^{-\ln \frac{1}{w_2}} = w_1 + w_2 = 1$$

$$\frac{w_1}{w_2} = e^{-\frac{S_1 - S_2}{k}} = \frac{e^{-\frac{S_1}{k}}}{e^{-\frac{S_2}{k}}} \quad w_1 + w_2 = 1 \quad \frac{w_1}{w_1 + w_2} = \frac{e^{-\frac{S_1}{k}}}{e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}}} = w_1, \quad w_2 = \frac{e^{-\frac{S_2}{k}}}{e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}}}$$

Quantum theory in its present state must be considered as a representation of a the ~~experimental~~ facts in a very wide field of experience. The fundamental laws of q. th. derive their justification from this firm fundament. They are a system of mathematical prescriptions to be applied to the numerical results of measurements.

To consider q. th as a satisfactory theory this system should be derived from ~~a~~ general physical assumption. The following paper tries to make a first step in this direction by looking for a closer connection between statistics thermodynamics and q. th.

According to q. th. the laws of nature are such that the information we can get for a system by measurements is limited in principle. This fact is closely connected with the third law of thermodynamics (Nernst th.). By measurements on the System we diminish its entropy. In classical theory we can do this infinitely, we can measure as exact as we like and diminish the entropy of the system as much as we like. The Nernst th. states that we can only extract from the system a finite amount of entropy and correspondingly the qu. th. states that we can get only a limited amount of information by measurements on the system. For a free one dim. particle $\Delta p \Delta q \sim h$, the Volume V in the phasespace cannot be made smaller than h.

If we adopt the classical connection between entropy S and ϑ. $S = k \ln \vartheta$ that means that the entropy cannot be made smaller than $S_0 = k \ln h$. For a harmonic resonator and the classical connection between energy $\mathcal{E}$ and ν the energy cannot be smaller than $h\nu$. Now its zero point energy is $\frac{1}{2} h\nu$. We can perhaps formulate our principle so that ϑ is dimished allways by h.? That's different from the old theory because is would have the consequence that we cannot determine the phase and the energy at the same time.

Question: If we have a resonator in the lowest state and we measure afterwards the phase or the impuls what is the result? Same question for particle in a box. Qu. th. give the answer. How derive this answer from a general principle by means of entropy formulation?

First let us try to make the assumption that at high temperatures we have the classical distribution of coordinates and impulses and see if get the right sequence of energies from our assumption:

1) Particle in box, one dimens. $\vartheta = \int p\, dq = 2l\sqrt{2m\mathcal{E}}$, $\vartheta_1 - \vartheta_2 = (n_1 - n_2)h$

Qu. th.: $\lambda = \frac{2l}{n} = \frac{h}{p}$, $\mathcal{E} = \frac{p^2}{2m}$, $p = n\frac{h}{2l}$, $\mathcal{E} = n^2 \frac{h^2}{8l^2 m}$, $\mathcal{E}_1 - \mathcal{E}_2 = (n_1^2 - n_2^2)\frac{h^2}{8l^2 m}$

$2l(\sqrt{2m\mathcal{E}_1} - \sqrt{2m\mathcal{E}_2}) = (n_1 - n_2)h$, $= 2l\left(n_1\frac{h}{2l} - n_2\frac{h}{2l}\right) = (n_1 - n_2)h$. That means if we take the qu. th. expression we get $\vartheta_1 - \vartheta_2 = (n_1 - n_2)h$. Can reverse the procedure?

$V = v^2$

$\frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 \quad q_1 \; q_2$ $\qquad \mathcal{C} \sim \frac{(2\pi)^{\frac{3n}{2}}}{n!}$

$\frac{1}{2m}(p_1^2 + p_2^2) = E, \; p_1^2 + p_2^2 = 2mE, \; V = v^n \frac{(2\pi)^{\frac{3}{2}n}}{3n(3n-2)\cdots 1}(2mE)^{\frac{3}{2}n} = v^n \mathcal{C} (2mE)^{\frac{3}{2}n}$

$S = nk \ln v + \frac{3}{2}nk \ln kT + ns_0 = nk \ln v + \frac{3}{2} nk \ln kT + ns_0$

$= k \ln v^n (kT)^{\frac{3n}{2}} + \mathcal{C}, \; = k \ln V + \mathcal{C}, \; \mathcal{C} = ns_0$

$P = U - TS, \; S = \frac{U}{T} - \frac{P}{T} = \frac{U}{T} + k \ln \int e^{-\frac{\varepsilon}{kT}} dv$

$s = \frac{3}{2}k + k \ln \frac{(2\pi m kT)^{\frac{3}{2}} v^n}{h^3} \; S = nk \ln v + \frac{3}{2} nk \ln kT + nk \ln \frac{(2\pi m)^{\frac{3}{2}}}{h^3}$

$V = v^n \mathcal{C} (2mE)^{\frac{3}{2}n}, \; k \ln V = nk \ln v + \frac{3}{2} n \ln 2mE + k \ln \mathcal{C}$

$E = n \frac{3}{2} kT \quad k \ln V = nk \ln v + \frac{3}{2} nk \ln kT + \frac{3}{2} nk \ln (2m\, n \frac{3}{2}) + k \ln \mathcal{C}$

$\mathcal{C} \sim \frac{\pi^{\frac{3}{2}n}}{n!} \quad \ln \mathcal{C} = \frac{3}{2} n \ln \pi - n \ln n + n + \text{konst}, \; k \ln V = nk \ln v + nk \ln \frac{(3m\pi)^{\frac{3}{2}}}{n} + \text{konst}$

$$d x^3 = 3x^2 dx$$

$$\int_0^\pi y\, 2\sin^2 y \cos y\, dy = \int_0^\pi 2y \sin^2 y\, d\sin y = \int_0^\pi \frac{2}{3} y\, d\sin^3 y = \cancel{\frac{2}{3} y \sin^3 y \Big|_0^\pi} - \int_0^\pi \frac{2}{3} \sin^3 y\, dy$$

$$\int_0^\pi \sin^3 y\, dy = -\int_0^\pi \sin^2 y\, d\cos y = -\int_0^\pi (1-\cos^2 y)\, d\cos y = -\cos y + \frac{1}{3}\cos^3 y \Big|_0^\pi = 2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3}$$

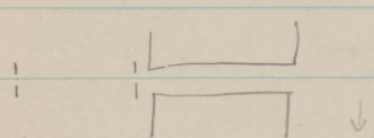
$$= \frac{1}{3}\cos y(\cos^2 y - 3) = -\frac{1}{3}\cos y(2 + 1 - \cos^2 y) = -\frac{1}{3}\cos y(\sin^2 y + 2)\Big|_0^\pi = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\psi(q,Q,t)=\sum_n c_n(Q,t)\,u_n(q,Q)$$

$$-\sum_{a=1}^{a=N}\frac{h^2}{2m^2}\sum_{k=1}^{k=3}\left[\frac{\partial^2}{\partial {x_k^a}^2} + V^a(Q) + \sum_{l=1}^{l=3}\frac{\partial V^{(a)}}{\partial Q_l}x_l^a + V(q_1\cdots q_f)\right]u(q,Q) = E_n(Q)\times u_n$$

$$-\frac{h}{i}\frac{\partial\psi}{\partial t} = (H_0\psi) + V(q_1\cdots q_f)\,\psi \qquad \psi=\sum_n c_n(Q,t)\,u_n(q,Q)$$

$$-\frac{h}{i}\frac{\partial c_n}{\partial t} = -$$



$$\cancel{F_y = \mu\frac{\partial H}{\partial y}} = \frac{\partial \mathcal{E}}{\partial y},\ m\ddot y = F_y,\ \ddot y = \frac{1}{m}\frac{\partial\mathcal{E}}{\partial y},\ y=\frac{1}{2m}\frac{\partial \mathcal{E}}{\partial y}t^2$$

$$\frac{p_y}{mv}\sim\frac{\lambda}{d}\quad \lambda=\frac{h}{mv}\quad \frac{p_y}{mv}\sim\frac{h}{mvd},\ p_y\sim\frac{h}{d},\ mv_y\sim\frac{h}{d},\ y\sim\frac{h}{md}t$$

$$\cancel{\frac{1}{2m}}\left(\frac{\partial\mathcal{E}_1}{\partial y}-\frac{\partial \mathcal{E}_2}{\partial y}\right)t^2 > \frac{h}{\cancel{md}}\cancel{t},\ d\,\frac{\partial(\mathcal{E}_1-\mathcal{E}_2)}{\partial y}t>h$$

$$\Delta(\mathcal{E}_1-\mathcal{E}_2)\times t>h$$

$m\ddot{x} = -a^2 x$, $x = Ae^{-i(2\pi\nu)t}$, $\dot{x} = A, -i2\pi\nu e^{-i(2\pi\nu)t}$, $\ddot{x} = A, +i^2(2\pi\nu)^2 e^{-i(2\pi\nu)t} = -(2\pi\nu)^2 x$

$m, -(2\pi\nu)^2 x = -a^2 x$, $(2\pi\nu)^2 m = a^2$

$$\mathcal{E} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2}a^2x^2 = \frac{p^2}{2m} + \frac{a^2}{2}q^2$$

$$\mathcal{E} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}a_1^2q_1^2 + \frac{1}{2}a_2^2q_2^2 + \lambda q_1q_2$$

$m_1 = m_2$ $a_1 = a_2$ $\mathcal{E} = \frac{1}{2m}(p_1^2+p_2^2) + \frac{1}{2}a^2(q_1^2+q_2^2) + \lambda q_1 q_2$

$\xi = \frac{1}{\sqrt{2}}(q_1+q_2)$, $\eta = \frac{1}{\sqrt{2}}(q_1-q_2)$, $q_1+q_2 = \xi\sqrt{2}$, $q_1-q_2 = \eta\sqrt{2}$ $q_1 = \frac{1}{\sqrt{2}}(\xi+\eta)$ $q_2 = \frac{1}{\sqrt{2}}(\xi-\eta)$ $q_1q_2 = \frac{1}{2}(\xi^2-\eta^2)$ $q_1^2+q_2^2 = \frac{1}{2}2(\xi^2+\eta^2)$

$\mathcal{E} = \frac{m^2}{2}\frac{1}{2m}2(\dot{\xi}^2+\dot{\eta}^2) + \frac{1}{2}a^2(\xi^2+\eta^2) + \lambda\frac{1}{2}(\xi^2-\eta^2)$ $p_1 = m\dot{q}_1 = \frac{m}{\sqrt{2}}(\dot{\xi}+\dot{\eta})$, $p_2 = m\dot{q}_2 = \frac{m}{\sqrt{2}}(\dot{\xi}-\dot{\eta})$

$= \frac{1}{2}m(\dot{\xi}^2+\dot{\eta}^2) + \frac{1}{2}(a^2+\lambda)\xi^2 + \frac{1}{2}(a^2-\lambda)\eta^2$

$\frac{a_1^2}{m_1} = \frac{a_2^2}{m_2} = 2\pi\nu$ $V = \frac{1}{2}(2\pi\nu_1)^2 q_1^2 + \frac{1}{2}(2\pi\nu_2)^2 q_2^2 = \frac{1}{2}\left(\frac{a^2}{M}+\lambda\right)q_1'^2 + \frac{1}{2}\left(\frac{a^2}{M}-\lambda\right)q_2'^2$

$\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_2\dot{q}_2^2 = \frac{1}{2}\dot{q}_1'^2 + \frac{1}{2}\dot{q}_2'^2$ $\dot{q}_1' = \sqrt{m_1}\dot{q}_1$ $\dot{q}_2' = \sqrt{m_2}\dot{q}_2$ $q_1' = \sqrt{m_1}q_1$ $q_2' = \sqrt{m_2}q_2$

$p_1' = \sqrt{m_1}p_1$ $p_2' = \sqrt{m_2}p_2$

$\mathcal{E} = \frac{1}{2}p_1'^2 + \frac{1}{2}p_2'^2 + \frac{1}{2}(2\pi\nu)^2(q_1'^2+q_2'^2) + \frac{\lambda}{\sqrt{m_1m_2}}q_1'q_2'$ $p' = \frac{\partial\mathcal{L}}{\partial\dot{q}'}$ $\mathcal{L} = \frac{1}{2}m\dot{q}^2 = \frac{1}{2}\dot{q}'^2$ $p' = \dot{q}'$

$q_1' = \frac{1}{\sqrt{2}}(\xi'+\eta')$, $q_2' = \frac{1}{\sqrt{2}}(\xi'-\eta')$, $q_1'^2+q_2'^2 = \xi'^2+\eta'^2$, $q_1'q_2' = \frac{1}{2}(\xi'^2-\eta'^2)$ $\dot{q}_1' = \frac{1}{\sqrt{2}}(\dot{\xi}'+\dot{\eta}')$, $\dot{q}_2' = \frac{1}{\sqrt{2}}(\dot{\xi}'-\dot{\eta}')$

$$\mathcal{E} = \frac{1}{2}(\dot{\xi}'^2+\dot{\eta}'^2) + \frac{1}{2}\left[(2\pi\nu)^2+\frac{\lambda}{\sqrt{m_1m_2}}\right]\xi'^2 + \frac{1}{2}\left[(2\pi\nu)^2-\frac{\lambda}{\sqrt{m_1m_2}}\right]\eta'^2$$

$p = \frac{l}{2h} = \frac{l}{4\pi\hbar}$ $p\frac{\hbar}{l} = \frac{1}{4\pi} = 0.07958 \cong 0.08$ $\frac{2\pi l}{\frac{h}{2\pi}}\frac{\pi}{\pi^2\pi} = \frac{8l}{h\pi^2} = \frac{8l}{\hbar 2\pi^3} = \frac{4}{\pi^3}\frac{l}{\hbar}$, $p\frac{\hbar}{l} = \frac{4}{\pi^3} = \frac{4}{31} = 0.129$

$p^2\frac{l^2}{\hbar^2}$ $p = 3\frac{\pi\hbar}{l}$ $p^2\frac{l^2}{\hbar^2} = 9\frac{\pi^2\hbar^2}{l^2}\frac{l^2}{\hbar^2} = 9\pi^2$, $n=1$ $P = \frac{2\pi l}{\hbar}\frac{1+\cos p\frac{l}{\hbar}}{\left(\pi^2-p^2\frac{l^2}{\hbar^2}\right)^2}$ $p = z\frac{\hbar}{l}\pi$, $p\frac{l}{\hbar} = z\pi$

$P = \frac{2\pi l}{\hbar}\frac{1+\cos z\pi}{\pi^4(1-z^2)^2}$ $z = 1+\varepsilon$, $z^2 = 1+2\varepsilon$, $1-z^2 = -2\varepsilon$, $(1-z^2)^2 = 4\varepsilon^2$ $\cos(\pi+\varepsilon\pi) = -1 + \frac{1}{2}\varepsilon^2\pi^2$ $\frac{\frac{1}{2}\varepsilon^2\pi^2}{\pi^4 4\varepsilon^2} = \frac{1}{2}\frac{1}{4\pi^2}$