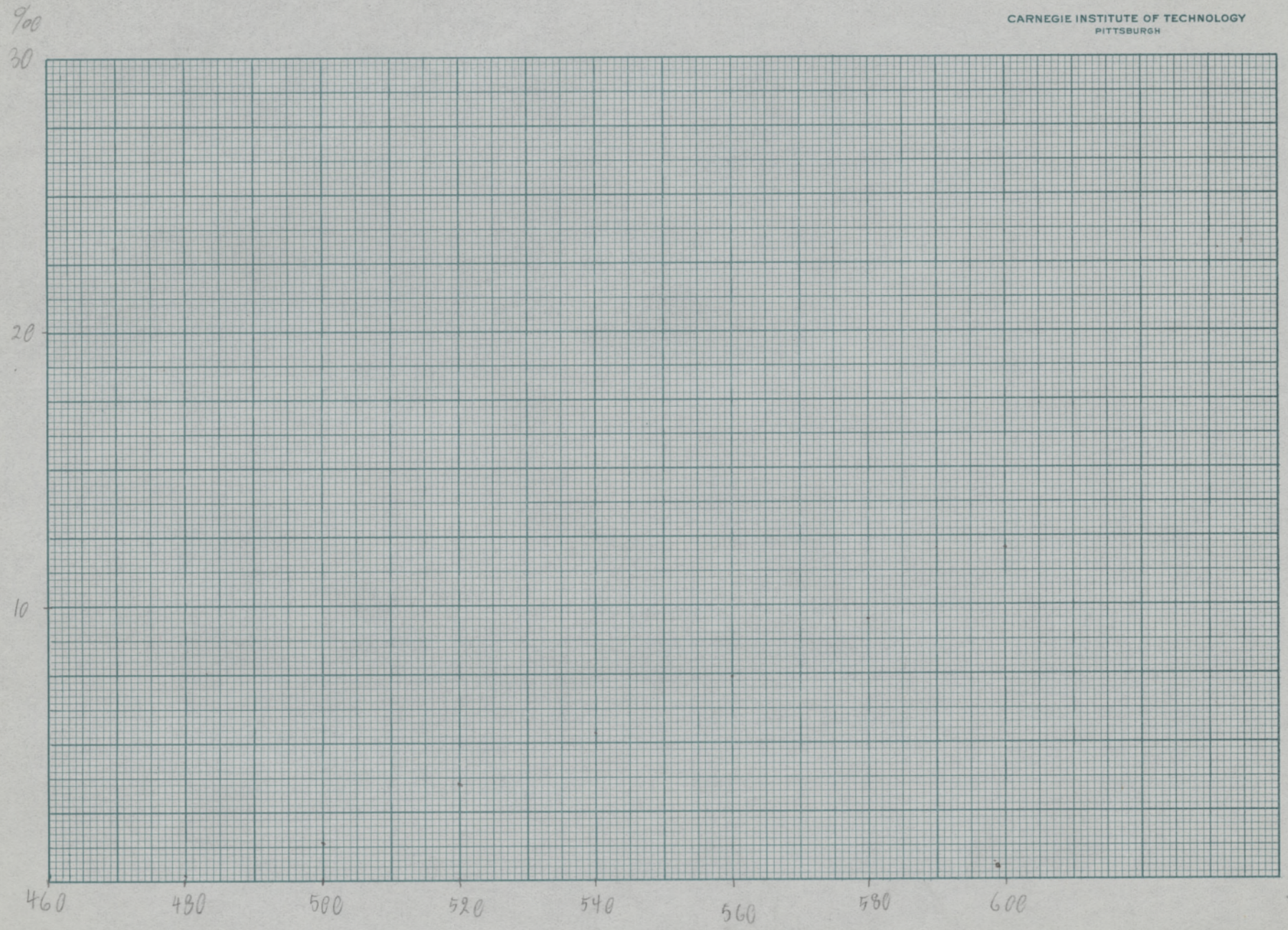
CARNEGIE INSTITUTE OF TECHNOLOGY
PITTSBURGH



LOG OF EXPERIMENT No. _____

Observers _____ Date Dec 21, 1942

Carnegie Institute of Technology Magnetic Deflection S = 2.0 mm/100

Time	Pos.	I.	Cur.	Scatt	MV	Sens	$\frac{I_{corr}}{I_0}$ for scatt
							$I_0 = \frac{460}{0.31} \times 2.8 = 4160$
957	76	402	0		8.40	0.4	
	87.5		~~460~~				
		11	0	11		1	
1030		11½	460	"			½ 0.1 ‰
		15½	480	"			4½ 1.1
		26½	500	"			15½ 3.7₂
		38½	520	"			27½ 6.60
		55	540	"			44 10.6
	→	74	560	"			63 15.2
		88½	580	"			77½ 18.6
		105(?)	600	"			94 22.6
1057					8.57		
							$I_0 = \frac{490 \times 2.8}{3.1} = 4430$
		89	580	12			77 17.4 ‰
	→	75	560	"			63 14.2
		58	540				46 10.4
		41	520				29 6.54
		28½	500				16½ 3.7₃
		15	480				3 0.68
		12?	460				0 0

$I_0 = 480\ A$

$\frac{\Delta I}{I_0} = \frac{20}{480} = \frac{1}{24}$ $\frac{1}{\beta} = \frac{8}{24} = \frac{1}{3} = 0{,}333$ $\frac{\gamma}{4_0} = 2{,}3\,‰$

$\frac{40}{480} = \frac{2}{24}$	$\frac{2}{3} = 0{,}667$	5,1‰
$\frac{3}{24}$	1,000	8.2‰
$\frac{4}{24}$	1.333	12,3‰
$\frac{5}{24}$	1,667	16,2‰
$\frac{6}{24}$	2,00	19.0‰

x 6.0 5.4 ~~4.8~~ 4.2 ~~3.6~~ 3.0 2.7 ~~2.4~~ 2.1 ~~1.8~~ 1.5 ~~1.2~~ 0.9 ~~0.6~~ 0.3

~~1.2~~ ~~0.8~~ ~~0.4~~ 1.0 1.4 0.2

	0.2	0.00135
	0.3	0.01040
/	0.4	0.03283
/	0.6	0.11332
/	0.8	0.22920
	0.9	0.29627
	1.0	0.36788
/	1.2	0.52152
	1.4	0.68536
	1.5	0.77012
/	1.6	0.85642
/	1.8	1.03275
	2.0	1.21306
	2.1	1.30442
/	2.4	1.58218
	2.7	1.86430
	3.0	2.14959
/	3.6	2.72689
	4.2	3.31015
/	4.8	3.89431
	5.4	4.48413
	6.0	5.07888

1.03275
−0.11332
0.91943
−0.52152
0.39791

0.52152
0.22920
0.29232
−0.03283
0.25949

$T = 453°$ $1‰ = 43$ / $1‰ = 4.0$ $\quad T = 440°$ $1‰ = 2,1$

500 A	3,6	4.0	3.3	3,6	5,0	4,2	4.65	4,6
540 "	11,2	12,0	10,2	11.1	13,1	11,7	11.9	12,2
580 "	17,7	18,05	18,15	18.0	20,2	19,3	18,7	19,4

$$\delta_\alpha = \delta_{\alpha_0} \frac{\Delta I}{I_0} \quad \beta = \frac{\delta_\alpha}{\delta_{\alpha_0}} = \frac{b}{\delta_{\alpha_0}} \frac{1}{\frac{\Delta I}{I_0}} \quad \frac{1}{\beta} = \frac{\delta_{\alpha_0}}{b} \frac{\Delta I}{I_0}$$

$I_0 = 460$ A $\quad T = 440°$, $\frac{\delta_\alpha}{\delta_{\alpha_0}} = 0.121$ $\frac{\delta_{\alpha_0}}{b} = 8.27$ $\quad T = 453°$ $\frac{\delta_\alpha}{\delta_{\alpha_0}} = 0.125$ $\frac{\delta_{\alpha_0}}{b} = 8$ $I_0 = 465$ A

I	$\frac{\Delta I}{I_0}$	$\frac{1}{\beta}$	$\frac{\gamma}{\gamma_0}$
500	$\frac{40}{460} = 0.087$	0,114	5.8
540	$\frac{80}{460} = 0.174$	1,44	13,9
580	$\frac{120}{460} = 0.261$	2.16	20.4

$I_0 = 465$ A

500	$\frac{35}{465} = 0.0753$	0,622	4,9	0.94
540	$\frac{75}{465} = 0,161$	1.33	12.7	0.96
580	$\frac{115}{465} = 0.247$	2.04	19,45	1.00

I	$\frac{\Delta I}{I_0}$	$\frac{1}{\beta}$	$\frac{\gamma}{\gamma_0}$	
500	0.0753	0.602	4,8	0,45
540	0.161	1.289	12,2	0.91
580	0.247	1.98	19.0	0,95

1)

$$S = \frac{E}{T} + k \ln \int + C$$

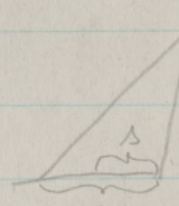
$$P = E - TS = E - E - kT \ln \int - CT = -kT \ln \int - CT$$

$$P_1 - P_2 = -kT_1 \ln \int_{T_1} + kT_2 \ln \int_{T_2} - C(T_1 - T_2)$$

$$C = k \ln h \qquad P_1 - P_2 = -kT_1 \ln \frac{\int_{T_1}}{h} + kT_2 \ln \frac{\int_{T_2}}{h}$$

$a e^{-\frac{b}{x}}$	x	$a=1$ $b=3$	$a=2$ $b=6$	$a=2$ $b=4.2$	$a=2$ $b=4$	$a=2$ $b=4.5$	$a=2$ $b=5$
	1	e^{-3} 0.05	$2e^{-6}$ 0.005	$2e^{-4.2}$ 0.03	$2e^{-4} = 0.037$	$2e^{-4.5}$ 0.022	$2e^{-5} = 0.0135$
	2	$e^{-1.5}$ 0.22	$2e^{-3}$ 0.10	$2e^{-2.1}$ 0.245	$2e^{-2} = 0.27$	$2e^{-2.25}$ 0.21	$2e^{-2.5} = 0.164$
	3	e^{-1} 0.36	$2e^{-2}$ 0.27	$2e^{-1.4}$ 0.49	$2e^{-1\frac{1}{3}} = 0.53$	$2e^{-1.5}$ 0.45	$2e^{-1\frac{2}{3}} = 0.396$

$$\gamma = \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} \gamma_0 e^{-x} x \left(1 - a e^{-bx}\right) dx, \quad x = \frac{\Delta\alpha}{\delta}, \quad \gamma_0 = \gamma_0^0 \left(1 - \frac{\delta}{\lambda_0}\right)$$



$$\delta = \frac{\Delta\alpha}{x}, \quad \gamma_0 = \gamma_0^0 \left(1 - \frac{\Delta\alpha}{\lambda_0} \frac{1}{x}\right)$$

$$\gamma = \gamma_0^0 \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-x} x \left(1 - a e^{-bx}\right) dx - \gamma_0^0 \frac{\Delta\alpha}{\lambda_0} \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-x} \left(1 - a e^{-bx}\right) dx$$

$$\frac{\gamma}{\gamma_0^0} = \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-x} x \, dx - \frac{\Delta\alpha}{\lambda_0} \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-x} dx - a \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-(1+b)x} x \, dx + a \frac{\Delta\alpha}{\lambda_0} \int_{\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-(1+b)x} dx \qquad (1+b)x = y$$

$$\int_{x_0}^{\infty} e^{-x} dx = -e^{-x} \Big|_{x_0}^{\infty} = e^{-x_0}$$

$$\int_{x_0}^{\infty} e^{-x} x \, dx = \int_{\infty}^{x_0} x \, de^{-x} = x e^{-x} \Big|_{\infty}^{x_0} - \int_{\infty}^{x_0} e^{-x} dx = x e^{-x} + e^{-x} \Big|_{\infty}^{x_0} = (1 + x_0) e^{-x_0}$$

$$\frac{\gamma}{\gamma_0} = \left(1 + \frac{\Delta\alpha}{\lambda_0}\right) e^{-\frac{\Delta\alpha}{\lambda_0}} - \frac{\Delta\alpha}{\lambda_0} e^{-\frac{\Delta\alpha}{\lambda_0}} - \frac{a}{(1+b)^2} \int_{(1+b)\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-y} y \, dy + \frac{a}{1+b} \frac{\Delta\alpha}{\lambda_0} \int_{(1+b)\frac{\Delta\alpha}{\lambda_0}}^{\infty} e^{-y} dy = e^{-\frac{\Delta\alpha}{\lambda_0}} - \frac{a}{(1+b)^2} \left[1 + (1+b)\frac{\Delta\alpha}{\lambda_0}\right] e^{-(1+b)\frac{\Delta\alpha}{\lambda_0}} + \frac{a}{1+b} \frac{\Delta\alpha}{\lambda_0} e^{-(1+b)\frac{\Delta\alpha}{\lambda_0}}$$

$$\frac{\gamma}{\gamma_0} = e^{-\frac{\Delta\alpha}{\lambda_0}} - \frac{a}{(1+b)^2} e^{-(1+b)\frac{\Delta\alpha}{\lambda_0}} = e^{-\frac{\Delta\alpha}{\lambda_0}} \left[1 - \frac{a}{(1+b)^2} e^{-b\frac{\Delta\alpha}{\lambda_0}}\right]$$

2)

$\frac{\Delta I}{I_0} = \frac{1}{16\beta}\left[f((3+n)\beta) - f((2+n)\beta) - f((1+n)\beta) + f(n\beta)\right]$ $f(x) = x e^{-\frac{1}{x}}\left[1 - \frac{\alpha}{(1+\alpha)^2} e^{-\frac{b-1}{x}}\right] = f_0(x)\left[1 - \varepsilon(x)\right]$

$s = 3b + nb$, $nb = \sigma$, $\frac{b}{\beta} = \beta$, $s_\beta = \frac{b}{\beta} = s_{\alpha_0}\frac{\Delta I}{I_0}$, $\frac{\Delta I}{I_0} \cdot \frac{s_\alpha}{s_{\alpha_0}} = \frac{b}{s_{\alpha_0}\beta}$

$\underline{b = 2.15}$

$\beta = 1.2$, $n = 1$ $\sigma = 2.15$ $s_\alpha = \frac{b}{\beta} = \frac{2.15}{1.2} = 1.79$, $s_{\alpha_0} = 17.2$, $\frac{\Delta I}{I_0} = \frac{2.15}{17.2 \cdot 1.2} = 0.104$ $\frac{1}{16,\beta} = \frac{1}{19,2}$

$I_0 = 440\,A$, $\Delta I = 0.104 \times 440 = 49$ $I = 519\,A$, $a = 2$ $b = 4.2$ $\frac{a}{(1+a)^2} = \frac{2}{27.04} = \frac{1}{13.52} = 0.07365$

$19.2\ \frac{\Delta I}{I_0} = f_0(4.8) - 0.07365 \times f_0(4.8)\, e^{-\frac{4.2}{4.8}} = 3.89731$

$- f_0(3.6) - 0.07365 \times - f_0(3.6)\, e^{-\frac{4.2}{3.6}} = -2.72689$

$- f_0(2.4) - 0.07365 \times - f_0(2.4)\, e^{-\frac{4.2}{2.4}} = -1.58218$

$+ f_0(1.2) - 0.07365 \times f_0(1.2)\, e^{-\frac{4.2}{1.2}} = 0.52152$

$1.58218 + 2.72689 = 4.30907$

$3.89731 + 0.52152 = 4.41883$

$4.41883 - 4.30907 = 0.10976 = 5.72‰$

$0.0382 / 10.0417 = 3.74‰$

$\frac{4.2}{4.8} = \frac{7}{8} = 0.875$, $e^{-0.875} = 0.41684 \times 3.89731 = 1.625$

$\frac{4.2}{3.6} = \frac{7}{6} = 1.16\frac{3}{3}$ e^{-} $= 0.31141 \times 2.72689 = 0.849$

$\frac{4.2}{2.4} = \frac{7}{4} = 1.75$ $= 0.11377 \times 1.58218 = 0.275$

$\frac{4.2}{1.2} = \frac{7}{2} = 3.5$ $= 0.03020 \times 0.52152 = 0.016$

$1.641 - 1.124 = 0.517 \times 0.07365 = 0.0382$

$a = 2$ $b = 5$ $\frac{a}{(1+a)^2} = \frac{2}{36} = \frac{1}{18} = 0.0556$

$\frac{5}{4.8} = 1.0417$ $e^{-1.0417} = 0.35285 \times 3.8973 = 1.375$

$\frac{5}{3.6} = 1.3889$ $e^{-1.3889} = 0.24935 \times 2.7269 = 0.679$

$\frac{5}{2.4} = 2.0833$ $e^{-2.0833} = 0.12452 \times 1.5822 = \frac{0.197}{0.876}$

$\frac{5}{1.2} = 4.1667$ $e^{-4.1667} = 0.01550 \times 0.5215 = 0.008$

$1.383 - 0.876 = 0.507 \times 0.0556 = \frac{0.1097}{0.0282} = 4.25‰$

0.08156

3)

$\beta = 0.6 \quad n = 1 \quad \sigma = 2.15 \quad s_\alpha = \frac{b}{\beta} = s_{\alpha_0} \frac{\Delta I}{I_0}, \quad \frac{\Delta I}{I_0} = \frac{b}{s_{\alpha_0} \beta} = \frac{2.15}{17.2 \cdot 0.6} = \frac{1}{8 \cdot 0.6} = \frac{1}{4.8} = 0.2083\frac{1}{3}$

$I_0 = 440 \quad \Delta I = \pm \frac{94}{4} = 98 \quad I = 568 \quad \frac{1}{16\beta} = \frac{1}{96}$

$9.6 \frac{y}{y_0} = f_0(2.4) = 1.58218$

$f_0(1.8) = 1.03244$

$f_0(1.2) = 0.52152$

1.55429

$f_0(0.6) = 0.11333$

1.58218

1.69551

-1.55429

$0.14122 : 9.6 = 14.72\,‰$

$\frac{13.87}{14.72} = 0.941$

$a = 2 \quad b = 4.8 \quad \frac{a}{(1+b)^2} = \frac{2}{5.8^2} = \frac{2}{33.64} = \frac{1}{16.82} = 0.05945$

$\frac{b}{x} = \frac{4.8}{2.4} = 2 \quad e^{-\frac{b}{x}} = e^{-2} = 0.135^{798}34 \times 1.582 = 0.2141$

$\frac{4.8}{1.8} = 2.66\frac{2}{3} \qquad 0.0694^{232}8 \times 1.033 = 0.0418$

$\frac{4.8}{1.2} = 4 \qquad 0.01832 \times 0.522 = 0.0096$

0.0814

$\frac{4.8}{0.6} = 8 \qquad 0.00034 \times 0.113 = 0.0004$

0.2145

-0.0814

$0.1331 \times 0.05945 = 0.14122 - 0.00791$

$0.13331 : 9.6 = 13.87\,‰$

4)

x	1	2	3
$2e^{-\frac{4.8}{x}}$	$2e^{-4.8}$	$2e^{-2.4}$	$2e^{-1.6}$
	0.016	0.18	0.40 .

$a=2 \quad b=4.8 \quad \frac{a}{(1+B)^2} = \frac{2}{5.8^2} = 0.05945$

$e^{-\frac{1}{x}} \cdot (1 - a e^{-\frac{b}{x}})$

$\beta = 1.2 \quad n=1 \quad \sigma = 2.15 \quad \frac{\Delta I}{I_0} = \frac{b}{I_{x_0}\beta} = \frac{1}{8 \cdot 1.2} = \frac{1}{9.6} = 0.10417$

$I_0 = 470 \quad \Delta I = \frac{47}{1.96} = 49 \quad I = 519 \text{ A} \quad \frac{1}{16\beta} = \frac{1}{19.2}$

$19.2 \frac{\gamma}{\gamma_0} = f_0(4.8) = 3.89431$

$f_0(3.6) = 2.72689$

$f_0(2.4) = 1.58218 - 4.30907$

$f_0(1.2) = 0.52152 \quad 4.41883 - 4.30907 = 0.10976 : 19.2 = 5.72‰$

$\frac{4.11}{5.72} = 0.719$

$a=2 \quad b=4.8$

$\frac{4.8}{4.8} = 1$	$e^{-1} = 0.36788 \times 3.8943$	$1.47152 - 0.03778 = 1.43374$	1.43374
$\frac{4.8}{3.6} = 1.33\frac{1}{3}$	0.26360×2.7269	$0.65900 - 0.05272 - 0.20301 - 0.00916 = 0.39411$	0.71881
$\frac{4.8}{2.4} = 2$	0.13534×1.5822		0.20412
$\frac{4.8}{1.2} = 4$	0.01832×0.5215		0.00955
			1.44329
			$-0.92293 = 0.52036 \times 0.05945 \cdot 0.0031$

$\frac{0.10976}{0.03093} = 0.00783 : 19.2 = 4.11‰$

5.4 1 (2.15

6.8 init

7.2 (2.0)

$\frac{\gamma}{\gamma_0} = f((3+n)\beta) - f((2+n)\beta) - f((1+n)\beta) + f(n\beta)$

4 3 2 1

5)

$$\frac{I}{I_0} = \frac{2b}{s_\alpha} e^{-\frac{s_\alpha}{s}} \left(\frac{s_\alpha}{s}\right)^3$$

$s = 2s_{\alpha_0}$ $\frac{I}{I_0} = \frac{2b}{s_{\alpha_0}} e^{-0.5}\frac{1}{8} = \frac{2b}{s_{\alpha_0}}\frac{0.60653}{8} = \frac{2b}{s_{\alpha_0}} 0.07582$ $\frac{b}{s_{\alpha_0}} = 0.125$ $\frac{I}{I_0} = \frac{0.07582}{4} = 0.01895 = 1.90\%$

$0.002369 \cdot 8$

$$I = I_0, \quad s = 2s_{\alpha_0}$$

$s_\alpha = \frac{1}{4} s_{\alpha_0}$ $\frac{I}{I_0} = \frac{1}{8} \frac{2b}{s_{\alpha_0}} e^{-0.5\frac{1}{4}} \frac{1}{4^3}\frac{1}{8} = \frac{b}{s_{\alpha_0}} e^{-0.125}\frac{1}{648} = \frac{0.88251}{642}$ $\frac{0.88251 : 4096}{8192\ \ 6331\ \ 4096\ \ 2235} = 0.00021545 = 0.21545\%$

$s_\alpha = \frac{n}{4} s_{\alpha_0}$ $\frac{I}{I_0} = \frac{1}{8}\frac{2b}{s_{\alpha_0}}\frac{4}{n} e^{-\frac{n}{4}0.5}\frac{1}{8}\frac{n^3}{4^3} = \frac{1}{8}\frac{1}{n} e^{-\frac{n}{8}}\frac{1}{8}\frac{n^3}{4^3} = \frac{n^2}{4096} e^{-\frac{n}{8}}$

$n = 2$ $\frac{I}{I_0} = \frac{1}{1024} e^{0.25} = \frac{0.777880 : 1024}{\frac{41}{6}\ \frac{68}{200}\ \frac{124}{56}} = 0.00076055$ $= 0.76055\%$

$n = 3$ $\frac{I}{I_0} = \frac{9}{4096} e^{-0.375} = \frac{9 \times 0.68730}{4096} = \frac{6.18570 : 4096}{\frac{6873}{40\ 97}\ \frac{20480}{4177}}$ $= 0.0015010$ $= 1.50102\%$

$n = 4$ $\frac{I}{I_0} = \frac{e^{-0.5}}{256} = \frac{0.60653 : 256}{\frac{5\ 13}{375}\ \frac{368}{1773}\ \frac{1536}{237}}$ $= 0.00236927$ $= 2.36927\%$

$n = 5$ $\frac{I}{I_0} = \frac{e^{-0.625} \times 25}{4096} = \frac{0.53526 \times 25}{4096} = \frac{13.3815 : 4096}{\frac{13288}{1.0935}\ \frac{8192}{2743}\ \frac{24576}{3685}\ \frac{3686}{39}}$ $= 0.0032669$ $= 3.26696\%$

$n = 6$ $\frac{I}{I_0} = \frac{e^{-0.75} \times 9}{1024} = \frac{0.47237 \times 9}{1024} = \frac{4.2513 : 1024}{\frac{4096}{1553}\ \frac{1024}{529}\ \frac{513}{17}}$ $= 0.0041516$ $= 4.15166\%$

$n = 7$ $\frac{I}{I_0} = \frac{e^{-0.875} \times 49}{4096} = \frac{0.41686 \times 49}{4096} = \frac{20.42614 : 4096}{\frac{16384}{4042}\ \frac{36864}{5577}\ \frac{4\ 3768}{2806}}$ $= 0.0049868$ $= 4.98686\%$

$n = 8$ $\frac{I}{I_0} = \frac{1}{2}\frac{e^{-1} \times 64}{4096} = \frac{0.36788}{128} = \frac{0.36788 : 128}{\frac{256}{1118}\ \frac{1024}{0948}\ \frac{896}{52}}$ $= 0.0028740$ $= \frac{2.87406\%}{2.01583\%}$

$$\frac{(n+1)^2}{4096} e^{-\frac{n}{8}-\frac{1}{8}} + \frac{(n-1)^2}{4096} e^{-\frac{n}{8}+\frac{1}{8}} = \frac{e^{-\frac{n}{8}}}{4096}\left[(n+1)^2 e^{-\frac{1}{8}} + (n-1)^2 e^{+\frac{1}{8}}\right] = \frac{e^{-0.5}}{256}\frac{[\ ]}{16}$$

$$e^{-\frac{1}{8}} = 1 - \frac{1}{8} + \frac{1}{2}\frac{1}{8^2} - \frac{1}{6}\frac{1}{8^3} + \frac{1}{24}\frac{1}{8^4} \qquad e^{+\frac{1}{8}} = 1 + \frac{1}{8} + \frac{1}{2}\frac{1}{8^2} + \frac{1}{6}\frac{1}{8^3} + \frac{1}{24}\frac{1}{8^4}$$

$$[\ ] = n^2\left(e^{\frac{1}{8}} + e^{+\frac{1}{8}}\right) + 2n\left(e^{-\frac{1}{8}} - e^{+\frac{1}{8}}\right) + \left(e^{-\frac{1}{8}} + e^{\frac{1}{8}}\right) = (n^2+1)\left(e^{-\frac{1}{8}} + e^{+\frac{1}{8}}\right) - 2n\left(e^{+\frac{1}{8}} - e^{-\frac{1}{8}}\right)$$

$$= (n^2+1)\left(2 + \frac{1}{8^2} + \frac{1}{12}\frac{1}{8^4}\right) - 2n\left(\frac{1}{4} + \frac{1}{3}\frac{1}{8^3}\right) \qquad n^2 e^{+\frac{1}{8}}\left[\left(1+\frac{1}{n^2}\right)\left(1+e^{-\frac{1}{4}}\right) - \frac{2}{n}\left(1-e^{-\frac{1}{4}}\right)\right]$$

$$= 17 \times \left(2.015625 + \frac{1}{12}\underset{0.0000203}{0.000244}1\right) - 8 \times 0.25 + \underset{0.000651}{\frac{1}{1536}}$$

$$\frac{[\ ]}{16} = 1.0625 \times 2.015645 - \frac{1}{2} \times 0.250651$$

0.125000
0.000625
3125
400
2.141623
− 0.123326
2.016630 × 2.36927 = 4.738554
0.02369
0.01260
0.00232
4.77415

$$\frac{477498}{473854}\cdot\left(1 + \frac{0.02944}{4.73854}\right)\times 2 = 2 + \frac{0.05888}{4.73854} = 2.0124$$

$$e^{+\frac{1}{8}} = e^{0.123} = 1.1331 \qquad [\ ] = \left(1+\frac{1}{16}\right)\left(1.77880 - \frac{1}{2}\,0.22120\right)$$

0.111115
− 0.11060
1.67820
0.17880
0.03300
23100
2.01513 × 2.36927 = 4.77455
291

4.73867
0.02369
1185
4.77455

$$\frac{(n+2)^2}{4096} e^{-\frac{n}{8}-\frac{1}{4}} + \frac{(n-2)^2}{4096} e^{-\frac{n}{8}+\frac{1}{4}} = \frac{e^{-\frac{n}{8}}}{4096}\left[(n+2)^2 e^{-\frac{1}{4}} + (n-2)^2 e^{+\frac{1}{4}}\right] = \frac{e^{-\frac{n}{8}}}{4096}\left[(n^2+4)\left(e^{+\frac{1}{4}} + e^{-\frac{1}{4}}\right) - 4n\left(e^{+\frac{1}{4}} - e^{-\frac{1}{4}}\right)\right]$$

$$= \frac{n^2 e^{-\frac{n}{8}}}{4096}\left[\left(1+\frac{4}{n^2}\right)\left(1+e^{-0.5}\right) - \frac{4}{n}\left(1-e^{-0.5}\right)\right]e^{0.25} = \frac{e^{-0.5}}{256}\left[\left(1+\frac{1}{4}\right)(1+0.60653) - 1 + 0.60653\right] 1.2840$$

$$\left[0.25 + 0.60653 \times 2\frac{1}{4}\right] \times 1.2840$$

1.21306
0.15163
1.61469 × 1.2840
0.32294
1.2918
6.46
2.07324
1.0366

0.46053
4.15166
4.91221
4.73854 = 1 + 0.17367
4.73854 = 1.0366

$$\frac{(n+3)^2}{4096}e^{-\frac{n}{8}-\frac{3}{8}}+\frac{(n-3)^2}{4096}e^{-\frac{n}{8}+\frac{3}{8}}=\frac{e^{-0.5}}{4096}\left[(n+3)^2e^{-\frac{3}{8}}+(n-3)^2e^{+\frac{3}{8}}\right]=\frac{e^{-0.5}}{256}\left[(1+\tfrac{3}{4})^2e^{-\frac{3}{4}}+(1-\tfrac{3}{4})^2\right]e^{0.375}$$

$4+\frac{1}{2}-1$ over $(2-\frac{1}{2})^2$

$$=\frac{e^{-0.5}}{256}\left[\left(3+\tfrac{1}{16}\right)\times 0.472237+\tfrac{1}{16}\right]1.4550$$

1.41711
0.02952
0.06256
1.50913 × 1.455
0.60365 2
0.67525 6
0.00752 56
2.19556
1.09778

4.99686
0.21345
5.20231

$$\frac{5.20231}{4.773854}=1+\frac{0.428377}{4.773854}=1.0978$$

1.00000
2.01513
2.04329
2.19556
7.28396 × 2.369217
14.56792
2.18519
0.43704
0.06556
146
51
17.25768
2.87406
20.13174 = 2.013%

10)

$\frac{b}{s_x} = 1.5$ $b = 1.5 \times 1.8 = 2.7$ $\frac{s}{s_x} = \frac{3b+b}{s_x} = 4.5 + 1.2 = 5.7$ $b = 1.2 \times 1.8 = 2.16$ $\frac{s_x}{b \times 16} = \frac{1}{27}$

$\frac{b}{s_x} = 1.2$ $b = 1.2 \times 1.8 = 2.16$ $\frac{s}{s_x} = 3.6 + 1.2 = 4.8$ $\frac{s_x}{b \times 16} = \frac{1}{1.2 \times 16} = \frac{1}{19.2}$

	x	$\frac{1}{x}$	$e^{-\frac{1}{x}}$	$3\frac{1}{x}$	$e^{-3\frac{1}{x}}$	$x e^{-\frac{1}{x}}$	$x e^{-\frac{1}{x}} \times \frac{1}{16} e^{-3\frac{1}{x}}$
$\frac{s}{s_x} = 4.8$	0.208333	0.81194 3.24776 649552 3.89731	0.625000	0.53526	3.89731 0.52152 4.41883	0.1301 26 0.1327 −0.1024 0.0303
$\frac{s}{s_x} - \frac{b}{s_x} = 3.6$	0.277778	0.75747 2.27241 45448 2.72689	0.833333	0.43460	2.72689 1.58218 4.30907	0.0740 284 0.1024
$\frac{s}{s_x} - 2\frac{b}{s_x} = 2.4$	0.416667	0.65924 1.31848 26370 1.58218	1.250000	0.28650	1.58218	0.0284
$\frac{s}{s_x} - 3\frac{b}{s_x} = 1.2$	0.833333	0.43460 8692 0.52152	2.500000	0.08208	0.52152	0.0026

$s_x = 3.6$ $\frac{b}{s_x} = \frac{2.16}{3.6} = 0.6$ $\frac{s}{s_x} = 2.4$ $\frac{s_x}{b} = \frac{1}{0.6 \times 16} = \frac{1}{9.6}$

4.41883
−4.30907
0.10976 : 19.2 = 0.00573 5.73 ‰
−0.0303
0.0795 : 19.2 = 0.00414 4.14 ‰

1.69551
−1.55429
0.14122 : 9.6 = 0.01471 14.71 ‰
−0.01352
0.12770 : 9.6 = 0.01330 13.30 ‰

	x	$\frac{1}{x}$	$e^{-\frac{1}{x}}$	$3\frac{1}{x}$	$e^{-3\frac{1}{x}}$	$x e^{-\frac{1}{x}}$	$x e^{-\frac{1}{x}} \times \frac{1}{16} e^{-3\frac{1}{x}}$
$\frac{s}{s_x}$	2.4	0.416667	0.65924 1.31848 26370 1.58218	1.250000	0.28650	1.58218 0.11333 1.69551	0.02834 0.00005 0.02839 −0.01487 0.01352
$\frac{s}{s_x} - \frac{b}{s_x}$	1.8	0.555556	0.57376 0.45901 1.03277	1.666667	0.18888	1.03277 0.52152 1.55429	0.01220 0.00267 0.01487
$\frac{s}{s_x} - 2\frac{b}{s_x}$	1.2	0.833333	0.43460 0.08692 0.52152	2.500000	0.08208	0.52152	0.00267
$\frac{s}{s_x} - 3\frac{b}{s_x}$	0.6	1.666667	0.18888 0.11333	5.000000	0.00674	0.11333	0.00005

11)

$S_\alpha = 5.4$ $\frac{b}{S_\alpha} = \frac{2.16}{5.4} = 0.4$ $\frac{S}{S_\alpha} = \frac{3.6+6}{S_\alpha} = \frac{4.6}{S_\alpha} = 1.6$ $\frac{S_\alpha}{6} = \frac{1}{0.4 \times 16} = \frac{1}{6.4}$

	$\times \frac{1}{x}$	$e^{-\frac{1}{x}}$	$\times e^{-\frac{1}{x}}$
$\frac{S}{S_\alpha} = 1.6$	0.625000	0.53526 32116 0.85642	0.85642 0.03283 0.88925
$\frac{S}{S_\alpha} - \frac{b}{S_\alpha} = 1.2$	0.833333	0.43460 0.08692 0.52152	0.52152 0.22920 0.75042
0.8	1.250000	0.28650 22920	0.22920
0.4	2.500000	0.08208 3 28 32	0.03283

0.88925
−0.75042
0.13853 : 64 = 0.02165 2.165%

40.55
32.11
8.44 · 0.09/0.5 = 32.11/19.95 = 1.84

$\beta = 0.4$

$n = 0$

$\frac{\gamma}{\gamma_0} = \frac{1}{6.4}[f(1.2) - f(0.8) - f(0.4)] = \frac{0.25949}{6.4} = 0.040545$ 40.55‰

$n = \frac{1}{2}$ $\frac{1}{0.4} = 2.5$

$\frac{1}{6.4}[f(1.4) - f(1.0) - f(0.6) + f(0.2)] = \frac{0.20551}{6.4} = 0.032110$ 32.11‰

32.110
14.504
46617 : 2
23.304
21.645
44952 : 2 22.48‰

$n = 1$

$\frac{1}{6.4}[f(1.6) - f(1.2) - f(0.8) + f(0.4)] = \frac{0.13853}{6.4} = 0.021645$ 21.65‰

$n = 1\frac{1}{2}$

1.8 1.4 1.0 0.6 $= \frac{0.09283}{6.4} = 0.014504$ 14.50‰

$n = 2$

2.0 1.6 1.2 0.8 $= \frac{0.06432}{6.4} = 0.010050$ 10.05‰

$16\ \frac{\Delta F}{F_0} = \frac{s_\alpha}{6}\left[f\left([3+n]\frac{b}{s_\alpha}\right) - f\left([2+n]\frac{b}{s_\alpha}\right) - f\left([1+n]\frac{b}{s_\alpha}\right) + f\left(n\frac{b}{s_\alpha}\right)\right],\ f(x) = x e^{-x}$

$s = 3b + nb,\ nb = \sigma,\ \frac{b}{s_\alpha} = \beta,\ s_\alpha = \frac{b}{\beta} = s_{\alpha_0}\frac{\Delta I}{I_0},\ \frac{\Delta I}{I_0} = \frac{s_\alpha}{s_{\alpha_0}} = \frac{b}{\beta}\frac{1}{s_{\alpha_0}}$

$\frac{\Delta F}{F_0} = \pm 6\,\frac{1}{\beta}\left[f([3+n]\beta) - f([2+n]\beta) - f([1+n]\beta) + f(n\beta)\right]$

$\beta = 1.2$

$n = 1$

$\frac{\Delta F}{F_0} = \frac{1}{19.2}\left[f(4.8) - f(3.6) - f(2.4) + f(1.2)\right] = \frac{0.10946}{19.2} = 0.0057166$ 5,72 ‰

$n = 0$

$\frac{\Delta F}{F_0} = \frac{1}{19.2}\left[f(3.6) - f(2.4) - f(1.2)\right] = \frac{1}{19.2}\begin{bmatrix}2.72689\\ -1.58218\\ -1.14471\\ -0.52152\\ 0.62319\end{bmatrix} = \frac{0.62319}{19.2} = 0.032454$ 3,246 ‰

$n = \frac{1}{2}$

$\frac{1}{19.2}\left[f(4.2) - f(3.0) - f(1.8) + f(0.6)\right] = \frac{0.24113}{19.2} = 0.012558$ 12,56 ‰

$n = 1\frac{1}{2}$

$\frac{1}{19.2}\left[f(5.4) - f(4.2) - f(3.0) + f(1.8)\right] = \frac{0.06014}{19.2} = 0.003132$ 3,13 ‰

$n = 2$

$\frac{1}{19.2}\left[f(6.0) - f(4.8) - f(3.6) + f(2.4)\right] = \frac{0.03686}{19.2} = 0.001920$ 1,92 ‰

12,558 $\frac{1}{1.2} = 0.833$

3,132 / 15.69 : 2 / 7.845 / 5715 / 18.56 : 2 / 6.78 — 6.78 ‰

$\beta = 0.6$

$n = 0$

41,45 / 25,91 / 15,54 — 6,218 / 0,109 / 0,5 = 3,39 — 25,91 / 29,3

$\frac{\Delta F}{F_0} = \frac{1}{9.6}\left[f(1.8) - f(1.2) - f(0.6)\right] = \frac{0.39791}{9.6} = 0.041448$ 41,45 ‰

$n = \frac{1}{2}$

$f(2.1) - f(1.5) - f(0.9) + f(0.3) = \frac{0.24843}{9.6} = 0.025909$ 25,91 ‰

$\frac{1}{0.6} = 1,667$

25,909 / 8,961 / 34,87 : 2 / 17,435 / 14,711 / 32,146 : 2 / 16,07 ‰

$n = 1$

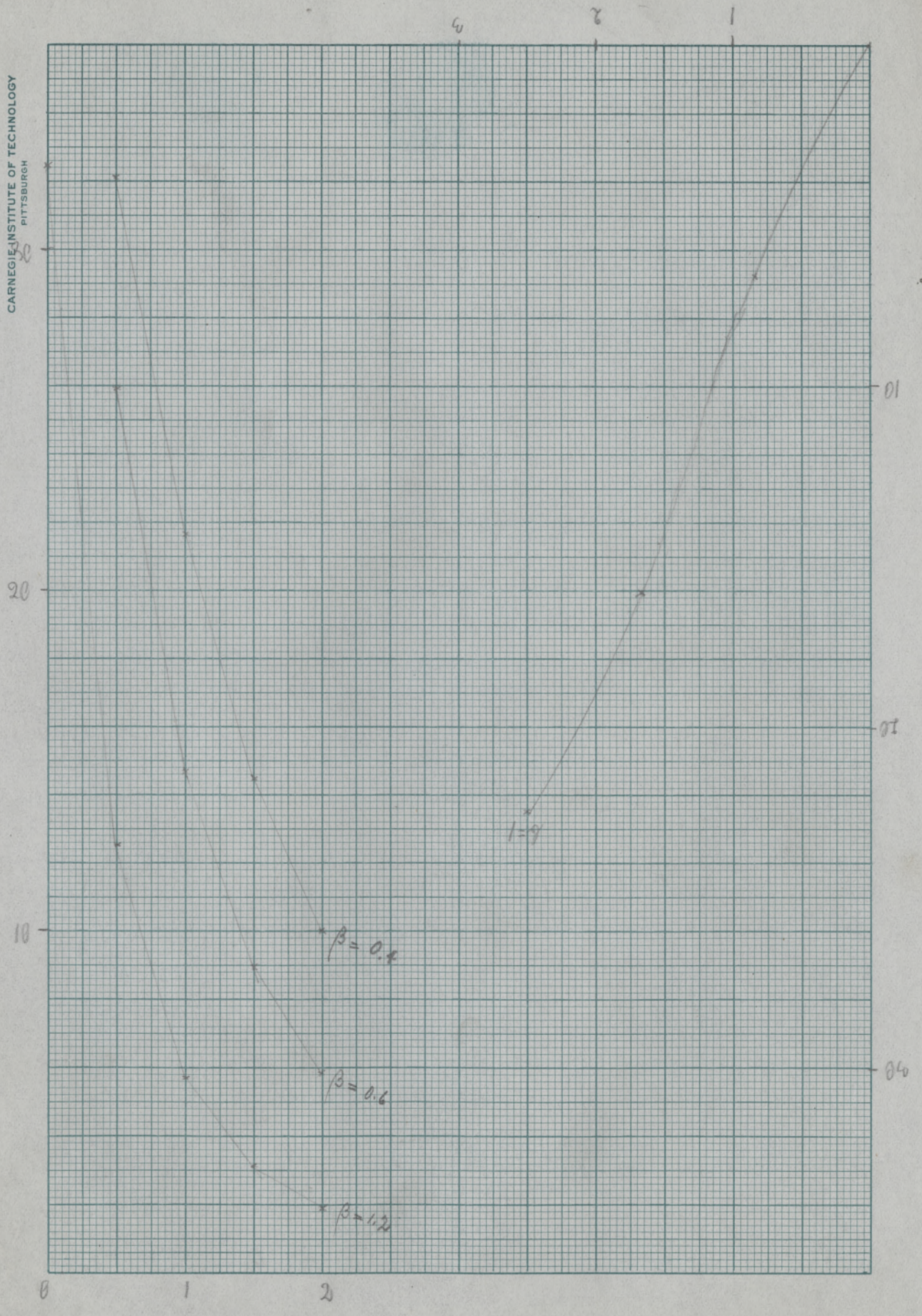
$f(2.4) - f(1.8) - f(1.2) + f(0.6) = \frac{0.14123}{9.6} = 0.014711$ 14,71 ‰

$n = 1\frac{1}{2}$

$f(2.7) - f(2.1) - (1.5) + f(0.9) = \frac{0.08603}{9.6} = 0.008961$ 8,96 ‰

$n = 2$

$f(3.0) - f(2.4) - f(1.8) + f(1.2) = \frac{0.05618}{9.6} = 0.005852$ 5,85 ‰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30
20
10
0
1
2
β = 0.4
β = 0.6
β = 0.2
1-β

$\frac{b}{s_{\alpha_0}} = \frac{2.15}{17.2} = 0.125$ $\frac{1}{\beta} = \frac{\frac{\Delta I}{I_0}}{\frac{b}{s_\alpha}} = 8\frac{\Delta I}{I_0}$

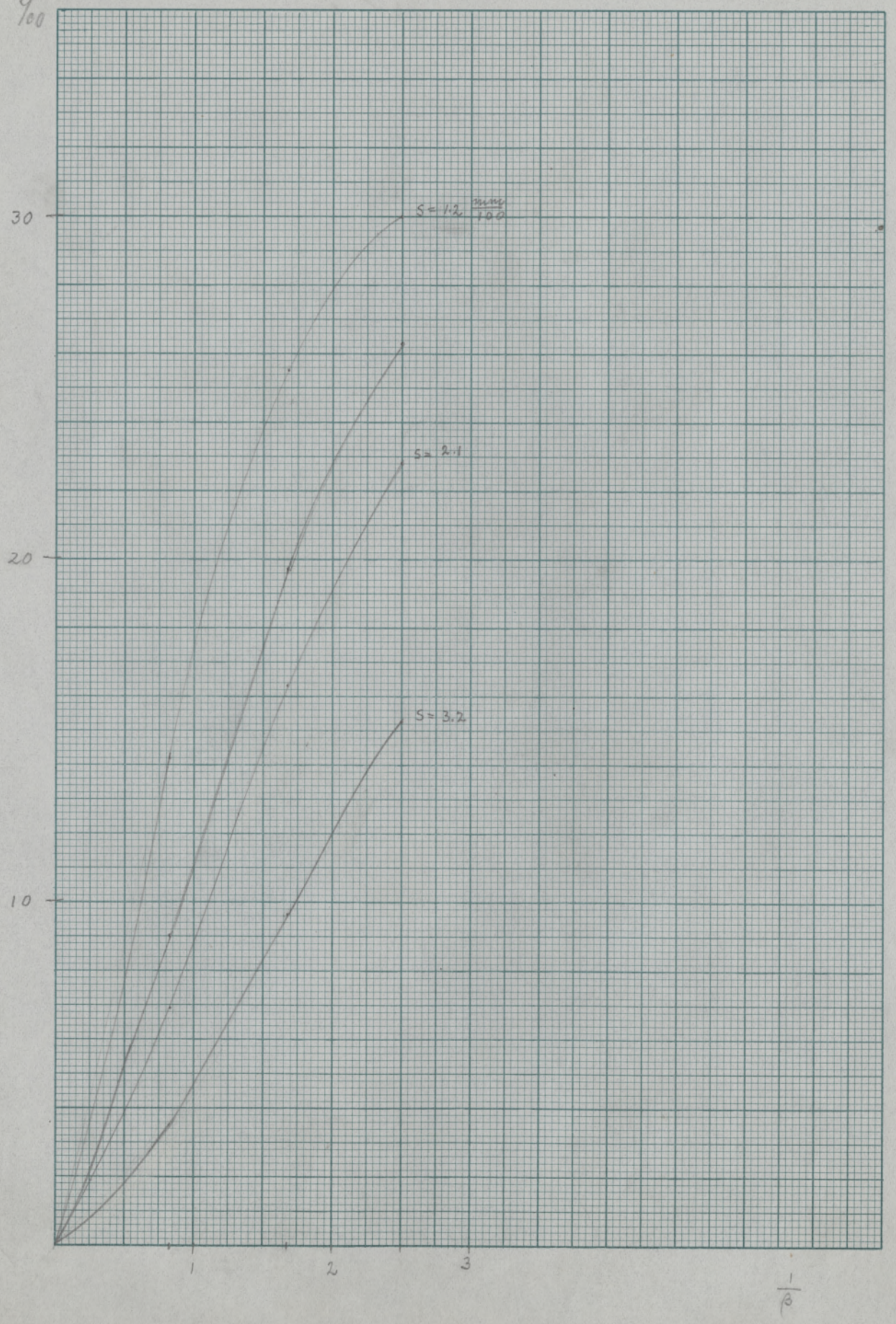
σ distance of centre of wire from edge of beam

$I_0 = 470A$

I	$\sigma = 3.2\frac{mm}{100}$ obs	calc	$\sigma = 2.1\frac{mm}{100}$ obs	calc	$\sigma = 1.2\frac{mm}{100}$ obs	calc		$8\frac{\Delta I}{I_0}$
460A	0	0	0	0	0	0		0
480 "	0.2	0.5	0.8 (0.9)	1.2	2.9	2.4	0.5	$\frac{8}{47} = 0.170_{5}$
500 "	1.4	1.8	3.6 (3.7)	3.9	9.7$_5$	7.9	1.8	0.511
520 "	3.5	3.6	7.1 (6.6)	7.2	15.6	14.7	0.9	$0.85\frac{1}{5}$
540 "	5.3	6.1	11.2 (10.5)	11.0	21.2	20.3	0.9	1.192
560 "	7.4	8.6	14.4 (14.7)	14.8	25.0	24.1	0.9	1.53_{27}
580 "	9.5	11.0	17.7 (18.0)	18.0	28.2	27.0	1.2	1.87_{33}
600 "	12.0$_5$	13.6	20.7	20.7	30.2	29.0	1.2	2.214

$I_0 = 465A$

460	0	0	0	0	0	0		0
480	0.2	0.9	0.8	1.9	2.9	3.7		$\frac{8\times15}{465} = \frac{8}{31} = 0.258$
500	1.4	2.3	3.6	4.8	9.7$_5$	9.6		$\frac{8\times35}{465} = \frac{8\times7}{93} = 0.086_1$
520	3.5	4.3	7.1	8.2	15.6	16.7		$11\times0.0861 = 0.947$
540	5.3	6.8	11.2	12.2	21.2	21.6		$15\times0.0861 = 1.291$
560	7.4	9.4	14.4	16.0	25.0	25.2		$19\times0.0861 = 1.636$
580	9.5	11.9	17.7	19.0	28.2	27.8		$23\times0.0861 = 1.980$
600	12.0$_5$	14.3	20.7	21.6	30.2	29.5		$27\times0.0861 = 2.323$



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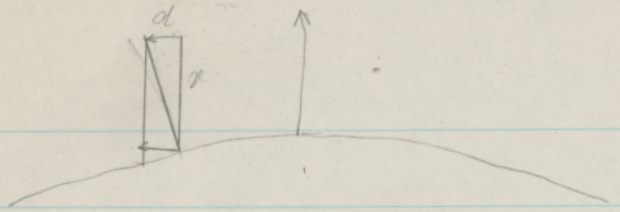


$$F = mg\left(1+\frac{\Delta I}{I_0}\right)\frac{d}{r} \qquad s_x = s_{x_0}\left(1+\frac{\Delta I}{I_0}\right)\frac{d}{r} = 23\frac{d}{r}\ \frac{mm}{100}\ \frac{d}{r} = 0.1 \qquad s_x = 0.023\ mm$$

$s_x < 0.1\ mm$

$\sigma = b$

$I_0 = 480\ A$

$I = 580 \quad \frac{\Delta I}{I_0} = \frac{100}{480} = 0.208 \quad \frac{1}{\beta} = \frac{0.208}{0.1480} = 1.41 \quad \frac{\Delta T}{T_0} = 13.6‰$

$540 \quad \frac{60}{480} = 0.125 \quad \frac{1}{\beta} = \quad = 0.845 \quad 11.2‰$

$I_0 = 440\ A$

$I = 580 \quad \frac{110}{440} \quad = 1.58 \quad 15.4‰$

$540 \quad \frac{40}{440} = 0.1480 \quad \frac{1}{\beta} = \frac{0.1485}{0.1480} \quad = 1.005 \quad 8.8‰$

I_0

Lower beam: $b = 2.56$

$b = 2.56$, $\beta = \frac{b}{b_\alpha}$, $b_\alpha = \frac{b}{\beta} = \frac{2.56}{0.4} = 6.4$ $\frac{\Delta I}{I_0} = \frac{b_\alpha}{b_{\alpha 0}} = \frac{6.4}{17} = 0.344$

$b_\alpha = 4.264$ $= 0.249$

$b_\alpha = 2.133$ $= 0.125$

$\frac{2}{2.56} = 0.782$ $\frac{1}{2.56} = 0.391$ $\frac{3}{2.56} = 1.173$

$\beta = 1.2$

16.40 12.56 5.42 4.7

16.4 | 12.56 (28.96) | 5.42 (18.28) | 4.7 (10.42)

0.390 | 0.500 | 1.800 | 1.140

1.45 | 0.78

$\frac{14.48 \times 0.110 + 9.14 \times 0.5 + 3.27 \times 0.14}{0.782} = \frac{1.593 + 4.57 + 0.886}{0.782} = \frac{7.05}{0.782} = 9.01‰$

$\beta = 0.6$

29.30 25.91 14.71 12.6

0.391 | 0.500 | 1.000 | 1.173

2.5 | 10.00

$\frac{24.6 \times 0.109 + 20.3 \times 0.5 + 13.66 \times 0.173}{0.782} = \frac{3.01 + 10.15 + 2.34}{15.53} = \frac{15.53}{0.782} = 19.7‰$

$\beta = 0.4$

33.95 32.11 21.63 19.01

0.391 | 0.500 | 1.000 | 1.173

2.97 | 14.81

$\frac{33.03 \times 0.109 + 26.88 \times 0.5 + 20.33 \times 0.173}{0.782} = \frac{3.60 + 13.44 + 3.51}{20.55} = \frac{20.55}{0.782} = 26.3‰$

$\frac{\Delta I}{I_0} = \frac{b_\alpha}{b_{\alpha 0}} = \frac{b}{b_{\alpha 0} b} = \frac{b}{b_{\alpha 0}} \frac{1}{\beta}$

Jan 6, 1943: $b_{\alpha 0} = 17.3$ $\frac{b}{b_{\alpha 0}} = 0.1480$ $I_0 = 4401$ $\Delta I = 1101$ $\frac{\Delta I}{I_0} = \frac{11}{47} = 0.234$ $\frac{1}{\beta} = \frac{0.234}{0.1480} = 1.58$ $\frac{Y}{Y_0} = 18.4‰$

$I_0 = 480$ $\Delta I = 100$ $\frac{10}{48} = 0.208$ $\frac{1}{\beta} = \frac{0.208}{0.1480} = 1.41$ $16.2‰$

0.1435 $\frac{1}{\beta} = 1.17$

0.043 0.29

0.119 0.76 $8.1‰$

$I_0 = 440\ A$

$\frac{\Delta I}{I_0} = \frac{10}{440} = \frac{1}{44}$ $\frac{1}{\beta} = \frac{\frac{\Delta I}{I_0}}{\frac{\delta}{\beta_{\alpha_0}}} = \frac{8}{47} = 0{,}170$ $\frac{\gamma}{\gamma_0} = 1{,}1‰$

$\frac{30}{440}$ $= \frac{1.48}{0.510}$ $3{,}4‰$

$\frac{50}{440}$ $= 0{,}85$ $6{,}8‰$

$\frac{70}{440}$ $= 1{,}19$ $10{,}5‰$

$\frac{90}{440}$ $= 1{,}53$ $14{,}6‰$

$\frac{110}{440}$ $= 1{,}87$ $18{,}0‰$

$\frac{130}{440}$ $= 2{,}21$ $20{,}6‰$

$I_0 = 460A$

$\frac{b}{I_{20}} = 0{,}125 \quad \frac{1}{\beta} = \frac{\Delta I}{I_0} / \frac{b}{I_{20}}$

$\frac{\Delta I}{I_0} = \frac{20}{460} = \frac{1}{23} \quad \frac{1}{\beta} = \frac{1}{23 \cdot 0{,}125} = \frac{8}{23} = 0{,}348 \quad \frac{4}{H_0} = 2{,}4‰$

$\frac{40}{460} = \frac{2}{23} \quad \frac{1}{\beta} = \frac{16}{23} = 0{,}69 \quad 5{,}4‰$

$= \frac{3}{23} \quad = 1{,}044 \quad 8{,}5‰$

$= \frac{4}{23} \quad = 1{,}392 \quad 13{,}0‰$

$= \frac{5}{23} \quad = 1{,}74 \quad 16{,}4‰$

$= \frac{6}{23} \quad = 2{,}09 \quad 19{,}4‰$

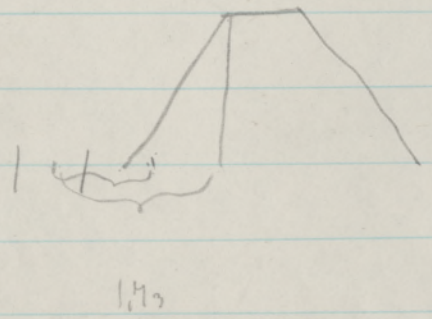
$= \frac{7}{23} \quad = 2{,}44 \quad 22{,}3‰$

$\frac{13}{47} \quad \frac{12}{48} \quad 0{,}25$

$9{,}35 \quad 1{,}91$

$-8{,}88 \quad 1{,}45$

$0{,}47 \quad 0{,}46$



1,73

1,2	0,2	1,2	2,2	0,093	0,56	1,02
2,1						
3,2	1,1	2,1	3,1	0,51	0,97₅	1,44
	2,2	3,2	4,2	1,02	1,49	1,95

2,1

12,3
6,0
3,4

12,3
3,4
15,7 : 2
7,85
6,00
12,85 : 2
6,9

β = 1,2 : 6,9

25,7
15,1
9,5

25,7
9,5
35,2
17,6
1,9
16,3₅

β = 0,6 : 16,3₅

31,9
22,1
15,2

31,9
15,2
47,1
23,5₆
9,1
45,6₂
22,8

β = 0,4

1,2

28,2
11,5
5,6

28,2
5,6
33,8 : 2
16,9
11,5
28,4 : 2
14,2

38,55
24,4
14,4

38,55
14,4
52,95 : 2
26,47
24,4
50,87 : 2
25,44

37,12
30,7
21,4

37,12
21,4
58,52 : 2
29,26
30,70
59,96 : 2
29,98

3,2

5,6
3,1
2,05

5,6
2,05
7,65 : 2
3,83
3,1
6,93 : 2
3,47

14,4
9,0
6,05

14,4
6,05
20,45 : 2
10,23
9,00
19,23 : 2
9,62

21,4
14,6
10,4

21,4
10,4
31,8 : 2
15,9
14,6
30,5 : 2
15,25

41,45
25,91
15,54
31,08
2,90
38,55

40,55
32,11
18,44
36,88
3,43

40,55
3,43
37,12

X	$\frac{1}{X}$	$e^{-\frac{1}{X}}$	$X \cdot e^{-\frac{1}{X}}$
0.2	5.000000	0.00674	0.001348
0.3	3.333333	0.03567	0.010701
0.4	2.500000	0.08208	0.032832
0.6	1.666667	0.18887	0.113322
0.8	1.250000	0.28650	0.22920
0.9	1.111111	0.32919	0.296271
1.0	1.000000	0.36788	0.36788
1.2	0.833333	0.43460	0.52152
1.4	0.714286	0.48954	0.685356
1.5	0.666667	0.51341	0.770115
1.6	0.625000	0.53526	0.856416
1.8	0.555556	0.57375	1.03275
2.0	0.500000	0.60653	1.21306
2.1	0.476190	0.62115	1.30442
2.4	0.416667	0.65924	1.58218
2.7	0.370370	0.69048	1.86430
3.0	0.333333	0.71653	2.14959
3.6	0.277778	0.75747	2.72689
4.2	0.238095	0.78813	3.31015
4.8	0.208333	0.81194	3.89731
5.4	0.185185	0.83095	4.48713
6.0	0.166667	0.84648	5.07888

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