$$dn = n_0\, d\mu_z \left[\int_{|\mu_z-\mu_0|}^{\infty} e^{-\xi^2} d\xi + \int_{\mu_z+\mu_0}^{\infty} e^{-\xi^2} d\xi\right], \quad s_\alpha = s_{\alpha\alpha}\mu_z, \quad ds_\alpha = s_{\alpha\alpha}\, d\mu_z$$

$$dJ = \frac{dn}{s} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^2}{s^2} = \frac{dn}{s} e^{-y} y^2 = n_0 \frac{ds_\alpha}{s_{\alpha\alpha}} [\;] e^{-y} y^2 \frac{1}{s}, \quad \frac{s_\alpha}{s} = y$$

$$J = \frac{n_0}{s_{\alpha\alpha}} \int_0^{\infty} e^{-y} y^2 \left[\int_{|\sigma y-\mu_0|}^{\infty} e^{-\xi^2} d\xi + \int_{\sigma y+\mu_0}^{\infty} e^{-\xi^2} d\xi\right] dy, \quad \mu_z = \frac{s_\alpha}{s_{\alpha\alpha}} = y\frac{s}{s_{\alpha\alpha}} = \sigma y$$

$$\frac{s_{\alpha\alpha}}{n_0} J = \sqrt{\pi}\int_0^{\infty} e^{-y} y^2 F\, dy, \quad \sqrt{\pi} F = \left[\sqrt{\pi} - \left(\int_0^{|\sigma y-\mu_0|} e^{-\xi^2} d\xi + \int_0^{\sigma y+\mu_0} e^{-\xi^2} d\xi\right)\right] = \sqrt{\pi}\left[1 - \frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\int + \frac{2}{\sqrt{\pi}}\int\right)\right]$$

$$F = 1 - \frac{1}{2}\left[\Phi(\sigma y-\mu_0) + \Phi(\sigma y+\mu_0)\right] = 1 - \frac{1}{\sqrt{\pi}}\left[\int_0^{|\sigma y-\mu_0|} e^{-\xi^2} d\xi + \int_0^{|\sigma y+\mu_0|} e^{-\xi^2} d\xi\right]$$

$$\frac{d(\sqrt{\pi}F)}{d\sigma} = -\left[y e^{-(\sigma y-\mu_0)^2} + y e^{-(\sigma y+\mu_0)^2}\right] \qquad = -\left[-y e^{-(\sigma y-\mu_0)^2} + y e^{-(\sigma y+\mu_0)^2}\right]$$

$\sigma y - \mu_0 > 0,\ y > \frac{\mu_0}{\sigma}$ $\qquad$ $\mu_0 - \sigma y > 0,\ y < \frac{\mu_0}{\sigma}$

$\frac{\mu_0}{\sigma} < y < \infty$ $\qquad$ $0 < y < \frac{\mu_0}{\sigma}$

$$\frac{s_{\alpha\alpha}}{n_0}\frac{dJ}{d\sigma} = -\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 \left[e^{-(\sigma y-\mu_0)^2} + e^{-(\sigma y+\mu_0)^2}\right] dy + \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^3 \left[e^{-(\sigma y-\mu_0)^2} - e^{-(\sigma y+\mu_0)^2}\right] dy$$

$$\mu_0 = 0: -\int_0^{\infty} e^{-y} y^3\, 2 e^{-\sigma^2 y^2} dy + 0, \quad \sigma = 0: -2\int_0^{\infty} e^{-y} y^3 dy = -2\times 6 = -12.$$

$$\sigma \ll 1: -\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 e^{-\mu_0^2} e^{-\sigma^2 y^2}\left[e^{2\sigma y\mu_0} + e^{-2\sigma y\mu_0}\right] dy + \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^3 e^{-\mu_0^2} e^{-\sigma^2 y^2}\left[e^{2\sigma y\mu_0} - e^{-2\sigma y\mu_0}\right] dy$$

$e^{x} = 1 + x + \frac{x^2}{2}$ $\quad$ $e^{x} + e^{-x} = 2 + x^2$ $\quad$ $\left[e^{2\sigma y\mu_0} + e^{-2\sigma y\mu_0}\right] = (2 + 4\sigma^2 y^2 \mu_0^2)(1 - \sigma^2 y^2) = 2 + \sigma^2 y^2(4\mu_0^2 - 2)$

$e^{-x} = 1 - x + \frac{x^2}{2}$ $\quad$ $e^{x} - e^{-x} = 2x$ $\quad$ $\left[e^{2\sigma y\mu_0} - e^{-2\sigma y\mu_0}\right] = 4\sigma y\mu_0(1 - \sigma^2 y^2) = 4\sigma y \mu_0$

$$= -e^{-\mu_0^2}\left[2\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy + \sigma^2(4\mu_0^2 - 2)\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^5 dy - 4\sigma\mu_0 \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^4 dy\right]$$

$$\mathcal{F} = \left[1 - \frac{1}{\sqrt{\pi}}\left(\int_0^{\mu_0+\sigma y} e^{-\xi^2} d\xi + \int_0^{\mu_0-\sigma y} e^{-\xi^2} d\xi\right)\right] \qquad \sigma \ll 1$$

$$\frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = -\left[e^{-(\mu_0+\sigma y)^2} y - e^{-(\mu_0-\sigma y)^2} y\right] = -y\, e^{-(\mu_0^2+\sigma^2 y^2)}\left[e^{-2\mu_0\sigma y} - e^{+2\mu_0\sigma y}\right]$$

$$= y\, e^{-(\mu_0-\sigma y)^2}\left[1 - e^{-4\mu_0\sigma y}\right]$$

$$\lim_{\sigma=0} \frac{d\mathcal{F}}{d\sigma} = 0$$

$$\sqrt{\pi}\,\mathcal{F} = \int_{|\mu_0-\mu_z|}^{\infty} e^{-\xi^2} d\xi + \int_{\mu_0+\mu_z}^{\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}\left[2 - \frac{2}{\sqrt{\pi}}\int_0^{|\mu_0-\mu_z|} e^{-\xi^2} d\xi - \frac{2}{\sqrt{\pi}}\int_0^{\mu_0+\mu_z} e^{-\xi^2} d\xi\right]$$

$$= \sqrt{\pi}\left\{1 - \frac{1}{2}\left[\frac{2}{\sqrt{\pi}}\int_0^{\mu_0-\mu_z} e^{-\xi^2} d\xi + \frac{2}{\sqrt{\pi}}\int_0^{|\mu_0+\mu_z|} e^{-\xi^2} d\xi\right]\right\}$$

$$\mathcal{F} = 1 - \frac{1}{2}\left[\phi(|\mu_0-\mu_z|) + \phi(\mu_0+\mu_z)\right].$$

$$\mu_0 = 0 \quad \mathcal{F} = 1 - \phi(\mu_z)$$

$$\frac{n_0}{2} = \int_0^{\infty} d\mu_z \int_{\mu_z}^{\infty} e^{-\xi^2} d\xi =$$

$$0 < \sigma y < \mu_0$$

$$\frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = y\, e^{-(\mu_0+\sigma y)^2}\left[e^{-(\mu_0-\sigma y)^2 + (\mu_0+\sigma y)^2} - 1\right] = y\, e^{-(\mu_0+\sigma y)^2}\left[e^{4\mu_0\sigma y} - 1\right]$$

$$= y\left[e^{-(\mu_0-\sigma y)^2} - e^{-(\mu_0+\sigma y)^2}\right] = y\left[e^{-\mu_0^2} e^{+2\sigma y} e^{-\sigma^2 y^2} - e^{-\mu_0^2} e^{-2\sigma y} e^{-\sigma^2 y^2}\right]$$

$$= y\, e^{-(\mu_0+\sigma y)^2}\left[e^{4\sigma y} - 1\right] = y\, e^{-(\mu_0-\sigma y)^2}\left[1 - e^{-4\sigma y}\right]$$

$$\sigma \ll 1 \quad \frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = y\, e^{-\mu_0^2}\left[e^{+2\sigma y} - e^{-2\sigma y}\right] e^{-\sigma^2 y^2} = y\, e^{-\mu_0^2}\, 4\sigma y$$

$$\mu_0 = 0, \quad \mathcal{F} = \left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\sigma y} e^{-\xi^2} d\xi\right] \quad \frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = -2y\, e^{-\sigma^2 y^2}$$

$$\mu_0 < \sigma y < \infty$$

$$\mathcal{F} = 1 - \frac{1}{\sqrt{\pi}}\left[\int_0^{\sigma y-\mu_0} + \int_0^{\sigma y+\mu_0}\right], \quad \frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = -y\left[e^{-(\sigma y-\mu_0)^2} + e^{-(\sigma y+\mu_0)^2}\right] = -y\, e^{-(\sigma y-\mu_0)^2}\left[1 + e^{-4\sigma y}\right]$$

$\sigma \ll 1$ 1

$$\frac{s\alpha}{n_0}\frac{dJ}{ds} = -e^{-\mu_0^2}\Big[2\Big(6+6\frac{\mu_0}{\sigma}+3\frac{\mu_0^2}{\sigma^2}+\frac{\mu_0^3}{\sigma^3}\Big)e^{-\frac{\mu_0}{\sigma}} - 4\sigma\mu_0\Big\{24-\Big(24+24\frac{\mu_0}{\sigma}+12\frac{\mu_0^2}{\sigma^2}+4\frac{\mu_0^3}{\sigma^3}+\frac{\mu_0^4}{\sigma^4}\Big)e^{-\frac{\mu_0}{\sigma}}\Big\}\Big]$$

$$= -e^{-\mu_0^2}\left[2\frac{\mu_0^3}{\sigma^3}e^{-\frac{\mu_0}{\sigma}} - 4\sigma\mu_0\cdot 24\right] = \sigma\mu_0 e^{-\mu_0^2}\left[96 - \frac{2}{\mu_0^2}\frac{\mu_0^4}{\sigma^4}e^{-\frac{\mu_0}{\sigma}}\right]$$

$$\frac{d\, x^4 e^{-x}}{dx} = 4x^3 e^{-x} - x^4 e^{-x},\quad x_m = 4,\quad 4^4 e^{-4} = 256\times 0.0183 = 4.69\cdot 2$$
$$9.38$$

$$= -e^{-\mu_0^2}\left[2\frac{\mu_0^3}{\sigma^3}e^{-\frac{\mu_0}{\sigma}} - 4\sigma\mu_0\left\{24 - \frac{\mu_0^4}{\sigma^4}e^{-\frac{\mu_0}{\sigma}}\right\}\right]$$

$$= e^{-\mu_0^2}\left[96\sigma\mu_0 - (2+4\mu_0^2)\frac{\mu_0^3}{\sigma^3}e^{-\frac{\mu_0}{\sigma}}\right]$$

$$\sqrt{\pi}F = \left[\int_{\mu_0-\mu_z}^{\infty} e^{-\xi^2}d\xi + \int_{\mu_0+\mu_z}^{\infty} e^{-\xi^2}d\xi\right] = \left[1-\frac12\left(\frac{2}{\sqrt\pi}\int_0^{\mu_0-\mu_z}e^{-\xi^2}d\xi + \int_0^{\mu_0+\mu_z}e^{-\xi^2}d\xi\right)\right]\sqrt\pi$$

$\mu_0>\mu_z$ $F = 1-\frac12\left[\Phi(\mu_0-\mu_z)+\Phi(\mu_0+\mu_z)\right]$, $\mu_z=\mu_0$, $F = 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0}e^{-\xi^2}d\xi$, $\mu_z=0$, $F=1-\frac{2}{\sqrt\pi}\int_0^{\mu_0}e^{-\xi^2}d\xi$

$\mu_0<\mu_z$ $F = 1-\frac12\left[\Phi(\mu_z-\mu_0)+\Phi(\mu_z+\mu_0)\right]$, $\mu_0=0$, $F=1-\frac{2}{\sqrt\pi}\int_0^{\mu_z}e^{-\xi^2}d\xi$

$\frac{dF}{d\mu_z} = \frac{1}{\sqrt\pi}\left[e^{-(\mu_0-\mu_z)^2} - e^{-(\mu_0+\mu_z)^2}\right]$, $\mu_0>\mu_z$ $\mu_z=\mu_0$ $\frac{dF}{d\mu_z} = \frac{1}{\sqrt\pi}\left[1-e^{-4\mu_0^2}\right]$

$\frac{dF}{d\mu_z} = -\frac{1}{\sqrt\pi}\left[e^{-(\mu_z-\mu_0)^2} + e^{-(\mu_z+\mu_0)^2}\right]$ $\mu_0<\mu_z$ $\frac{dF}{d\mu_z} = -\frac{1}{\sqrt\pi}\left[1+e^{-4\mu_0^2}\right]$

Näherungsweise: $\overset{\mu_0>\mu_z}{\frac{dF}{d\mu_z}} = \frac{1}{\sqrt\pi}(1-\alpha)$, $\overset{\mu_0<\mu_z}{\frac{dF}{d\mu_z}} = -\frac{1}{\sqrt\pi}(1+\alpha)$, $F_{(\mu_z=\mu_0)} = 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0}e^{-\xi^2}d\xi$

$\mu_0>\mu_z$

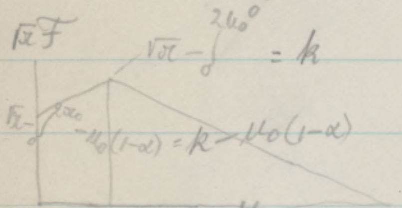
$$F = 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0} -\mu_0\frac{1}{\sqrt\pi}(1-\alpha) + \mu_z\frac{1}{\sqrt\pi}(1-\alpha) = 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0} -(\mu_0-\mu_z)\frac{1}{\sqrt\pi}(1-\alpha)$$

$\mu_0<\mu_z$

$$F = 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0} -(\mu_z-\mu_0)\frac{1}{\sqrt\pi}(1+\alpha),\quad 1-\frac{1}{\sqrt\pi}\int_0^{2\mu_0} = (\mu_z^m-\mu_0)\frac{1}{\sqrt\pi}(1+\alpha),\quad \mu_z^m = \mu_0 + \frac{\sqrt\pi - \int_0^{2\mu_0}}{1+\alpha}$$

$$\int_0^\infty \sqrt{\pi}\,F\,d\mu_z = \int_0^{\mu_0} d\mu_z \left[\int_{\mu_0-\mu_z}^{\infty} + \int_{\mu_0+\mu_z}^{\infty}\right] + \int_{\mu_0}^{\infty} d\mu_z \left[\int_{\mu_z-\mu_0}^{\infty} + \int_{\mu_z+\mu_0}^{\infty}\right]$$

$$(\mu_0=0)\qquad = \int_0^\infty d\mu_z\, 2\int_{\mu_z}^\infty e^{-\xi^2}d\xi = 2\int_0^\infty d\xi\, e^{-\xi^2}\xi = \int_0^\infty e^{-\xi^2}d\xi^2 = -e^{-\xi^2}\Big|_0^\infty = 1$$



$$\mu_z^m = \mu_0 + \frac{\sqrt{\pi}-\int_0^{2\mu_0}}{1+\alpha} = \mu_0 + \frac{k}{1+\alpha}$$

$$\left(\sqrt{\pi}-\int_0^{2\mu_0} - \mu_0(1-\alpha) + \sqrt{\pi}-\int_0^{2\mu_0}\right)\frac{1}{2}\mu_0 + \left(\sqrt{\pi}-\int_0^{2\mu_0}\right)\frac{1}{2}\left(\cdot\ \frac{\sqrt{\pi}-\int_0^{2\mu_0}}{1+\alpha}\right) = 1$$

$$\mu_0\sqrt{\pi} - \mu_0\int_0^{2\mu_0}e^{-\xi^2}d\xi - \frac{1}{2}\mu_0^2(1-\alpha) + \frac{1}{2}\frac{\left(\sqrt{\pi}-\int_0^{2\mu_0}e^{-\xi^2}d\xi\right)^2}{1+\alpha} = 1$$

$$\sqrt{\pi}-\int_0^{2\mu_0}e^{-\xi^2}d\xi = k,\quad \mu_0 k - \frac{1}{2}\mu_0^2(1-\alpha) + \frac{1}{2}\frac{k^2}{1+\alpha} = 1$$

$$[k + k - \mu_0(1-\alpha)]\frac{1}{2}\mu_0 + k\frac{1}{2}\frac{k}{1+\alpha} = k\mu_0 - \frac{1}{2}\mu_0^2(1-\alpha) + \frac{1}{2}\frac{k^2}{1+\alpha} = 1$$

$$\frac{2k\cancel{\mu_0}}{\mu_0^2}(1+\alpha) - (1-\alpha^2) + \frac{k^2}{\mu_0^2} = \frac{2}{\mu_0^2}(1+\alpha),\quad \frac{2k}{\mu_0} + 2\frac{k}{\mu_0}\alpha - 1 + \alpha^2 + \frac{k^2}{\mu_0^2} = \frac{2}{\mu_0^2} + \frac{2}{\mu_0^2}\alpha$$

$$\alpha^2 + 2\alpha\left(\frac{k}{\mu_0}-\frac{1}{\mu_0^2}\right) + \frac{k^2}{\mu_0^2} + 2\frac{k}{\mu_0} = 1 + \frac{2}{\mu_0^2},\quad \left[\alpha + \left(\frac{k}{\mu_0}-\frac{1}{\mu_0^2}\right)\right]^2 = \cancel{\frac{k^2}{\mu_0^2}} + \frac{2k}{\mu_0^3} - \frac{1}{\mu_0^4} + \cancel{\frac{k^2}{\mu_0^2}} + 2\frac{k}{\mu_0} - 1 + \frac{2}{\mu_0^2}$$

$$k = \sqrt{\pi} - \int_0^{2\mu_0} e^{-\xi^2}d\xi = \frac{\sqrt{\pi}}{2}\left(2 - \frac{2}{\sqrt{\pi}}\int_0^{2\mu_0} e^{-\xi^2}d\xi\right),\quad \alpha + \frac{k}{\mu_0} - \frac{1}{\mu_0^2} = \sqrt{1+\frac{2}{\mu_0^2}+\frac{1}{\mu_0^4} - \frac{2k}{\mu_0}\left(1+\frac{1}{\mu_0^2}\right)}$$

$$\alpha = \frac{1}{\mu_0^2} - \frac{k}{\mu_0} \pm \sqrt{\left(1+\frac{1}{\mu_0^2}\right)^2 - \frac{2k}{\mu_0}\left(1+\frac{1}{\mu_0^2}\right)}$$

$$\sqrt{\left(1+\frac{1}{\mu_0^2}\right)\left(1+\frac{1}{\mu_0^2}-\frac{2k}{\mu_0}\right)}$$

$\mu_0 = 0.5\quad k = 0.88623\quad \begin{array}{r} 2.0000 \\ -0.8427 \\ \hline 1.1573 \end{array} = 1.0256$

$\frac{1}{\mu_0} = 2\quad \frac{1}{\mu_0^2} = 4\quad \frac{k}{\mu_0} = 2.0512$

$25 - 5.$

$\alpha = \begin{array}{r} 4.0000 \\ -2.0512 \\ \hline 1.9488 \\ 2.1185 \\ \hline 4.0673 \end{array} \pm 2.1185$

$5(5 - 4.1024)$

$5 \times 0.8976 = 4.488$

3)

$$\mu_z < \mu_0 : \sqrt{\pi} F = \sqrt{\pi} - \int_0^{2\mu_0} - (\mu_0 - \mu_z)(1-\alpha) = k - (\mu_0 - \mu_z)(1-\alpha)$$

$$\mu_z > \mu_0 : \sqrt{\pi} F = \sqrt{\pi} - \int_0^{2\mu_0} - (\mu_z - \mu_0)(1+\alpha) = k - (\mu_z - \mu_0)(1+\alpha), \quad \mu_z^m = \mu_0 + \frac{k}{1+\alpha}$$

$\mu_0 = 0.5$, $k = 1.0256$, $\alpha = 4.0673$,

$$\left[k - \frac{1}{2} \mu_0 (1-\alpha)\right] \mu_0 + k \frac{1}{2} \frac{k}{1+\alpha} = \sqrt{\pi}$$

$$k\mu_0 - \frac{1}{2}\mu_0^2(1-\alpha) + \frac{1}{2}\frac{k^2}{1+\alpha} = \sqrt{\pi}, \quad 2\frac{k}{\mu_0}(1+\alpha) - (1-\alpha^2) + \frac{k^2}{\mu_0^2} = \frac{2\sqrt{\pi}}{\mu_0^2}(1+\alpha)$$

$$2\frac{k}{\mu_0} + \alpha 2 \frac{k}{\mu_0} - 1 + \alpha^2 + \frac{k^2}{\mu_0^2} = \frac{2\sqrt{\pi}}{\mu_0^2} + \alpha \frac{2\sqrt{\pi}}{\mu_0^2} \qquad \sqrt{\pi} = 1.772_5$$

$$\alpha^2 + 2\alpha\left(\frac{k}{\mu_0} - \frac{\sqrt{\pi}}{\mu_0^2}\right) + \frac{k^2}{\mu_0^2} + 2\frac{k}{\mu_0} = \frac{2\sqrt{\pi}}{\mu_0^2} + 1$$

$$\alpha^2 - 2\alpha\left(\frac{\sqrt{\pi}}{\mu_0^2} - \frac{k}{\mu_0}\right) + \frac{\pi}{\mu_0^4} - 2\frac{k\sqrt{\pi}}{\mu_0^3} + \frac{k^2}{\mu_0^2} - \frac{\pi}{\mu_0^4} + 2\frac{k\sqrt{\pi}}{\mu_0^3} = \frac{2\sqrt{\pi}}{\mu_0^2} + 1 - 2\frac{k}{\mu_0}$$

$$\left[\alpha - \left(\frac{\sqrt{\pi}}{\mu_0^2} - \frac{k}{\mu_0}\right)\right]^2 = \frac{\pi}{\mu_0^4} - 2\frac{k\sqrt{\pi}}{\mu_0^3} + 2\frac{k\sqrt{\pi}}{\mu_0^2} - 2\frac{k}{\mu_0} + 1$$

$$\alpha = \frac{\sqrt{\pi}}{\mu_0^2} - \frac{k}{\mu_0} - \sqrt{\frac{\pi}{\mu_0^4} - 2k\left(\frac{\sqrt{\pi}}{\mu_0^3} - \frac{\sqrt{\pi}}{\mu_0^2} + \frac{1}{\mu_0}\right) + 1}$$

$\mu_0 = 0.5$ $k = 1.0256$, $\frac{k}{\mu_0^2} = 4$, $\frac{k}{\mu_0} = 2.0512$, $\frac{1}{\mu_0^4} = 16$, $\left(\frac{\sqrt{\pi}}{\mu_0^3} - \frac{\sqrt{\pi}}{\mu_0^2} + \frac{1}{\mu_0}\right) =$ 1.7725(8−4)
1.7725.4
7.09
+2.00
9.09×2.0512
18.18
4.65
18.645

$\frac{\pi}{\mu_0^4} =$ 31.416
18.850
51.266
−18.645
$\sqrt{32.62_1} = 5.71$

$\alpha = 7.09_0 - 2.0512 - 5.71$

$$e^{-(2\mu_0)^2} = e^{-1} = 0.3679 \qquad \frac{dF}{d\mu_z} = \frac{0.6321}{1.772_5} = 0.356_5$$

$$= \frac{1.368}{1.772_5} = 0.772$$

$\mu_z = \mu_0$

$F = 1 - \frac{1}{2}\Phi(2\mu_0) = \begin{array}{r} 1.0000 \\ -0.4213 \\ \hline 0.5787 \end{array}$ $\sqrt{\pi}F_{\mu_z=\mu_0} = \sqrt{\pi} - \frac{\sqrt{\pi}}{2}\frac{2}{\sqrt{\pi}}\int_0^{2\mu_0} = \sqrt{\pi} - \int_0^{2\mu_0} = k = 0.5787\sqrt{\pi} = 1.0256$

$\frac{d(F\sqrt{\pi})}{d\mu_z} \overset{\mu_z<\mu_0}{=} 1-\alpha$, $-\frac{d(F\sqrt{\pi})}{d\mu_z} \overset{\mu_z>\mu_0}{=} 1+\alpha$, $\sqrt{\pi}F = k - \mu_0(1-\alpha) + \mu_z(1-\alpha) = k - (\mu_0 - \mu_z)(1-\alpha)$

$\sqrt{\pi}F = k - (\mu_z - \mu_0)(1+\alpha)$, $\mu_z^m = \mu_0 + \frac{k}{1+\alpha}$

Fläche 1: $[k - \frac{1}{2}\mu_0(1-\alpha)]\mu_0$, $\mu_0 = 0.5$ $\alpha = 0.2$, $k = 1$ $[\,] = 1 - \frac{1}{2}0.5\cdot0.8 = 0.8$ $\cdot 0.5$

2:

$\frac{s_{\alpha\alpha}}{n_0}\mathcal{F} = \int_0^\infty e^{-y}y^2\sqrt{\pi}F\,dy = \mathcal{F}'$ $\frac{d\mathcal{F}'}{d\sigma} = \int_0^{\mu_0} e^{-y}y^2\frac{d(\sqrt{\pi}F)}{d\sigma}dy + \int_{\mu_0}^\infty e^{-y}y^2\frac{d(\sqrt{\pi}F)}{d\sigma}dy$

$\sigma = \frac{s}{s_{\alpha\alpha}}$, $\sigma y = \frac{s}{s_{\alpha\alpha}}\frac{s_\alpha}{s} = \frac{s_\alpha}{s_{\alpha\alpha}} = \mu_z$, $\frac{d(F\sqrt{\pi})}{d\sigma} = \frac{d(F\sqrt{\pi})}{d(\sigma y)}\frac{d(\sigma y)}{d\sigma} = y\frac{d(F\sqrt{\pi})}{d\mu_z}$

$$\frac{d\mathcal{F}'}{d\sigma} = (1-\alpha)\int_0^{\frac{\mu_0}{\sigma}} e^{-y}y^3dy - (1+\alpha)\int_{\frac{\mu_0}{\sigma}}^{\frac{\mu_0+\frac{k}{1+\alpha}}{\sigma}} e^{-y}y^3dy = 0$$

$\alpha = e^{-(2\mu_0)^2}$, $\mu_0 = 0.5$, $\alpha = 0.3679$ $\frac{k}{1+\alpha} = \frac{1.0259}{1.3679} = 0.751$

$\sigma = 0.1$ $0.6321\underbrace{\int_0^5 e^{-y}y^3dy}_{4.41} - 1.3679\int_5^{12.5} e^{-y}y^3dy$ $= \begin{array}{r} 2.79 \\ -2.18 \\ \hline 0.61 \end{array}$

$\sigma = 0.1666$ $0.6321\underbrace{\int_0^3}_{2.12}$ $-1.3679\overset{1.59}{\int_3^{7.5}}$ $=$ $\begin{array}{r} 1.34 \\ -4.80 \\ \hline -3.46 \end{array}$

$3.88 - 0.37 = 3.51$

$\sigma = 0.125$ $0.6321\int_0^4$ 3.40 $-1.3679\int_4^{10}$ $2.60 - 0.06 = 2.54$ $\begin{array}{r} 2.15 \\ -3.48 \\ \hline -1.33 \end{array}$

$$\frac{d\mathcal{F}'}{d\sigma} \cong (1-\alpha)\int_0^{\frac{\mu_0}{\sigma}} e^{-y}y^3dy - (1+\alpha)\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y}y^3dy = \int_0^\infty e^{-y}y^3dy - 2\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y}y^3dy - \alpha\int e^{-y}y^3dy$$

$= (1-\alpha)\,6$ $-2\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y}y^3dy = 3.7926 - 2\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y}y^3dy = 0$ für σ für $\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y}y^3dy = 1.8963$ $\frac{\mu_0}{\sigma} = 4.6$, $\sigma = \frac{0.5}{4.6} = 0.108$

$$s_{\alpha\alpha}\mathcal{F} = \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^2 \sqrt{\pi}\, dy - \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^2 \left[\int_0^{\mu_0-\sigma y} + \int_0^{\mu_0+\sigma y}\right] dy + \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^2 \sqrt{\pi}\, dy - \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^2 \left[\int_0^{\sigma y-\mu_0} + \int_0^{\sigma y+\mu_0}\right] dy$$

$$= \sqrt{\pi}\int_0^{\infty} e^{-y} y^2 dy - \left\{\int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^2 \left[\int_0^{\mu_0-\sigma y} + \int_0^{\mu_0+\sigma y}\right] dy + \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^2 \left[\int_0^{\sigma y-\mu_0} + \int_0^{\sigma y+\mu_0}\right] dy\right.$$

$$\int_0^x e^{-\xi^2} d\xi = \int_0^x \left(1-\xi^2+\frac{1}{2!}\xi^4-\frac{1}{3!}\xi^6+-\cdots\right)d\xi = x - \frac{x^3}{1!\,3} + \frac{x^5}{2!\,5} - \frac{x^7}{3!\,7} + - \cdots \frac{x^{2n+1}}{n!(2n+1)}(-1)^n \cdots$$

$x_1 = a+b \quad x_2 = a-b$

$$= \begin{matrix} a+b \\ a-b \end{matrix} - \frac{1}{3}\begin{pmatrix} a^3+3a^2b+3ab^2+b^3 \\ a^3-3a^2b+3ab^2-b^3 \end{pmatrix} + \frac{1}{10}\begin{pmatrix} a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5 \\ a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5 \end{pmatrix}$$

$$= 2a - \frac{2}{3}(a^3+3ab^2) + \frac{2}{10}(a^5+10a^3b^2+5ab^4)$$

$$\int_0^{\mu_0-\sigma y} + \int_0^{\mu_0+\sigma y} = \cancel{2\mu_0} - \frac{2}{3}\mu_0(\mu_0^2+3\sigma^2y^2) + \frac{2}{10}\mu_0(\mu_0^4+10\mu_0^2\sigma^2y^2+5\sigma^4y^4)$$

$$-\cancel{2}\mu_0\frac{1}{3}\mu_0^2 \quad -2\mu_0\sigma^2y^2 \quad +\cancel{2\mu_0}\frac{\mu_0^4}{10} \quad +2\mu_0\mu_0^2\sigma^2y^2$$

$$\int_0^{\sigma y-\mu_0} + \int_0^{\sigma y+\mu_0} = 2\sigma y - \frac{2}{3}\sigma y(\sigma^2y^2 \pm 3\mu_0^2) + \frac{2}{10}\sigma y(\sigma^4y^4+10\sigma^2y^2\mu_0^2+5\mu_0^4)$$

$$s_{\alpha\alpha}\mathcal{F} = 2\sqrt{\pi} - \left\{2\mu_0\left(1-\frac{\mu_0^2}{3}+\frac{\mu_0^4}{10}\right)\int_0^{\frac{\mu_0}{\sigma}} e^{-y}y^2dy - 2\mu_0(1-\mu_0^2)\sigma^2\int_0^{\frac{\mu_0}{\sigma}} e^{-y}y^4dy\right.$$

$$\left. + 2\sigma\left(1-\mu_0^2+\frac{1}{2}\mu_0^4\right)\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y}y^3dy - 2\sigma^3\left(\frac{1}{3}-\mu_0^2\right)\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y}y^5dy\right\}$$

$$= 2\sqrt{\pi} - \left\{2\mu_0(\ )\left[2-\left(\frac{\mu_0^2}{\sigma^2}+2\frac{\mu_0}{\sigma}+2\right)e^{-\frac{\mu_0}{\sigma}}\right] - 2\mu_0(\ )\sigma^2\left[24-\left(\frac{\mu_0^4}{\sigma^4}+4\frac{\mu_0^3}{\sigma^3}+12\frac{\mu_0^2}{\sigma^2}+24\frac{\mu_0}{\sigma}+24\right)e^{-\frac{\mu_0}{\sigma}}\right]\right.$$

$$\left. +2\sigma(\ )\left(\frac{\mu_0^3}{\sigma^3}+3\frac{\mu_0^2}{\sigma^2}+6\frac{\mu_0}{\sigma}+6\right)e^{-\frac{\mu_0}{\sigma}} - 2\sigma^3(\ )\left(\frac{\mu_0^5}{\sigma^5}+\cdots\qquad+120\right)e^{-\frac{\mu_0}{\sigma}}\right\}$$

$$ {}_{s_{\alpha\alpha}}\mathcal{Y}_{\sigma=0} = \int_0^\infty e^{-y} y^2 \left[2\int_{\mu_0}^\infty e^{-\xi^2} d\xi\right] dy = 4\left[\frac{\sqrt{\pi}}{2} - \int_0^{\mu_0} e^{-\xi^2} d\xi\right] = 2\sqrt{\pi}\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\mu_0} e^{-\xi^2} d\xi\right] $$

$$ {}_{s_{\alpha\alpha}}\mathcal{Y}_{\sigma=0} \simeq \int_0^\infty e^{-y} y^2 \left[2\frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi}\,\mu_0}\right] dy = 2\sqrt{\pi}\, e^{-\sqrt{\pi}\,\mu_0} \qquad \sqrt{\pi} = 1.772_5 $$

$$ {}_{s_{\alpha\alpha}}\mathcal{Y}_{\sigma=0} \simeq \int_0^\infty e^{-y} y^2 \left[2\frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\mu_0\right)\right] dy = 2\sqrt{\pi}\left(1 - \frac{\sqrt{\pi}}{2}\mu_0\right) \qquad \frac{\sqrt{\pi}}{2} = 0.886_2 $$

$\mu_0 =$	0.1	0.5	1.0
$1-\Phi =$	0.8875	0.4795	0.1573
$e^{-\sqrt{\pi}\mu_0} =$	0.8376	0.4123	0.1699
$1-\frac{\sqrt{\pi}}{2}\mu_0 =$	0.9114	0.5569	0.1138

$e^{+x} - e^{-x} = 2x + \frac{x^3}{3}$

$1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$-1 + x - \frac{x^2}{2} + \frac{x^3}{6}$

$$\frac{s_{\alpha\alpha}}{\sqrt{\pi}} \mathcal{Y} = \int_0^\infty e^{-y} y^2 \mathcal{F}\, dy, \quad \frac{s_{\alpha\alpha}}{\sqrt{\pi}} \frac{d\mathcal{Y}}{d\sigma} = \int_0^\infty e^{-y} y^2 \frac{d\mathcal{F}}{d\sigma}\, dy$$

$$\sigma \ll 1, \quad \frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = 4\sigma e^{-\mu_0^2} y^2, \quad \frac{s_{\alpha\alpha}}{\cancel{\sqrt{\pi}}} \frac{d\mathcal{Y}}{d\sigma} = 4\sigma e^{-\mu_0^2} \int_0^\infty e^{-y} y^4\, dy = 96\sigma e^{-\mu_0^2}$$

$$\frac{d\mathcal{F}\sqrt{\pi}}{d\sigma} = y e^{-\mu_0^2}\left[4\sigma y + \frac{8}{3}\sigma^3 y^3\right](1-\sigma^2 y^2) = 4\sigma y^2 e^{-\mu_0^2}\left(1 + \frac{2}{3}\sigma^2 y^2 - \sigma^2 y^2\right)(1-\sigma^2 y^2)$$

$$= 4\sigma y^2 e^{-\mu_0^2}\left(1 - \frac{1}{3}\sigma^2 y^2\right) = 4\sigma e^{-\mu_0^2} y^2 - \frac{4}{3}\sigma^3 e^{-\mu_0^2} y^4$$

$$\frac{s_{\alpha\alpha}}{\cancel{\sqrt{\pi}}} \frac{d\mathcal{Y}}{d\sigma} = 4\sigma e^{-\mu_0^2} 24 - \frac{4}{3}\sigma^3 e^{-\mu_0^2} 720 = 96 e^{-\mu_0^2}(\sigma - 10\sigma^3)$$

$(\sigma y - \mu_0)^2 = (\mu_0 - \sigma y)^2$

$$s_{\alpha\alpha}\frac{d\mathcal{Y}}{d\sigma} = \int_0^\infty e^{-y} y^2 \frac{d(\mathcal{F}\sqrt{\pi})}{d\sigma}\, dy = \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^3 (1 - e^{-4\sigma y}) e^{-(\mu_0 - \sigma y)^2} dy - \int_{\frac{\mu_0}{\sigma}}^\infty e^{-y} y^3 (1 + e^{-4\sigma y}) e^{-(\sigma y - \mu_0)^2} dy$$

$$= \int_0^\infty e^{-y} y^3 e^{-(\mu_0 - \sigma y)^2} dy - 2\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y} y^3 e^{-(\mu_0-\sigma y)^2} dy - \int_0^\infty e^{-y} y^3 e^{-(\mu_0+\sigma y)^2} dy$$

$$\sigma y \ll 1 \quad e^{-(\mu_0-\sigma y)^2} - e^{-(\mu_0+\sigma y)^2} = 1 - (\mu_0 - \sigma y)^2 - 1 + (\mu_0 + \sigma y)^2 = 4\sigma y$$

$$\int_{\frac{\mu_0}{\sigma}}^\infty e^{-y} y^3 e^{-(\mu_0-\sigma y)^2} dy \simeq \int_{\frac{\mu_0}{\sigma}}^\infty e^{-y} y^3\, dy = \left[\left(\frac{\mu_0}{\sigma}\right)^3 + 3\left(\frac{\mu_0}{\sigma}\right)^2 + 6\left(\frac{\mu_0}{\sigma}\right) + 6\right] e^{-\frac{\mu_0}{\sigma}} \simeq \left(\frac{\mu_0}{\sigma}\right)^3 e^{-\frac{\mu_0}{\sigma}}$$

$$\sigma \ll 1; \quad s_{\alpha\alpha}\frac{d\mathcal{Y}}{d\sigma} = 96\sigma - 2\left(\frac{\mu_0}{\sigma}\right)^3 e^{-\frac{\mu_0}{\sigma}}$$

-1.68

$\mu_0 = 0.5, \quad \sigma = 0.1, \quad 9.6 - 2 \times 125 \times 0.006738$

$\sigma = 0.2 \quad 19.2 - 2 \times \frac{125}{8} \times 0.08208$

$10.26 : 4 = 2.565$

1.68

$\mu_0 = 0.1 \quad \sigma = 0.02 \quad \frac{\mu_0}{\sigma} = 5 \quad 1.92 - 2 \times 125 \times 0.006738$

$\sigma = 0.04 \quad = 2.5 \quad 3.84 - 2 \times \frac{125}{8}\, 0.08208$

$\mu_0 = 0$

$$dn = n_0\, d\mu_z \int_{\mu_z}^{\infty} e^{-\xi^2} d\xi, \quad \int_{\mu_z<\mu_0} dn = n_0 \int_{0}^{\infty}{}_{\mu_z>\mu_0} d\mu_z \int_{\mu_z}^{\infty} e^{-\xi^2} d\xi = n_0 \int_0^{\infty} d\xi\, e^{-\xi^2} \int_0^{\xi} d\mu_z = n_0 \frac{1}{2}$$

$$dn = \frac{1}{2} n_0\, d\mu_z \left[\int_{\mu_0-\mu_z}^{\infty} + \int_{\mu_0+\mu_z}^{\infty}\right] = \frac{1}{2} n_0\, d\mu_z \left[\int_{\mu_z-\mu_0}^{\infty} + \int_{\mu_z+\mu_0}^{\infty}\right], \ \mu_0 = 0: 2\cdot[\,] = 2\int_{\mu_z}^{\infty},$$ Integralgebiet für erstes Integral ist Null

$$dJ = \frac{dn}{s} e^{-\frac{s_x}{s}} \frac{s_x^2}{s^2}, \ \frac{s_x}{s_{xx}} = \frac{\mu_z}{\mu_1} = \mu_z, \ \frac{s_x}{s} = y, \ s_x = sy, \ \mu_z = \frac{s}{s_{xx}} y = \sigma y, \ d\mu_z = \sigma\, dy = \frac{s}{s_{xx}} dy, \ y = \frac{\mu_z}{\sigma}$$

$$dJ = \frac{1}{2} n_0 \frac{1}{s_{xx}} dy \overset{F(y)}{[\;]} e^{-y} y^2, \quad J = \frac{n_0}{2 s_{xx}} \left\{ \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^2 \left[\int_{\mu_0-\sigma y}^{\infty} + \int_{\mu_0+\sigma y}^{\infty}\right] dy + \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^2 \left[\int_{\sigma y-\mu_0}^{\infty} + \int_{\sigma y+\mu_0}^{\infty}\right] dy \right\}$$

$$J = \frac{n_0}{2 s_{xx}} \int_0^{\infty} e^{-y} y^2 F(y)\, dy, \quad \frac{dJ}{d\sigma} = \frac{n_0}{2 s_{xx}} \int_0^{\infty} e^{-y} y^2 \frac{dF(y)}{d\sigma} dy$$

$$\mu_z < \mu_0 \quad \frac{dF}{d\mu_z} = +\left[e^{-(\mu_0-\mu_z)^2} - e^{-(\mu_0+\mu_z)^2}\right] = \left[1 - e^{-4\mu_0^2}\right] \text{ für } \mu_z = \mu_0 \qquad \frac{dF}{d\sigma} = \frac{dF}{d\mu_z}\frac{d\mu_z}{d\sigma} = y \frac{dF}{d\mu_z}$$

$$\mu_z > \mu_0 \quad \frac{dF}{d\mu_z} = -\left[e^{-(\mu_z-\mu_0)^2} - e^{-(\mu_z+\mu_0)^2}\right] = -\left[1 + e^{-4\mu_0^2}\right] \text{ für } \mu_z = \mu_0$$

Näherung: streng y_m

$$\frac{dJ}{d\sigma} = \frac{n_0}{2 s_{xx}} \left[(1 - e^{-4\mu_0^2}) \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^3 dy - (1 + e^{-4\mu_0^2}) \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy\right]$$

$$= \frac{n_0}{2 s_{xx}} \left[\int_0^{\infty} e^{-y} y^3 dy - \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy - \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy - e^{-4\mu_0^2} \int_0^{\infty} e^{-y} y^3 dy\right]$$

$$= \frac{n_0}{2 s_{xx}} \left[6 (1 - e^{-4\mu_0^2}) - 2 \int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy\right]$$

$\frac{dJ}{d\sigma} = 0$ für den Wert von σ für den $\int_{\frac{\mu_0}{\sigma}}^{\infty} e^{-y} y^3 dy = 3 (1 - e^{-4\mu_0^2})$ ist

$\mu_0 = 0.2$, $4\mu_0^2 = 0.16$, $e^{-4\mu_0^2} = \overset{1\,4\,79}{0.8521}$, $3(1-e^{-4\mu_0^2}) = 0.4437$, $\frac{\mu_0}{\sigma} = 7.15$, $\sigma = \frac{0.2}{7.15} = 0.028$

$\mu_0 = 0.5$, $4\mu_0^2 = 1$, $e^{-4\mu_0^2} = \overset{6\,3\,2\,1}{0.3679}$, $3(1-e^{-4\mu_0^2}) = 1.8963$, $\frac{\mu_0}{\sigma} = 4.6$, $\sigma = \frac{0.5}{4.6} = 0.108$

$\mu_0 = 1$, $4\mu_0^2 = 4$, $e^{-4\mu_0^2} = \underset{0.98168}{0.01832}$, $3(1-e^{-4\mu_0^2}) = 2.9550$, $\frac{\mu_0}{\sigma} = \underset{(69)}{3.7}$, $\sigma = \frac{1}{3.7} = 0.270$

5)

$$\int_{y_0}^{\infty} e^{-y} y^3 \, dy = (y_0^3 + 3y_0^2 + 6y_0 + 6)\, e^{-y_0} = \mathcal{F}(y_0)$$

$y_0 = 0 \quad \mathcal{F}(y_0) = \quad 6 \cdot 1 \qquad = 6$

$y_0 = 1 \quad \mathcal{F}(y_0) = (1+3+6+6)e^{-1} = 16 \times 0.36779 = \begin{array}{r} 3.679 \\ 2.207 \\ \hline 5.886 \end{array} = 5.886$

$y_0 = 2 \quad \mathcal{F}(y_0) = (8+12+12+6)e^{-2} = 38 \times 0.13534 = \begin{array}{r} 4.0602 \\ 1.0827 \\ \hline 5.1429 \end{array} = 5.143$

$y_0 = 3 \quad \mathcal{F}(y_0) = (27+27+18+6)e^{-3} = 78 \times 0.04979 \qquad = 3.88$

1.28

$y_0 = 4 \quad \mathcal{F}(y_0) = (64+48+24+6)e^{-4} = 142 \times 0.01832 \qquad = 2.60$

1.01

$y_0 = 5 \quad \mathcal{F}(y_0) = (125+75+30+6)e^{-5} = 236 \times 0.006738 \qquad = 1.59$

$y_0 = 6 \quad \mathcal{F}(y_0) = (216+108+36+6)e^{-6} = 366 \times 0.002479 \qquad = 0.91$

0.42

$y_0 = 7 \quad \mathcal{F}(y_0) = (343+147+42+6)e^{-7} = 538 \times 0.000912 \qquad = 0.49$

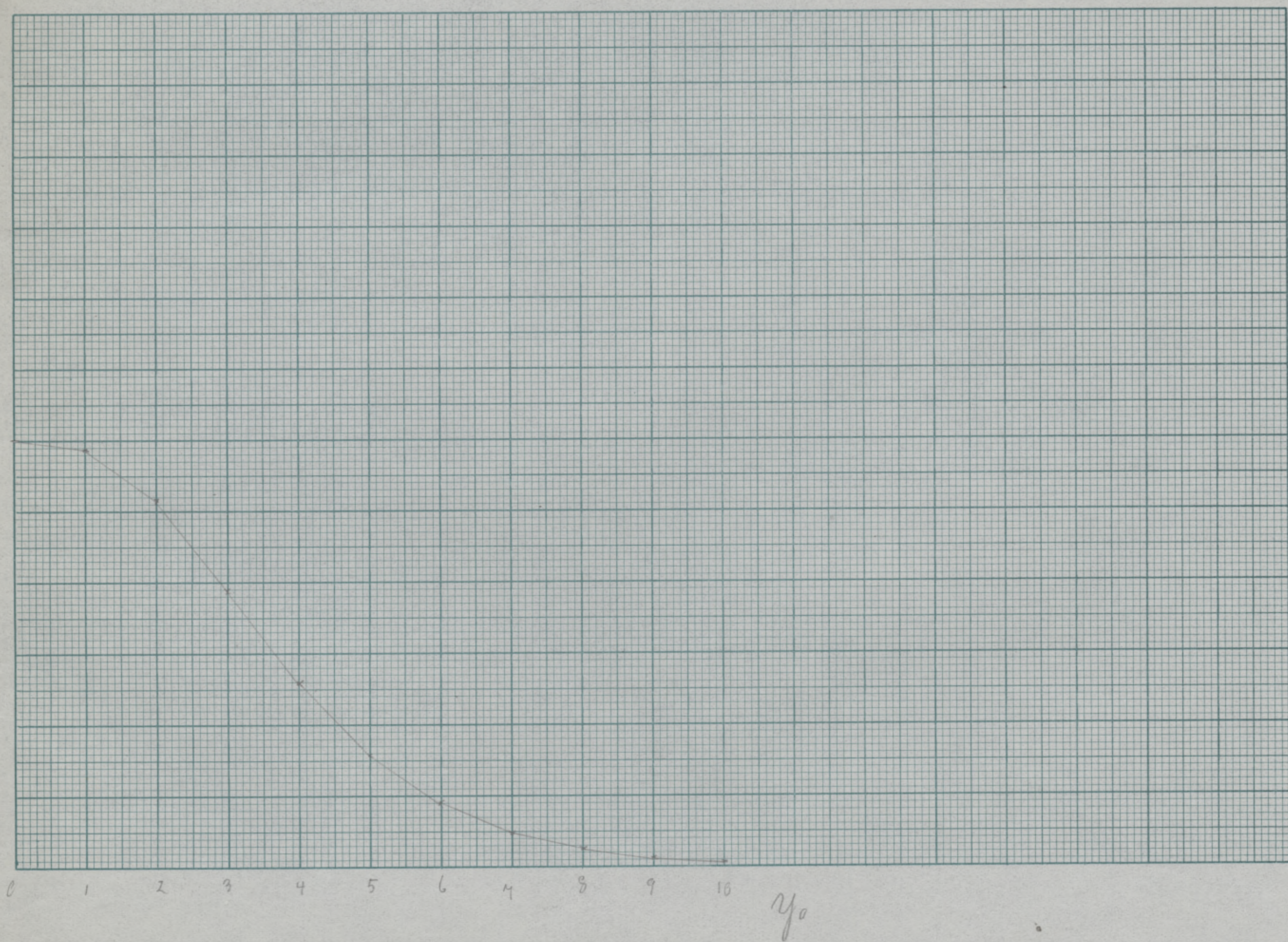
0.24

$y_0 = 8 \quad \mathcal{F}(y_0) = (512+192+48+6)e^{-8} = 758 \times 0.0003355 \qquad = 0.25$

$y_0 = 9 \quad \mathcal{F}(y_0) = (729+243+54+6)e^{-9} = 1032 \times 0.0001234 \qquad = 0.13$

$y_0 = 10 \quad \mathcal{F}(y_0) = (1000+300+60+6)e^{-10} = 1366 \times 0.0000454 \qquad = 0.06$

$$\int_{y_0}^{\infty} e^{-y} y^3 \, dy = (y_0^3 + 3y_0^2 + 6y_0 + 6)\, e^{-y_0}$$



$\underline{\underline{\mu_0 = 0.2}}$

$$\mathcal{F}_0 = \sqrt{\pi} - 0.4\left[1 - \frac{0.04}{3} + \frac{0.00016}{10}\right] \qquad \mathcal{F}_{\mu_0} = \sqrt{\pi} - 0.4\left[1 - \frac{0.16}{3} + 0.000256\right]$$

$$\begin{array}{r} 1.772\,454 \\ = -0.394\,673 \\ \hline 1.377\,781 \end{array} \quad \begin{array}{r} 1.000016 \\ -0.013333 \\ \hline 0.986683.4 \\ 0.3946732 \end{array} \qquad \begin{array}{r} 1.772\,454 \\ -0.378\,469 \\ \hline 1.393\,685 \\ -1.377\,781 \\ \hline 0.015\,904 \end{array} \quad \begin{array}{r} 1.000256 \\ -0.053333 \\ \hline 0.946923.4 \\ 0.3787692 \end{array}$$

$$\mathcal{F}_{\mu_0} - \mathcal{F}_0 = 0.008\,[2 - 0.12] = 0.008 \times 1.88 = 0.01504 \qquad \frac{\mathcal{F}_{\mu_0} - \mathcal{F}_0}{\mathcal{F}_0} = \frac{0.01504}{1.3778} = 1.09\%.$$

$$\mathcal{J} = \frac{1}{s_{\alpha\alpha}}\left\{\int_0^{\frac{1}{\sigma}\mu_0} e^{-y} y^2 \left[\int_{\mu_0-\sigma y}^{\infty} e^{-\xi^2} d\xi + \int_{\mu_0+\sigma y}^{\infty} e^{-\xi^2} d\xi\right] dy + \int_{\frac{1}{\sigma}\mu_0}^{\infty} e^{-y} y^2 \left[\int_{\sigma y-\mu_0}^{\infty} e^{-\xi^2} d\xi + \int_{\sigma y+\mu_0}^{\infty} e^{-\xi^2} d\xi\right] dy\right\}$$

$\underline{\underline{\sigma = 0}}$

$$s_{\alpha\alpha}\mathcal{J}_{\sigma=0} = \int_0^{\infty} e^{-y} y^2 \left[2\int_{\mu_0}^{\infty} e^{-\xi^2} d\xi\right] dy = 2\int_{\mu_0}^{\infty} e^{-\xi^2} d\xi \int_0^{\infty} e^{-y} y^2 dy = 4\left[\frac{\sqrt{\pi}}{2} - \int_0^{\mu_0} e^{-\xi^2} d\xi\right]$$

$$s_{\alpha\alpha}\mathcal{J}_{\sigma=0} = 2\sqrt{\pi}\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\mu_0} e^{-\xi^2} d\xi\right], \quad \mu_0 = 0.2 \quad s_{\alpha\alpha}\mathcal{J} = 2\sqrt{\pi}\,[1 - \overset{0.7773}{\Phi(0.2)}] = 2 \times \overset{3.545}{1.7725} \times 0.7773 = 2.757$$

$\underline{\underline{\sigma = 0.1}}$

$$s_{\alpha\alpha}\mathcal{J}_{\sigma=0.1} = \int_0^{10\mu_0} e^{-y} y^2 \left[\sqrt{\pi} - 2\mu_0 + \tfrac{2}{3}\mu_0^3 + 2\mu_0(\sigma y)^2\right] dy + \int_{10\mu_0}^{\infty} e^{-y} y^2 \left[\sqrt{\pi} - 2\sigma y(1-\mu_0^2) + \tfrac{2}{3}(\sigma y)^3\right] dy$$

$$= \int_0^{2} e^{-y} y^2 \left[\sqrt{\pi} - 0.4 + 0.0053 + 0.004\,y^2\right] dy + \int_2^{\infty} e^{-y} y^2 \left[\sqrt{\pi} - 0.192\,y + 0.00067\,y^3\right] dy$$

$$= \sqrt{\pi}\int_0^{\infty} e^{-y} y^2 dy - 0.3947\int_0^{2} e^{-y} y^2 dy + 0.004\int_0^{2} e^{-y} y^4 dy - 0.192\int_2^{\infty} e^{-y} y^3 dy + 0.00067\int_2^{\infty} e^{-y} y^5 dy$$

$$= 2\sqrt{\pi} - 0.3947\left[2 - \underbrace{\overbrace{(2+4+4)e^{-2}}^{10\times 0.13534}}_{0.6466}\right] + 0.004\left[\overset{22.737}{24} - \underbrace{\overbrace{(24+48+48+32+16)e^{-2}}^{=168\times 0.13534}}_{1.263}\right] - 0.192\,(6+\overset{38}{12}+12+8)e^{-2} + 0.00067\,\overset{872}{(32+80+160+240+240+120)}\,e^{-2}$$

$$= 3.545 - 0.2555 + 0.0050 - (\overset{6.12}{7.30} - 0.584)\,0.13534 = \begin{array}{r} 3.545 \\ -1.210 \\ \hline 2.335 \end{array}$$

$$\begin{array}{r} 0.960 \\ 0.250 \\ \hline 1.210 \end{array}$$

$\mu_0 = 0.2$

σy	$F(\mu_z = \sigma y)$	
0	0.7773	
0.1	0.7794_5	
0.2	0.7858	
0.3	0.6835	1023
0.4	0.5867	968
0.5	0.4968	899
0.6	0.4147	821
0.7	0.3413	734
0.8	0.2767	656
0.9	0.2210	557
1.0	0.1738	472
1.1	0.1345	
1.2	0.1025	
1.3	0.0768	
1.4	0.0567	
1.5	0.0411	
1.6	0.0293	
1.7	0.0205	
1.8	0.0142	
1.9	0.0096	
2.0	0.0064	
2.1	0.0041	
2.2	0.0027	
2.5	0.00063_5	

$$F = 1 - \frac{1}{2}\left[\Phi(\mu_0 - \mu_z) + \Phi(\mu_0 + \mu_z)\right]$$

μ_z

0.02
0.2009 $\quad$ 0.7774
0.2443
0.4452 : 2 = 0.2226

0.04
0.1790 $\quad$ 0.7776_5
0.2657
0.4447 : 2 = 0.2223_5

0.06
0.1569 $\quad$ 0.7781
0.2869
0.4438 $\quad$ 0.2219

0.08
0.1348 $\quad$ 0.7786_5
0.3079
0.4427 $\quad$ 0.2213_5

0.10
0.1125 $\quad$ 0.7794_5
0.3286
0.4411 $\quad$ 0.2205_5

$\sigma = 0.8$

y	σy	F	$y^2 e^{-y}$	P	
0	0	0.777_3	0	0	
0.5	0.4	0.5867	0.1516_3	0.0918	0.0229_5
1	0.8	0.2767	0.3679	0.1017	0.0254
1.5	1.2	0.1025	0.5020	0.0516	0.0129
2	1.6	0.0293	0.5414	0.0158	0.0039_5
2.5	2.0	0.0064	0.5129	0.0032_8	0.0008_2
3	2.4	0.001_0	0.4480	0.0004_5	0.0001_1
					0.0661

$$\int_x^{\infty} e^{-\xi^2} d\xi \simeq \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right) = \frac{\sqrt{\pi}}{2} - \frac{\pi}{4}x$$

$$\mu_0 < \frac{1}{\sqrt{\pi}}$$

$$s_{\alpha a} \mathcal{F} = \left\{ \int_0^{\frac{\mu_0}{\sigma}} e^{-y} y^2 \left[\sqrt{\pi} - \frac{\pi}{4} 2\mu_0\right] dy + \int_{\frac{\mu_0}{\sigma}}^{\frac{\frac{1}{2}\sqrt{\pi}+\mu_0}{\sigma}} e^{-y} y^2 \left[\frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}(\sigma y-\mu_0)\right)\right] dy + \int_{\frac{\mu_0}{\sigma}}^{\frac{\frac{1}{2}\sqrt{\pi}-\mu_0}{\sigma}} e^{-y} y^2 \left[\frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}(\sigma y+\mu_0)\right)\right] dy\right.$$

$$= \sqrt{\pi}\left(1-\frac{\sqrt{\pi}}{2}\mu_0\right)\left[2-\left(\frac{\mu_0^2}{\sigma^2}+2\frac{\mu_0}{\sigma}+2\right)e^{-\frac{\mu_0}{\sigma}}\right] + \frac{\sqrt{\pi}}{2}\left(1+\frac{\sqrt{\pi}}{2}\mu_0\right)\left[\left(\frac{\mu_0^2}{\sigma^2}+\cdots\right)e^{-\frac{\mu_0}{\sigma}} - \left(\frac{(\frac{1}{2}\sqrt{\pi}+\mu_0)^2}{\sigma^2}+\cdots\right)e^{-\frac{\frac{1}{2}\sqrt{\pi}+\mu_0}{\sigma}}\right]$$

$$-\frac{\pi}{4}\sigma\left[\left(\frac{\mu_0^3}{\sigma_0^3}\cdots\right)e^{-\frac{\mu_0}{\sigma}} - \left(\frac{(\frac{1}{2}\pi+\mu_0)^3}{\sigma^3}+\right)e^{-\frac{\frac{1}{2}\sqrt{\pi}+\mu_0}{\sigma}}\right] + \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}\mu_0\right)\left[\left(\frac{\mu_0^2}{\sigma^2}+\cdots\right)e^{-\frac{\mu_0}{\sigma}} - \left(\frac{(\frac{1}{2}\sqrt{\pi}-\mu_0)^2}{\sigma^2}+\cdots\right)e^{-\frac{\frac{1}{2}\sqrt{\pi}+\mu_0}{\sigma}}\right]$$

$$-\frac{\pi}{4}\sigma\left[\left(\frac{\mu_0^3}{\sigma^3}+\cdots\right)e^{-\frac{\mu_0}{\sigma}} - \left(\frac{(\frac{1}{2}\sqrt{\pi}-\mu_0)^3}{\sigma^3}+\cdots\right)e^{-\frac{\frac{1}{2}\sqrt{\pi}-\mu_0}{\sigma}}\right]$$

$\mu_0 = 0.2$

$$\int_{\mu_z-\mu_0}^{\infty} e^{-\xi^2} d\xi + \int_{\mu_z+\mu_0}^{\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2} - \int_0^{\mu_z-\mu_0} + \frac{\sqrt{\pi}}{2} - \int_0^{\mu_z+\mu_0} = \frac{\sqrt{\pi}}{2}\left[2-\left(\Phi(\mu_z-\mu_0)+\Phi(\mu_z+\mu_0)\right)\right]$$

$\mu_z = 0.2$	$\Phi_- + \Phi_+ =$ 0.0000 +0.4284	= 0.4284	2142 2046		
$\mu_z = 0.3$	0.1125 +0.5205	= 0.6330	3165 1936	110	28
$\mu_z = 0.4$	0.2227 0.6039	= 0.8266	4133 1798	138	19
0.5	0.3286 0.6778	= 1.0064	5032 1641	157	15
0.6	0.4284 0.7421	= 1.1705	5853 1469	172	5
0.7	0.5205 0.7969	= 1.3174	6587 1292	177	1
0.8	0.6039 0.8427	= 1.4466	7233 1114	178	8
0.9	0.6778 0.8802	= 1.5580	7790 944	170	11
1.0	0.7421 0.9103	= 1.6524	8262 785	159	15
1.1	0.7969 0.9340	= 1.7309	8655 641	144	16
1.2	0.8427 0.9523	- 1.7950	8975 513	128	18
1.3	0.8802 0.9661	= 1.8463	9232 403	110	19
1.4	0.9103 0.9763	= 1.8866	9433 312	91	

1.5	0.9340 0.9838	1.9178	9589 236	76	15
1.6	0.9523 0.9891	1.9414	9707 175	61	15
1.7	0.9661 0.9928	1.9589	9795 127	48	13
1.8	0.9763 0.9953	1.9716	9858 92	35	13
1.9	0.9838 0.9970	1.9808	9904 64	28	7
2.0	0.9891 0.9981	1.9872	9936 45	19	9
2.1	0.9928 0.9989	1.9917	9959 29	16	3
2.2	0.9953 0.9993	1.9946	9973		

$$\int_{\mu_0-\mu_z} + \int_{\mu_0+\mu_z} = \frac{\sqrt{\pi}}{2}\left[2-(\Phi+\Phi)\right]$$

0.0	0.4454	2227
0.2	0.4284	2142
0.1	0.4411	2206
2.5	0.99886 0.99984	1.99873 99936s

$\mu_0 = 0.2$

	$\sigma = 0.3$				$\sigma = 0.5$			
y	σy	F	P		σy	F	P	
0	0	0.7773	0		0	0.7773	0	
0.5	0.15	0.7826	0.1184	0.0296	0.25	0.7347	0.1111	0.0278
1.0	0.3	0.6835	$\overset{3698}{0.2514}$	0.0925	0.5	0.4968	$\overset{2939}{0.1828}$	0.0735
1.5	0.45	0.5418	$\overset{5234}{0.2720}$	0.1308	0.75	0.3090	$\overset{3378}{0.1558}$	0.0844
2.0	0.6	0.4147	$\overset{4962}{0.2242}$	0.1241	1.0	0.1738	$\overset{2491}{0.0941}$	0.0623
2.5	0.75	0.3090	$\overset{3824}{0.1582}$	0.0956	1.25	0.0897	$\overset{1401}{0.0460}$	0.0350
3.0	0.9	0.2210	$\overset{2572}{0.0990}$	0.0643	1.5	0.0411	$\overset{644}{0.0184}$	0.0161
4.0	1.2	0.1025	$\overset{129}{0.0300}$	0.0645	2.0	0.0064	$\overset{203}{0.0019}$	0.0051
5.0	1.5	0.0411	$\overset{341}{0.0041}$	0.0171	2.5	0.000635	0.0001	0.0010
6.0	1.8	0.0142	0.0001	$\underline{0.0021}$ 444	3.0			38
				0.621				0.305

y	σy	F	P		P	
0	$\sigma = 0.05$		0		0	
0.5	0.025	0.7778	0.1178	0.0295	0.1178	0.0295
1.0	0.05	0.7783	$\overset{4038}{0.2860}$	0.1009	$\overset{4038}{0.2860}$	0.1009
1.5	0.075	0.7788	$\overset{6772}{0.3912}$	0.1693	$\overset{6762}{0.3902}$	0.1691
2.0	0.1	0.7793	$\overset{8132}{0.4220}$	0.2033	$\overset{8112}{0.4210}$	0.2028
2.5	0.125	0.7810	$\overset{8220}{0.4000}$	0.2055	$\overset{8192}{0.3982}$	0.2048
3.0	0.15	0.7826	$\overset{7502}{0.3502}$	0.1876	$\overset{7462}{0.3480}$	0.1866
4.0	0.2	0.7858	$\overset{5804}{0.2302}$	0.2702	$\overset{5758}{0.2278}$	0.2879
5.0	0.25	0.7347	$\overset{3538}{0.1236}$	0.1769	$\overset{3588}{0.1318}$	0.1794
6.0	0.3	0.6835	$\overset{1846}{0.0610}$	0.0923	$\overset{2004}{0.0694}$	0.1002
7.0	0.35	0.6351	$\overset{894}{0.0284}$	0.0447	$\overset{1042}{0.0348}$	0.0521
8.0	0.4	0.5867	$\overset{410}{0.0126}$	0.0205	$\overset{515}{0.0167}$	0.0257
9.0	0.45	0.5417	$\overset{180}{0.0054}$	0.0090	$\overset{245}{0.0078}$	0.0122
10.0	0.5	0.4968	$\overset{77}{0.0023}$	$\underline{0.0039}$	$\overset{113}{0.0035}$	$\underline{0.0056}$
				1.514		1.557 [illegible] 1.555

$\mu_0 = 0.2$

$\sigma = 0.025$

y	σy	F	P	
0	0	0.777_3	0	
0.5	0.012_5	0.777_3	0.1177	0.0294
1	0.025	0.777_5	$\overset{4037}{0.2838}$	0.1009
1.5	0.037_5	0.777_6	$\overset{6460}{0.390_2}$	0.169_0
2	0.05	0.777_9	$\overset{8112}{0.421_0}$	0.2028
2.5	0.062_5	0.778_2	$\overset{8200}{0.399_0}$	0.205_0
3	0.075	0.778_5	$\overset{7476}{0.3486}$	0.1869
4	0.1	0.779_5	$\overset{5768}{0.228_2}$	0.2884
5	0.125	0.781_4	$\overset{3598}{0.131_6}$	0.179_9
6	0.15	0.782_8	$\overset{2014}{0.069_8}$	0.100_7
7	0.17_5	0.784_3	$\overset{1048}{0.0350}$	0.052_4
8	0.2	0.7858	$\overset{519}{0.0169}$	0.0260
9	0.225	0.7602	$\overset{245}{0.0076}$	0.0122
10	0.25	0.7346	$\overset{109}{0.0033}$	0.0054
12	0.3	0.6835	0.0006	0.0039
				1.563

$\mu_0 = 0.5$

$\sigma = 0.05$ | $\sigma = 0.15$

y	σy	F	P		σy	F	P	
0	0	0.479_5	0		0	0.479_5	0	
0.5	0.025	0.481_6	0.0731	0.0183	0.07_5	0.483_0	0.0732	0.0183
1	0.05	0.482_2	$\overset{2503}{0.1772}$	0.0626	0.15	0.490_3	$\overset{2534}{0.1802}$	0.0633
1.5	0.07_5	0.4830	$\overset{4196}{0.242_4}$	0.1049	0.22_5	0.5020	$\overset{4322}{0.2520}$	0.108_1
2	0.1	0.4838	$\overset{5046}{0.262_2}$	0.126_1	0.3	0.517_6	$\overset{5322}{0.2802}$	0.133_0
2.5	0.125	0.487_0	$\overset{5128}{0.2498}$	0.1280	0.37_5	0.538_2	$\overset{5560}{0.275_8}$	0.139_0
3	0.15	0.490_3	$\overset{4938}{0.244_0}$	0.123_5	0.45	0.561_0	$\overset{5246}{0.251_8}$	0.1319
4	0.2	0.4968	$\overset{3894}{0.145_7}$	0.194_7	0.6	0.503_7	$\overset{3992}{0.147_4}$	0.199_6
5	0.25	0.507_2	$\overset{2309}{0.085_5}$	0.115_4	0.7_5	0.4011	$\overset{2150}{0.067_6}$	0.107_5
6	0.3	0.5176	$\overset{1317}{0.0462}$	0.065_9	0.9	0.309_7	$\overset{952}{0.0276}$	0.0476
7	0.35	0.531_0	$\overset{400}{0.0238}$	0.035_0	1.05	0.233_3	$\overset{350}{0.0104}$	0.0190
8	0.4	0.5453	$\overset{355}{0.0117}$	0.017_7	1.2	0.169_2	$\overset{140}{0.0036}$	0.0070
9	0.45	0.561_0	$\overset{143}{0.0056}$	0.008_7	1.35	0.119_8	$\overset{48}{0.0012}$	0.0024
10	0.5	0.5786_5	$\overset{82}{0.0026}$	0.004_1	1.5	0.0810	$\overset{16}{0.0004}$	$\underset{345}{0.000_8}$
12	0.6	0.5036_5	$\overset{30}{0.0004}$	$\underset{476}{0.003_0}$				
				1.008				$0.9\overset{5}{7}8$

$u_e = 0.5$

$\sigma = 0.4$

y	σy	F	y^2e^{-y}	P	
0	0	0.479_5	0	0	
0.5	0.2	0.4968	0.1516_3	0.075_3	0.018_8
1.0	0.4	0.545_3	0.367_9	0.200_4	0.068_9
1.5	0.6	0.503_{65}	0.502_0	0.252_4	0.113_2
2.0	0.8	0.368_7	0.541_4	0.199_8	0.113_1
2.5	1.0	0.256_7	0.512_9	0.131_4	0.0828
3.0	1.2	0.169_2	0.448_0	0.075_8	0.051_8
4	1.6	0.0614	0.293_1	0.0180	0.046_9
5	2.0	0.017_2	0.168_{45}	0.002_9	0.010_5
6	2.4	0.0036	0.089_{24}	0.0003	0.001_6
				0.9563:2 0.47816 +0.01895 +0.0099 0.00145 0.50755	0.508

$\sigma = 0.8$

σy	F	P	
0	0.48	0	
0.4	0.545_3	0.0827	0.0207
0.8	0.368_7	0.135_4	0.054_5
1.2	0.169_2	0.0849	0.055_1
1.6	0.0614	0.0332	0.029_5
2.0	0.017_2	0.0088	0.010_5
2.4	0.003_6	0.0016	0.002_6
			8 23 0.1797 0.174

$\sigma = 1.2$

y	σy	F	P	
0	0	0.479_5	0	
0.5	0.6	0.5036_5	0.0763	0.014_1
1	1.2	0.169_2	0.0624	0.034_7
1.5	1.8	0.0336	0.0168	0.019_8
2	2.4	0.0036	0.001_9	0.007_2
2.5	3.0	0.0002_1	0.000_1	0.0005
				0.076

$\sigma = 1.0$

σy	F	P	
0	0.479_5	0.0	
0.5	0.5787	0.0878	0.0219_5
1	0.2567	0.0944	0.0455_5
1.5	0.0810	0.040_7	0.0338
2	0.0172	0.009_3	0.0125
2.5	0.0023_5	0.001_2	0.0026
3	0.0002_1	0.000_1	0.000_6 14
			0.117

$\underline{\mu_0 = 0.5}$

$$\mathcal{F} = 1 - \frac{1}{2}\left[\Phi(\mu_0 - \mu_z) + \Phi(\mu_0 + \mu_z)\right]$$

μ_z			$\mathcal{F}$
0	0.5205 0.5205 1.0410 : 2	0.5205	0.4795
0.1	0.4284 0.6039 1.0323	0.5161₅	0.4838₅
0.2	0.3286 0.6778 1.0064	0.5032	0.4968
0.3	0.2227 0.7421 0.9648	0.4824	0.5176
0.4	0.1125 0.7969 0.9094	0.4547	0.5453
0.5	0.0000 0.8427 0.8427	0.4213₅	0.5786₅
0.6	0.1125 0.8802 0.9927	0.4963₅	0.5036₅
0.7	0.2227 0.9103 1.1330	0.5665	0.4335
0.8	0.3286 0.9340 1.2626	0.6313	0.3687
0.9	0.4284 0.9523 1.3807	0.6903₅	0.3096₅
1.0	0.5205 0.9661 1.4866	0.7433	0.2567
1.1	0.6039 0.9763 1.5802	0.7901	0.2099
1.2	0.6778 0.9838 1.6616	0.8308	0.1692
1.3	0.7421 0.9891 1.7312	0.8656	0.1344
1.4	0.7969 0.9928 1.7897	0.8948₅	0.1051₅
1.5	0.8427 0.9953 1.8380	0.9190	0.0810
1.6	0.8802 0.9970 1.8772	0.9386	0.0614
1.7	0.9103 0.9981 1.9084	0.9542	0.0458
1.8	0.9340 0.9988 1.9328	0.9664	0.0336
1.9	0.9523 0.9993 1.9516	0.9758	0.0242
2.0	0.9661 0.9996 1.9657	0.9828	0.0172
2.4	0.9928 1.0000 1.9928	0.9964	0.0036
3.0	0.99959 0.99999 1.99958	0.99979	0.00021
2.5	0.99532 0.99998 1.99530	0.99765	0.00235

$$\mu_0 = 0.2$$

$$\int\limits_{|\mu_z - \mu_0|} + \int\limits_{\mu_z + \mu_0} = \sqrt{\pi}\left[1 - \tfrac{1}{2}(\Phi' + \Phi)\right] \qquad s_{\alpha\alpha}\mathcal{F} = \tfrac{1}{\sqrt{\pi}}\int_0^\infty \overbrace{e^{-y} y^2 \mathcal{F}(\mu_z = \sigma y)\,dy}^{P} \qquad \frac{s}{s_{\alpha\alpha}}\frac{s_\alpha}{s} = \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_z}{1}$$

$\sigma = 0.1$

μ_z	[ ]	y	σy	$\mathcal{F}(\sigma y)$	y^2	e^{-y}	$e^{-y}y^2$	P
0.0	0.777_3	0	0	0.777_3	0	1	0	0
0.1	0.779_4	1	0.1	0.779_4	1	0.3679	0.3679	0.286_7
0.2	0.785_8	2	0.2	0.785_8	4	0.1353_4	0.5414	0.425_4
0.3	0.6835	3	0.3	0.683_5	9	0.04978 479	0.4480	0.306_0
0.4	0.5867	4	0.4	0.5867	16	0.01832 10992 1832	0.2931	0.172_0
0.5	0.4968	5	0.5	0.4968	25	0.006738	0.1684_5	0.083_7
0.6	0.414_7	6	0.6	0.414_7	36	0.002479 1487 7437	0.0892_4	0.037_0
0.7	0.3413	7	0.7	0.3413	49	0.0009119 456	0.0447	0.0152_3
0.8	0.2767	8	0.8	0.2767	64	0.0003355 1342 20130	0.0215	0.0059_5
0.9	0.2210	9	0.9	0.2210	81	0.00012341 98728	0.0100	0.0022
1.0	0.1738	10	1.0	0.1738	100	0.00004540	0.0045	0.0008
1.1	0.134_5	$\frac{1}{2}$	0.05	0.778	$\frac{1}{4}$	0.6065	0.1516	0.1177
1.2	0.102_5	$\frac{3}{2}$	0.15	0.782	$2\frac{1}{4}$	0.2231 4462 558	0.5020	0.392_4
1.3	0.0768	$\frac{5}{2}$	0.25	0.735	$6\frac{1}{4}$	0.08208 49248 2052	0.5129	0.376_7
1.4	0.0567	$\frac{1}{10}$	0.01	0.777_3	$\frac{1}{100}$	0.9048	0.0090_5	0.007_0
1.5	0.0411							
1.6	0.0293							
1.7	0.0205							
1.8	0.0142							

y	y^2	e^{-y}	y^2e^{-y}				
0.1	0.01	0.9048	0.0090_5				
0.2	0.04	0.8187	0.0327_5				
0.3	0.09	0.7408	0.0666_7				
0.4	0.16	0.6703	0.1072_5				
0.5	0.25	0.6065	0.1516_3				
1.0	1	0.3679	0.3679	0.1840	0.1605	+0.0235	+0.0073
1.5	2.25	0.2231	0.5020				
2.0	4	0.1353_4	0.5414	0.4546	0.4861	−0.0315	−0.0078
2.5	6.25	0.0820_8	0.5129				
3.0	9	0.0497_8	0.4480	0.4947	0.5071	−0.0124	−0.0033
4.0	16	0.01832	0.2931	0.3706	0.3700	+0.0006	−0.0038
5.0	25	0.006738	0.1684_5	0.2308	0.2270	+0.0038	
6.0	36	0.002479	0.0892_4	0.1288	0.1253	+0.0035	
7.0	49	0.0009119	0.0447	0.0670	0.0647	+0.0023	
8.0	64	0.0003355	0.0215	0.0331	0.0318	+0.0013	
9.0	81	0.0001234	0.0100	0.0154	0.0150	+0.0004	−0.0494
10.0	100	0.0000454	0.0045	0.0073	0.0070	+0.0003	+0.0360
12.0	144	0.00000614	0.0009	1.9866	0.0055	+0.0360	−0.0134
				1.9922	2.0000		

0.0379 + 0.1299 = 0.1678 0.0253 + 0.1299 = 0.1552

0.2175 0.2608 0.4783

0.2636 0.2402 0.5038

−0.0038 + −0.0055 −0.0093 +0.0125 0.0032

$$\int_{10}^{\infty} e^{-y}y^2\,dy = (100+20+2)e^{-10} = 122 \times 0.0000454 = 0.00555$$

$$\int_0^1 e^{-y}y^2\,dy = 2-(2+2+1)e^{-1} = 2-5\times 0.3679 = 2.0000 - 1.8395 = 0.1605$$

$$\int_1^2 e^{-y}y^2\,dy = 1.8395-(2+4+4)e^{-2} = \qquad 1.8395 - 1.3534 = 0.4861$$

$$\int_2^3 = 1.3534-(2+6+9)e^{-3} = \qquad 0.04978\cdot 17 \quad 1.3534 - 0.8463 = 0.5071$$

$$\int_3^4 = 0.8463-(2+8+16)\cdot e^{-4} = \qquad 0.01832\cdot 26 \quad 0.8463 - 0.4763 = 3700$$

$$\int_4^5 = 0.4763-(2+10+25)e^{-5} = \qquad 0.006738\cdot 37 \quad 0.4763 - 0.2493 = 0.2270$$

$$\int_5^6 = 0.2493-(2+12+36)e^{-6} \qquad 0.2493 - 0.1240 = 0.1253$$

$$\int_6^7 0.1240-(2+14+49)e^{-7} \qquad 0.0009119\cdot 65 \quad 0.1240 - 0.0593 = 0.0647$$

$$\int_7^8 0.0593-(2+16+64)e^{-8} \qquad 0.0003355\cdot 82 \quad 0.0593 - 0.0275 = 0.0318$$

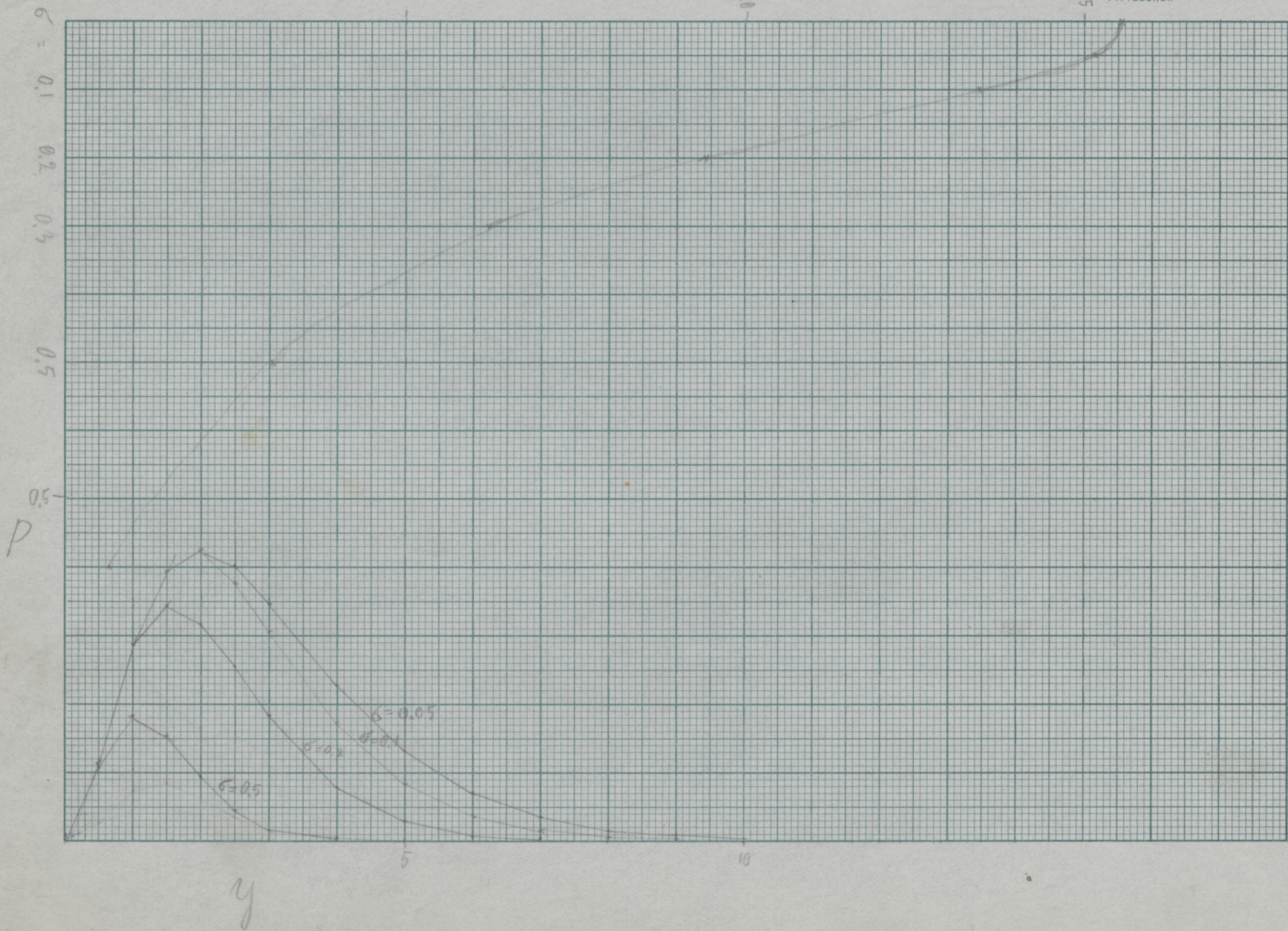
$$\int_8^9 0.0275-(2+18+81)e^{-9} \qquad 0.0001234\cdot 101 \quad 0.0275 - 0.0125 = 0.0150$$

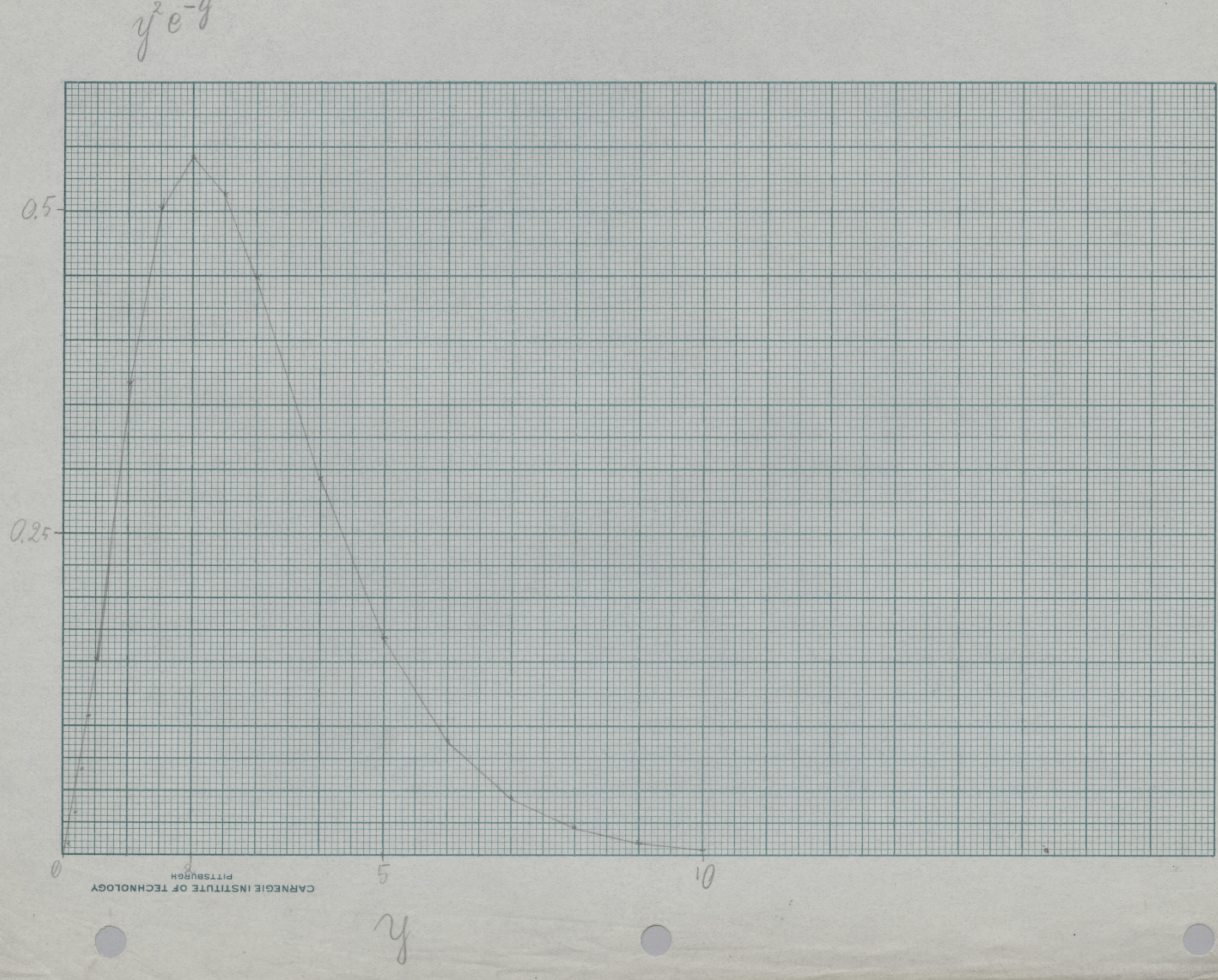
$$\int_9^{10} 0.0125-(2+20+100)e^{-10} \qquad 0.0000454\cdot 122 \quad 0.0125 - 0.0055 = 70$$

$\mu_e = 0.5$

$\sigma = 0.1$ | $\sigma = 0.2$

y	σy	F	y^2e^{-y}	P		σy	F	P	
0	0	0.479_5	0	0		0	0.479_5	0	
0.5	0.05	0.482_2	0.1516_3	0.0731	0.0183	0.1	0.4838_5	0.0734	0.0183_5
1.0	0.1	0.483_8	0.367_9	$\overset{2510}{0.1779}$	0.0628	0.2	0.496_8	$\overset{2561}{0.1827}$	0.0640
1.5	0.15	0.490_3	0.5020	$\overset{4239}{0.2460}$	0.1060	0.3	0.517_6	$\overset{4427}{0.260_0}$	0.1107
2.0	0.2	0.496_8	0.5414	$\overset{5152}{0.269_2}$	0.1288	0.4	0.545_3	$\overset{5554}{0.295_4}$	0.1388_5
2.5	0.25	0.507_2	0.5129	$\overset{5242}{0.260_0}$	0.1323	0.5	0.5786_5	$\overset{5916}{0.2962}$	0.1479
3	0.3	0.517_6	0.4480	$\overset{4918}{0.2318}$	0.1229	0.6	0.5036_5	$\overset{5216}{0.225_4}$	0.130_4
4	0.4	0.545_3	0.293_1	$\overset{3914}{0.1596}$	0.195_7	0.8	0.368_7	$\overset{3334}{0.108_0}$	0.1667
5	0.5	0.5786_5	0.168_{45}	$\overset{2540}{0.0974}$	0.128_5	1.0	0.256_7	$\overset{1513}{0.043_3}$	0.075_7
6	0.6	0.5036_5	0.089_{24}	$\overset{1424}{0.045_0}$	0.071_2	1.2	0.169_2	$\overset{584}{0.015_1}$	0.029_2
7	0.7	0.433_5	0.044_7	$\overset{644}{0.0194}$	0.032_2	1.4	0.1051_5	$\overset{198}{0.004_7}$	0.009_9
8	0.8	0.368_7	0.021_5	$\overset{274}{0.0049}$	0.013_7	1.6	0.061_4	$\overset{60}{0.001_3}$	0.0030
9	0.9	0.309_{65}	0.010_0	$\overset{110}{0.0031}$	0.005_5	1.8	0.033_6	$\overset{16}{0.000_3}$	0.0008
10	1.0	0.256_7	0004_5	$\overset{42}{0.001_2}$	0.002_1	2.0	0.017_2		
					466				366
					1.020_0				0.895_5


1.0
1.5
0.5
P
σ = 0.05
σ = 0.1
σ = 0.2
σ = 0.5
5
10
y



$y^2 e^{-y}$
0.5
0.25
0
2
5
10
y
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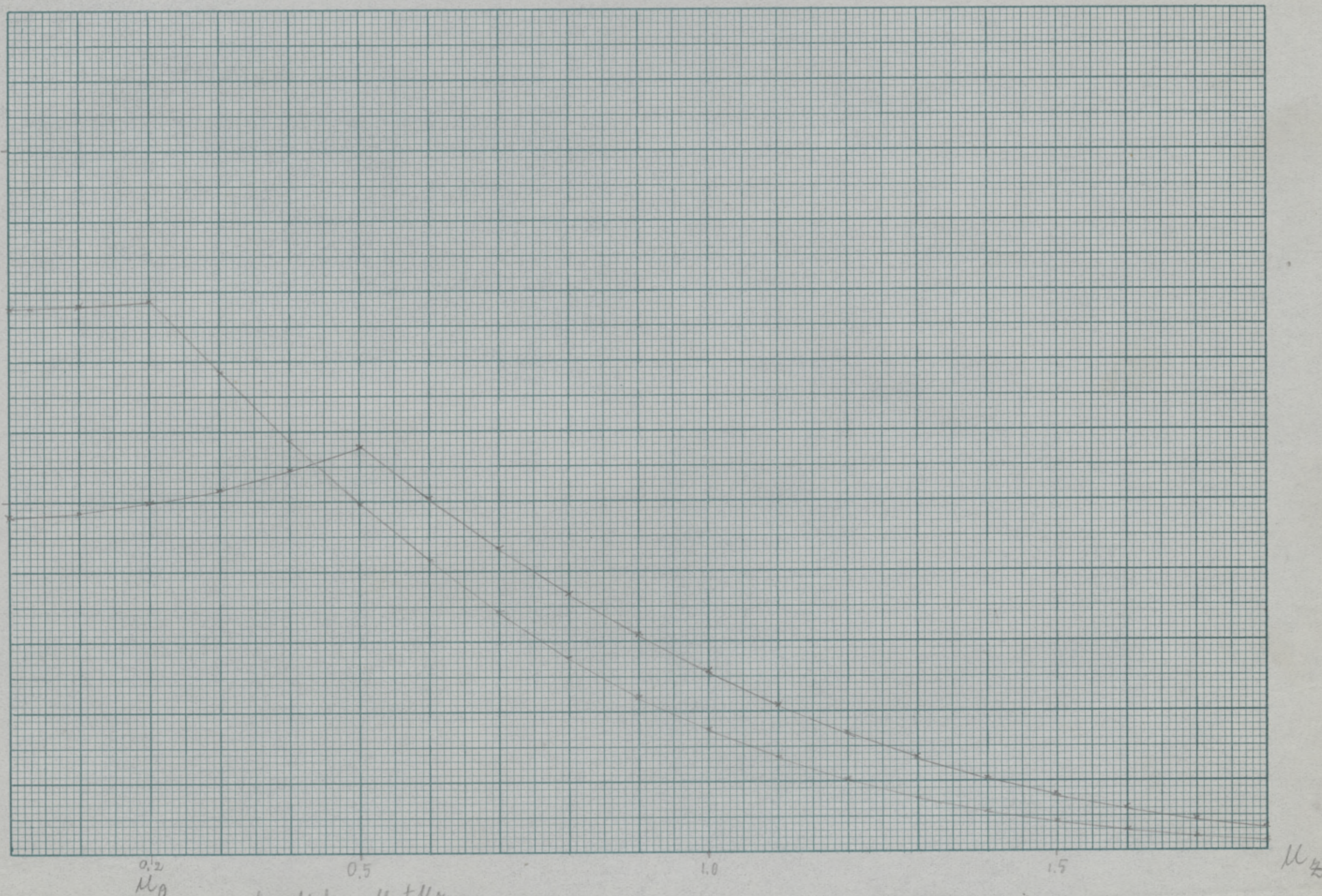
PITTSBURGH



$$F = \left[1 - \left[\frac{1}{2}\left(\int_0^{|\mu_0-\mu_z|} e^{-\xi^2}\,d\xi + \int_0^{\mu_0+\mu_z} e^{-\xi^2}\,d\xi\right)\right]\right]$$

$$\frac{\mu_0}{\mu_\alpha} = 1$$

$\sigma = 0$ $\quad \mathcal{F}_{\sigma=0} = \frac{n_0}{s_{\alpha\alpha}} \int_1^\infty e^{-x^2} dx = \frac{n_0}{s_{\alpha\alpha}} \frac{\sqrt{\pi}}{2} \left\{ 1 - \Phi(1) \right\} = \frac{n_0}{s_{\alpha\alpha}} \frac{\sqrt{\pi}}{2}\, 0.1573 = \frac{1}{4} \frac{n_0}{s_{\alpha\alpha}} \sqrt{\pi}\, 0.3146$

$\sigma = 0.2$

y	σy	$\mathcal{F}$
0	0	0.1573
0.1	0.02	0.1575
0.5	0.1	0.1615
1.0	0.2	0.1738
1.5	0.3	0.1941
2.0	0.4	0.2219
2.5	0.5	0.2567
3.0	0.6	0.2976
4.0	0.8	0.3941
5.0	1.0	0.5023
6.0	1.2	0.3896

$$\mu = \mu_0 + \mu_r^z , \quad d\mu = d\mu_r^z$$

$$dn = n_0 e^{-\frac{\mathcal{E}_r}{kT}} d\frac{\mathcal{E}_r}{kT} \qquad \int_0^\infty dn = n_0 \int_0^\infty e^{-x} dx = n_0 \left| -e^{-x} \right|_0^\infty = n_0$$

$$\mathcal{E}_r = \frac{h^2}{8\pi^2 J} n(n+1) \cong C n^2 , \quad \Delta \mathcal{E}_r = 2 C n \Delta n$$

$$dn = n_0 e^{-\frac{\mathcal{E}_r}{kT}} \frac{1}{2} \sin\vartheta \, d\vartheta \, d\frac{\mathcal{E}_r}{kT} \qquad 2\int_0^{\frac{\pi}{2}} \frac{1}{2} \sin\vartheta \, d\vartheta = \int_{\frac{\pi}{2}}^{0} d\cos\vartheta = \cos\vartheta \Big|_{\frac{\pi}{2}}^{0} = 1$$

$$\mu_r = \mu_r^\alpha \sqrt{\frac{\mathcal{E}_r}{kT}} , \quad \mu_r^z = \mu_r^\alpha \sqrt{\frac{\mathcal{E}_r}{kT}} \cos\vartheta$$

$$\frac{\mathcal{E}_r}{kT} = x^2 , \quad \cos\vartheta = y , \quad dn = -\frac{1}{2} n_0 e^{-x^2} dx^2 dy = -n_0 e^{-x^2} x \, dx \, dy$$

$$\mu_r^z = \mu_r^\alpha x y ,$$

$$\Delta n^z = -n_0 \iint_{\mu_r^z}^{\mu_r^z + \Delta\mu_r^z} e^{-x^2} x \, dx \, dy , \quad y = \frac{\mu_r^z}{\mu_r^\alpha} \frac{1}{x} , \quad dy = \frac{d\mu_r^z}{\mu_r^\alpha x} \quad (x \text{ konst.})$$

$$\Delta n^z = -n_0 \iint_{\mu_r^z}^{\mu_r^z + \Delta\mu_r^z} e^{-x^2} x \, dx \frac{d\mu_r^z}{\mu_r^\alpha x} = -\frac{n_0}{\mu_r^\alpha} \Delta\mu_r^z \int_{x = \frac{\mu_r^z}{\mu_r^\alpha}}^{x = \infty} e^{-x^2} dx$$

$$\left|\Delta n^z\right| = n_0 \left| \Delta \frac{\mu_r^z}{\mu_r^\alpha} \right| \int_{\frac{\mu_r^z}{\mu_r^\alpha}}^{\infty} e^{-x^2} dx , \qquad \int_0^\infty dy \int_y^\infty e^{-x^2} dx = \int_0^\infty dx \int_0^x e^{-x^2} dy = \int_0^\infty e^{-x^2} x \, dx$$

$$= \frac{1}{2} \left| -e^{-x^2} \right|_0^\infty = \frac{1}{2}$$

$$\int_0^\infty dy \int_y^\infty f(x\,y) \, dx = \int_0^\infty dx \int_0^x f(xy) \, dy$$

$$\mu = \mu_0 + \mu_r^{*}, \quad \Delta n = n_0 \Delta \frac{\mu_*}{\mu_\alpha} \int_{\frac{\mu_*}{\mu_\alpha}}^{\infty} e^{-x^2} dx = n_0 \Delta x \int_x^{\infty} e^{-\xi^2} d\xi$$

$$\frac{\mu_*}{\mu_\alpha} = \sqrt{\frac{\mathcal{E}_r}{kT}} \cos\vartheta = x = \sqrt{\frac{\mathcal{E}_r}{kT}} \text{ für } \vartheta = 0, \cos\vartheta = 1.$$

$$dn = \Delta n\, e^{-y} y\, dy, \quad y = \frac{v^2}{\alpha^2} = \frac{s_\alpha}{s}, \quad dy = -\frac{s_\alpha}{s^2} ds = -y^2 \frac{ds}{s_\alpha}$$

$$\frac{dn}{ds} = -\frac{\Delta n}{s_\alpha} e^{-y} y^3 = \Delta \mathcal{J} = \frac{e^{-y} y^3}{s_\alpha} n_0 \Delta x \int_x^{\infty} e^{-\xi^2} d\xi$$

$$\underline{\mu_0 = 0}$$

$$\mu = \mu_r^{*}, \quad \Delta n = n_0 \Delta x \int_x^{\infty} e^{-\xi^2} d\xi, \quad \Delta \mathcal{J} = \frac{e^{-y} y^3}{s_\alpha} n_0 \Delta x \int_x^{\infty} e^{-\xi^2} d\xi$$

$$x = \frac{\mu_r^{*}}{\mu_\alpha}, \quad s = \mathcal{C} \frac{\mu_r^{*}}{\mu_\alpha} \frac{\alpha^2}{v^2} = \mathcal{C} \frac{x}{y}, \quad s_{\alpha} = \mathcal{C}, \quad s_\alpha = \mathcal{C} x = s_{\alpha\alpha} X, \quad y = \frac{s_\alpha}{s} = \frac{s_{\alpha\alpha} X}{s}, \quad x = \frac{s}{s_{\alpha\alpha}} y$$

$$\Delta y = \frac{s_{\alpha\alpha}}{s} \Delta x$$

$$\Delta \mathcal{J} = \frac{n_0}{s} \frac{e^{-\frac{s_{\alpha\alpha} X}{s}} \left(\frac{s_{\alpha\alpha} X}{s}\right)^3}{\frac{s_{\alpha\alpha} X}{s}} \Delta x \int_x^{\infty} e^{-\xi^2} d\xi = \frac{n_0}{s} e^{-y} y^2 \frac{s}{s_{\alpha\alpha}} dy \int_{x = \frac{s}{s_{\alpha\alpha}} y}^{\infty} e^{-\xi^2} d\xi$$

$$\mathcal{J} = \frac{n_0}{s_{\alpha\alpha}} \int_0^{\infty} e^{-y} y^2 dy \int_{\frac{s}{s_{\alpha\alpha}} y}^{\infty} e^{-\xi^2} d\xi$$

$$\mu = \mu_0 + \mu_x^*$$

$$dJ = \frac{e^{-y} y^3}{s_\alpha} n_0 \, dx \int_x^\infty e^{-\xi^2} d\xi$$

$$x = \frac{\mu_x}{\mu_\alpha} \quad s = C \frac{\mu_0 + \mu_x}{\mu_0 + \mu_\alpha} \frac{\alpha^2}{v^2} = C\left(\frac{\mu_0}{\mu_\alpha} + x\right)\frac{1}{y}, \quad s_{\alpha\alpha} = C\left(\frac{\mu_0}{\mu_\alpha} + 1\right), \quad s = s_{\alpha\alpha} \frac{\frac{\mu_0}{\mu_\alpha} + x}{\frac{\mu_0}{\mu_\alpha} + 1} \frac{1}{y}$$

$$s_\alpha = s_{\alpha\alpha} \frac{\frac{\mu_0}{\mu_\alpha} + x}{\frac{\mu_0}{\mu_\alpha} + 1} \quad y = \frac{s_\alpha}{s} \quad dJ = \frac{n_0}{s} e^{-y} y^2 \, dx \int_x^\infty e^{-\xi^2} d\xi$$

$$y = \frac{s_{\alpha\alpha}}{s} \frac{\frac{\mu_0}{\mu_\alpha} + x}{\frac{\mu_0}{\mu_\alpha} + 1}, \quad \frac{\mu_0}{\mu_\alpha} + x = \frac{s}{s_{\alpha\alpha}} y \left(\frac{\mu_0}{\mu_\alpha} + 1\right), \quad dx = \frac{s}{s_{\alpha\alpha}}\left(\frac{\mu_0}{\mu_\alpha} + 1\right) dy$$

$$J = \frac{n_0}{s_{\alpha\alpha}}\left(\frac{\mu_0}{\mu_\alpha} + 1\right) \int_0^\infty e^{-y} y^2 \, dy \int_{x = \frac{s}{s_{\alpha\alpha}} y \left(\frac{\mu_0}{\mu_\alpha} + 1\right) - \frac{\mu_0}{\mu_\alpha}}^\infty e^{-\xi^2} d\xi$$

Vereinfachung

$$dn = n_0\, dx \int_x^\infty e^{-\xi^2} d\xi \,, \quad \int_0 dn = n_0 \int_0 dx \int_x^\infty e^{-\xi^2} d\xi = \frac{1}{2} n_0 \qquad x = \frac{\mu_\mathfrak{z}}{\mu_\alpha}$$

Mittleres Moment: $$\overline{\mu}_\mathfrak{z} = \frac{\int \mu_\mathfrak{z}\, dn}{\int dn} = \mu_\alpha \frac{\int x\, dn}{\frac{1}{2} n_0} = 2\mu_\alpha \int_0^\infty x\, dx \int_x^\infty e^{-\xi^2} d\xi = 2\mu_\alpha I = \mu_\alpha \frac{\sqrt{\pi}}{4}$$

$$I = \int_0^\infty e^{-\xi^2} d\xi \int_0^\xi x\, dx = \frac{1}{2}\int_0^\infty e^{-\xi^2} \xi^2 d\xi = -\frac{1}{4}\int_0^\infty \xi\, d e^{-\xi^2} = -\frac{1}{4}\xi e^{-\xi^2}\Big|_0^\infty + \frac{1}{4}\int_0^\infty e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{8}$$

$$\overline{\mu}^\mathfrak{z}_{\varepsilon_r} = \mu_{\varepsilon_r} \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin\vartheta \cos\vartheta\, d\vartheta = \frac{1}{2}\mu_{\varepsilon_r} \int_0 \sin\vartheta\, d\sin\vartheta = \frac{1}{2}\sin^2\vartheta \Big|_0^{\frac{\pi}{2}} = \frac{1}{2}\left|\mu_{\varepsilon_r}\right|$$

$$\mu_{\varepsilon_r} = \mu_\alpha \sqrt{\frac{\varepsilon_r}{kT}} \qquad \overline{\mu}_{\varepsilon_r} = \mu_\alpha \int_0^\infty e^{-\frac{\varepsilon_r}{kT}} \sqrt{\frac{\varepsilon_r}{kT}}\, d\frac{\varepsilon_r}{kT} = \mu_\alpha 2 \int_0 e^{-x^2} x^2 dx = \mu_\alpha 2 I = \frac{\sqrt{\pi}}{2}\mu_\alpha$$

$$\overline{\mu}_\mathfrak{z} = \frac{1}{2}\left|\overline{\mu}_{\varepsilon_r}\right| = \mu_\alpha \frac{\sqrt{\pi}}{4}$$

$$\int_x^\infty e^{-\xi^2} d\xi = \int_0^\infty e^{-\xi^2} d\xi - \int_0^x e^{-\xi^2} d\xi \cong \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi\right) = \frac{\sqrt{\pi}}{2}(1-\Phi) \cong \frac{\sqrt{\pi}}{2}(1 - cx)$$

$$dn = n_0\, dx \frac{\sqrt{\pi}}{2}(1 - cx) \quad \frac{1}{2} n_0 = n_0 \frac{\sqrt{\pi}}{2}\left[\int_0^{x_0} dx - c\int_0^{x_0} x\, dx\right] = n_0 \sqrt{\pi}\left[x_0 - \frac{c}{2}x_0^2\right]$$

$$x_0 - \frac{c}{2}x_0^2 = \frac{1}{\sqrt{\pi}} \,, \; c x_0 = 1 \,, \; x_0 - \frac{1}{2}x_0 = \frac{1}{\sqrt{\pi}} = \frac{1}{2}x_0 \,, \; x_0 = \frac{2}{\sqrt{\pi}} \,, \; c = \frac{\sqrt{\pi}}{2} \,, \; \int_x^\infty e^{-\xi^2} d\xi \doteq \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right)$$

$$\Phi \cong cx = \frac{\sqrt{\pi}}{2}x = 0.886\,227\, x$$

$$\frac{2}{\sqrt{\pi}} = 1.12838$$

$x =$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.1284
$\frac{\sqrt{\pi}}{2}x =$	0	0.089	0.177	0.266	0.354	0.443	0.532	0.620	0.709	0.798	0.886	0.975	1.000
$\Phi =$	0	0.113	0.223	0.329	0.428	0.521	0.604	0.678	0.742	0.797	0.843	0.880	0.890
$\frac{2}{\sqrt{\pi}}x =$	0	0.113	0.226	0.339	0.451	0.564							

$$\Phi = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi \,, \quad \int_x^\infty e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi\right) \cong \frac{\sqrt{\pi}}{2}(1 - 1.128\,38\, x) \text{ für } x \lesssim 1$$

$x = 0.3 \quad 0.3385$

$$dn = n_0\, d\frac{\mu_x}{\mu_\alpha}\int_{\frac{\mu_x}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi = n_0\, dx \int_x^\infty e^{-\xi^2} d\xi, \quad n_0\int_0^\infty dx \int_x^\infty e^{-\xi^2} d\xi = \frac{1}{2} n_0 \qquad x = \frac{\mu_x}{\mu_\alpha}$$

$$dn = n_0\, dx\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right), \quad n_0 \int_0^{\frac{2}{\sqrt{\pi}}} dx\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right) = n_0 \frac{\sqrt{\pi}}{2}\left[\frac{2}{\sqrt{\pi}} - \frac{\sqrt{\pi}}{2}\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\right)^2\right] = n_0\left[1 - \frac{1}{2}\right] = \frac{1}{2} n_0$$

$$\frac{\overline{\mu_x}}{\mu_\alpha} = \frac{\int x\, dn}{\int dn} = 2\int_0^{\frac{2}{\sqrt{\pi}}} x\, dx\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right) = \sqrt{\pi}\left[\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\right)^2 - \frac{\sqrt{\pi}}{2}\frac{1}{3}\left(\frac{2}{\sqrt{\pi}}\right)^3\right] = \sqrt{\pi}\left[\frac{2}{\pi} - \frac{1}{3}\frac{4}{\pi}\right] = \frac{2}{3\sqrt{\pi}}$$

$$\frac{\sqrt{\pi}}{4} = 0.443114 \qquad \frac{2}{3\sqrt{\pi}} = 0.37613$$

$$s = s_{\alpha\alpha}\frac{x}{y}, \quad y = \frac{s_{\alpha\alpha}}{s}x, \quad y_0 = \frac{1}{6}\frac{2}{\sqrt{\pi}}$$

$$\mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\int_0^{\frac{1}{6}\frac{2}{\sqrt{\pi}}} e^{-y} y^2\, dy\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\frac{s}{s_{\alpha\alpha}}y\right) = \frac{n_0}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2}\Big[\left[2 - (2 + 2y_0 + y_0^2)e^{-y_0}\right] -$$
$$- \frac{1}{y_0}\left[6 - (6 + 6y_0 + 3y_0^2 + y_0^3)e^{-y_0}\right]\Big]$$

$$\mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2}\left\{2 - \frac{6}{y_0} + \left(\frac{6}{y_0} + 4 + y_0\right)e^{-y_0}\right\}, \quad y_0 = \frac{s_{\alpha\alpha}}{s}\frac{2}{\sqrt{\pi}}$$

$$y_0 = 1, \quad \frac{s}{s_{\alpha\alpha}} = \frac{2}{\sqrt{\pi}} = 1.1284, \quad \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}} 0.8862\left\{-4 + 11 \times \underset{4.047}{\overset{3.679}{0.3679}}\right\} = \frac{n_0}{s_{\alpha\alpha}} 0.0416$$

$$y_0 \ll 1 \quad \sigma \gg 1, \quad \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\left\{2 - \frac{6}{y_0} + \left(\frac{6}{y_0} + 4 + y_0\right)\left(1 - y_0 + \frac{1}{2}y_0^2 - \frac{1}{6}y_0^3 + \frac{1}{24}y_0^4\right)\right\}$$

$$-6 - 4y_0 - y_0^2$$
$$+3y_0 + 2y_0^2 + \tfrac{1}{2}y_0^3 \qquad \tfrac{6}{12}$$
$$-y_0^2 - \tfrac{4}{6}y_0^3 \qquad \tfrac{8}{12}$$
$$+\tfrac{1}{4}y_0^3 \qquad \tfrac{3}{12}$$

$$\mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\left\{2 - \frac{6}{y_0} + \frac{6}{y_0} - 2 + \frac{1}{12}y_0^3\right\} = \frac{n_0}{s_{\alpha\alpha}}\frac{1}{12}y_0^3$$

3.42
2.736
0.0342
1.026
34.2 (1-0.1813) 62 0.046

$$\frac{s}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2} = 5, \quad y_0 = \frac{1}{5} = 0.2, \quad \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\overset{0.00067}{\frac{1}{1500}} \text{ [illegible] } \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}} 1.1284\left\{2 - 30 + (30 + 4 + 0.2)\,0.8187\right\} = \frac{n_0}{s_{\alpha\alpha}} 0.00051$$

$$\frac{s}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2} = 2, \quad y_0 = \frac{1}{2} = 0.5 \quad \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\overset{0.0102}{\frac{1}{96}}, \quad \mathcal{J} \qquad \left\{2 - 12 + (\overset{16.5}{12 + 4 + 0.5})\,0.6065\right\}$$

6.0650
3.6390
0.30325
10.00725
− 10.00000

$$\mathcal{J} = \frac{n_0}{s_{\alpha\alpha}} 0.0082$$

$$\frac{s}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2} = \frac{1}{6} \quad y_0 = 6 \qquad \mathcal{J} = \frac{n_0}{s_{\alpha\alpha}}\left\{2 - 1 + (\overset{11}{1 + 4 + 6})\underset{0.0248}{0.00248}\right\}$$

$$1 \quad + 0.0273$$

$dn = n_0' e^{-y} y\,dy$, $y = \frac{s_\alpha}{s}$, $dy = -\frac{s_\alpha}{s^2}\,ds$, $|ds| = \frac{s^2}{s_\alpha}\,dy = \frac{s_\alpha}{y^2}\,dy$

$\sigma = \frac{s}{s_{\alpha\alpha}}$

$dJ = \frac{dn}{ds} = \frac{n_0'}{s_\alpha} e^{-y} y^3$, $n_0' = n_0\, d\frac{\mu_z}{\mu_\alpha} \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2}dx$, $\frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_z}{\mu_\alpha}$, $s_\alpha = ys$, $\frac{\mu_z}{\mu_\alpha} = \frac{s}{s_{\alpha\alpha}}\, y = \sigma y$

$d\frac{\mu_z}{\mu_\alpha} = \sigma\, dy$, $n_0' = n_0 \sigma\, dy \int_{\sigma y}^{\infty} e^{-x^2}dx$ $\quad dJ = n_0 \frac{\sigma\, dy}{s\, y} e^{-y} y^3 \int_{\sigma y}^{\infty} e^{-x^2}dx$

$y = \frac{s_\alpha}{s} = \frac{s_{\alpha\alpha}\frac{\mu_z}{\mu_\alpha}}{s}$

$dJ = \frac{n_0}{s_{\alpha\alpha}} e^{-y} y^2\, dy \int_{\sigma y}^{\infty} e^{-x^2}dx$, $J = \frac{n_0}{s_{\alpha\alpha}} \int_0^{\infty} e^{-y} y^2\, dy \int_{\sigma y}^{\infty} e^{-x^2}dx$

$y_0 = \frac{1}{\sigma}\frac{2}{\sqrt{\pi}}$, $\frac{1}{y_0} = \frac{\sqrt{\pi}}{2}\sigma$

Annäh: $\int_{\sigma y}^{\infty} e^{-x^2}dx = \int_0^{\infty} e^{-x^2}dx - \int_0^{\sigma y} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}}\int_0^{\sigma y} e^{-x^2}dx\right) = \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\sigma y\right)$ ~ $y = 0$ bei $\sigma y_0 = \frac{2}{\sqrt{\pi}}$

$J = \frac{n_0}{s_{\alpha\alpha}} \int_0^{\frac{1}{\sigma}\frac{2}{\sqrt{\pi}}} e^{-y} y^2\, dy\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\sigma y\right) = \frac{n_0}{s_{\alpha\alpha}} \frac{\sqrt{\pi}}{2}\left\{2 - \frac{6}{y_0} + \left(\frac{6}{y_0} + 4 + y_0\right) e^{-y_0}\right\}$

$\sigma \ll 1$ $\quad y_0 \gg 1$ $\quad J = \frac{n_0 \sqrt{\pi}}{s_{\alpha\alpha}\, 2}\left\{2 - \frac{6}{y_0}\right\} = \frac{n_0}{s_{\alpha\alpha}}\left\{\sqrt{\pi} - \frac{3}{2}\pi\sigma\right\}$, $\sigma \gg 1$, $y_0 \ll 1$, $J = \frac{n_0\sqrt{\pi}}{s_{\alpha\alpha}\, 2}\frac{1}{12} y_0^3 = \frac{n_0}{s_{\alpha\alpha}} \frac{1}{3\pi} \frac{1}{\sigma^3}$

$\mu = \mu_0 + \mu_z$, $n_0' = n_0\, d\frac{\mu_z}{\mu_\alpha} \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2}dx$, $\frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_0 + \mu_z}{\mu_0 + \mu_\alpha} = \frac{\mu_0}{\mu_0 + \mu_\alpha} + \frac{\mu_z}{\mu_0 + \mu_\alpha} = \frac{y s}{s_{\alpha\alpha}}$

$\frac{d\mu_z}{\mu_0 + \mu_\alpha} = \frac{s}{s_{\alpha\alpha}}\, dy$, $\frac{\mu_\alpha}{\mu_0 + \mu_\alpha}\, d\frac{\mu_z}{\mu_\alpha} = \sigma\, dy$, $d\frac{\mu_z}{\mu_\alpha} = \frac{\mu_0 + \mu_\alpha}{\mu_\alpha}\sigma\, dy = \left(1 + \frac{\mu_0}{\mu_\alpha}\right)\sigma\, dy$

$dJ = \frac{n_0'}{s_{\alpha\alpha}} e^{-y} y^3 = \frac{n_0\left(1 + \frac{\mu_0}{\mu_\alpha}\right)\frac{s}{s_{\alpha\alpha}}\, dy}{s\, y} e^{-y} y^3 \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2}dx$, $\quad \frac{\mu_z}{\mu_0 + \mu_\alpha}\frac{\mu_\alpha}{\mu_\alpha} = \sigma y - \frac{\mu_0}{\mu_0 + \mu_\alpha}$

$\frac{\mu_z}{\mu_\alpha} = \left(1 + \frac{\mu_0}{\mu_\alpha}\right)\left(\sigma y - \frac{\mu_0}{\mu_0 + \mu_\alpha}\right)$
$= \left(1 + \frac{\mu_0}{\mu_\alpha}\right)\sigma y - \frac{\mu_0}{\mu_\alpha}$

$dJ = \frac{n_0}{s_{\alpha\alpha}}\left(1 + \frac{\mu_0}{\mu_\alpha}\right) e^{-y} y^2\, dy \int_{\left(1 + \frac{\mu_0}{\mu_\alpha}\right)\sigma y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2}dx$

$y = \frac{s_\alpha}{s} = \frac{s_{\alpha\alpha}}{s}\frac{\mu_0 + \mu_z}{\mu_0 + \mu_\alpha} = \frac{1}{\sigma}\frac{\frac{\mu_0}{\mu_\alpha} + \frac{\mu_z}{\mu_\alpha}}{\frac{\mu_0}{\mu_\alpha} + 1}$

$J = \frac{n_0}{s_{\alpha\alpha}}\left(1 + \frac{\mu_0}{\mu_\alpha}\right) \int_{\frac{1}{\sigma}\frac{\mu_0}{\mu_0 + \mu_\alpha}}^{\infty} e^{-y} y^2\, dy \int_{\left(1 + \frac{\mu_0}{\mu_\alpha}\right)\sigma y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2}dx$

Näherung

$$\mu=\mu_0+\mu_z\,,\quad dn = n_0'\, d\frac{\mu_z}{\mu_\alpha}\int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2}dx = n_0'\, d\frac{\mu_z}{\mu_\alpha}\,\frac{\sqrt\pi}{2}\left(1-\frac{\sqrt\pi}{2}\frac{\mu_z}{\mu_\alpha}\right)$$

größte μ_z zugehörige Wert ist $\frac{\mu_z^m}{\mu_\alpha}=\frac{2}{\sqrt\pi}$, $\mu_z^m=\frac{2}{\sqrt\pi}\mu_\alpha$

$$dJ = \frac{e^{-y}y^3}{s_\alpha}dn = \frac{e^{-y}y^3}{s_\alpha}\, n_0'\, d\frac{\mu_z}{\mu_\alpha}\frac{\sqrt\pi}{2}\left(1-\frac{\sqrt\pi}{2}\frac{\mu_z}{\mu_\alpha}\right),\quad y=\frac{s_\alpha}{s},\ s_\alpha = ys$$

$$dJ = \frac{n_0'}{s}e^{-y}y^2\frac{\sqrt\pi}{2}\left(1-\frac{\sqrt\pi}{2}\frac{\mu_z}{\mu_\alpha}\right)d\frac{\mu_z}{\mu_\alpha}\,,\quad \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{s}{s_{\alpha\alpha}}y = \frac{\mu_0+\mu_z}{\mu_0+\mu_\alpha}=\sigma y = \frac{\mu_0}{\mu_0+\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}\frac{\mu_\alpha}{\mu_0+\mu_\alpha}$$

$$\mu_z=0\,,\ \sigma y = \frac{\mu_0}{\mu_0+\mu_\alpha}\,,\ \mu_z^m=\frac{2}{\sqrt\pi}\mu_\alpha\,,\ \sigma y = \frac{\mu_0+\frac{2}{\sqrt\pi}\mu_\alpha}{\mu_0+\mu_\alpha}\,,\ \frac{\mu_z}{\mu_\alpha}=\frac{\mu_0+\mu_\alpha}{\mu_\alpha}\left(\sigma y - \frac{\mu_0}{\mu_0+\mu_\alpha}\right)$$

$$\frac{\mu_z}{\mu_\alpha} = \frac{\mu_0+\mu_\alpha}{\mu_\alpha}\sigma y - \frac{\mu_0}{\mu_\alpha} = \left(1+\frac{\mu_0}{\mu_\alpha}\right)\sigma y - \frac{\mu_0}{\mu_\alpha} = \sigma y + \frac{\mu_0}{\mu_\alpha}(\sigma y-1)$$

$$d\frac{\mu_z}{\mu_\alpha} = \left(1+\frac{\mu_0}{\mu_\alpha}\right)\sigma\, dy$$

$$dJ = \frac{n_0'}{s}e^{-y}y^2\frac{\sqrt\pi}{2}\left[1-\frac{\sqrt\pi}{2}\left(1+\frac{\mu_0}{\mu_\alpha}\right)\sigma y+\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}\right]\left(1+\frac{\mu_0}{\mu_\alpha}\right)\frac{s}{s_{\alpha\alpha}}dy$$

$$dJ = \frac{n_0'}{s_{\alpha\alpha}}\frac{\sqrt\pi}{2}\left(1+\frac{\mu_0}{\mu_\alpha}\right)e^{-y}y^2\left[1+\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}-\frac{\sqrt\pi}{2}\left(1+\frac{\mu_0}{\mu_\alpha}\right)\sigma y\right]dy$$

$$J = \frac{n_0'}{s_{\alpha\alpha}}\frac{\sqrt\pi}{2}\left(1+\frac{\mu_0}{\mu_\alpha}\right)\int_{\frac{1}{\sigma(1+\frac{\mu_\alpha}{\mu_0})}}^{\frac{1}{\sigma}\frac{1+\frac{2}{\sqrt\pi}\frac{\mu_\alpha}{\mu_0}}{1+\frac{\mu_\alpha}{\mu_0}}} e^{-y}y^2[\;]dy \qquad \frac{s}{s_{\alpha0}} = \frac{s}{s_\alpha}\frac{s_\alpha}{s_{\alpha0}} = \frac{s}{s_\alpha}\left(1+\frac{\mu_z}{\mu_0}\right) = \frac{s}{s_\alpha}\sigma y$$

$$y\sigma = \frac{s_\alpha}{s_{\alpha0}}\,,\ y=\frac{s_\alpha}{s}$$

$$\mu=\mu_0+\mu_z\,,\ \frac{s_\alpha}{s_{\alpha0}} = \frac{\mu_0+\mu_z}{\mu_0} = \sigma y = 1+\frac{\mu_z}{\mu_0} = 1+\frac{\mu_\alpha}{\mu_0}\frac{\mu_z}{\mu_\alpha}\,,\ \frac{\mu_z}{\mu_\alpha}=\frac{\mu_0}{\mu_\alpha}(\sigma y-1)\,,\ d\frac{\mu_z}{\mu_\alpha}=\frac{\mu_0}{\mu_\alpha}\sigma\, dy$$

$$\mu_z=0\,,\ \sigma y=1\,,\ \mu_z=\frac{2}{\sqrt\pi}\mu_\alpha\,,\ \sigma y = 1+\frac{2}{\sqrt\pi}\frac{\mu_\alpha}{\mu_0} \qquad \sigma=\frac{s}{s_{\alpha0}}$$

$$dJ = \frac{n_0'}{s_{\alpha0}}\frac{\sqrt\pi}{2}e^{-y}y^2\left[1-\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}\sigma y+\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}\right]\frac{\mu_0}{\mu_\alpha}dy \qquad \mu_z^m = \frac{2}{\sqrt\pi}\mu_\alpha = \mu_0\,,\ \frac{2}{\sqrt\pi}\frac{\mu_\alpha}{\mu_0}=1=\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}$$

$$J = \frac{n_0'}{s_{\alpha0}}\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}\int_{\frac{1}{\sigma}}^{\frac{1}{\sigma}\left(1+\frac{2}{\sqrt\pi}\frac{\mu_\alpha}{\mu_0}\right)} e^{-y}y^2\left[1+\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}-\frac{\sqrt\pi}{2}\frac{\mu_0}{\mu_\alpha}\sigma y\right]dy \qquad J=\frac{n_0'}{s_{\alpha0}}\int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}}e^{-y}y^2[2-\sigma y]\,dy$$

[illegible] $$d\mathcal{J} = \frac{dn}{s_\alpha} e^{-y} y^3 = \frac{e^{-y}y^3}{s_\alpha} n_0' \frac{d\mu_m}{\mu_\alpha} \int_{\frac{\mu_x}{\mu_\alpha}}^{\infty} e^{-x^2}dx \,, \quad y = \frac{s_\alpha}{s} \,, \quad s_\alpha = ys$$

$$= e^{-y} y^3 \frac{1}{ys} n_0' \frac{\mu_0}{\mu_\alpha} \frac{s}{s_{\alpha 0}} dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y - 1)}^{\infty} e^{-x^2} dx$$

$$\mathcal{J} = \frac{n_0'}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y - 1)}^{\infty} e^{-x^2} dx$$

$$\mu = \mu_0 - \mu_x \,, \quad \frac{s_\alpha}{s_{\alpha 0}} = \frac{\mu_0 - \mu_x}{\mu_0} = \sigma y = 1 - \frac{\mu_x}{\mu_0} = 1 - \frac{\mu_\alpha}{\mu_0}\frac{\mu_x}{\mu_\alpha} \,, \quad \frac{\mu_x}{\mu_\alpha} = \frac{\mu_0}{\mu_\alpha}(1-\sigma y) \,, \quad d\frac{\mu_x}{\mu_\alpha} = -\frac{\mu_0}{\mu_\alpha}\sigma dy$$

$$\mu_x = 0 \,, \quad \sigma y = 1 \,, \quad y = \frac{1}{\sigma} \,, \quad \mu_x^m = \frac{2}{\sqrt{\pi}}\mu_\alpha \,, \quad \sigma y = 1 - \frac{\mu_\alpha}{\mu_0}\frac{2}{\sqrt{\pi}} = 0 \quad \frac{\mu_\alpha}{\mu_0} = \frac{\sqrt{\pi}}{2}$$

$$dn = n_0' d\frac{\mu_x}{\mu_\alpha} \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\frac{\mu_x}{\mu_\alpha}\right) = -n_0' \cancelto{1}{\frac{\sqrt{\pi}}{2}\frac{\mu_0}{\mu_\alpha}} \frac{s}{s_{\alpha 0}} dy \left[1 - \cancelto{1}{\frac{\sqrt{\pi}}{2}\frac{\mu_0}{\mu_\alpha}}(1-\sigma y)\right]$$

$$d\mathcal{J} = \frac{e^{-y}y^3}{s_\alpha} dn = -\frac{e^{-y}y^3}{\cancel{s}\cancel{y}} n_0' \frac{\cancel{s}}{s_{\alpha 0}} dy\, \sigma \cancel{y} = -\frac{n_0'}{s_{\alpha 0}} \sigma e^{-y} y^3 dy$$

$$\mathcal{J}_2 = \frac{n_0'}{s_{\alpha 0}} \sigma \int_0^{\frac{1}{\sigma}} e^{-y} y^3 dy \qquad \mathcal{J}_1 = \frac{n_0'}{s_{\alpha 0}} 2 \int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^2 dy - \frac{n_0'}{s_{\alpha 0}} \sigma \int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^3 dy$$

$$\mathcal{J}_2 + \mathcal{J}_1 = \frac{n_0'}{s_{\alpha 0}} \left[2\int_0^{\frac{2}{\sigma}} e^{-y} y^2 dy - 2\int_0^{\frac{1}{\sigma}} e^{-y} y^2 dy - \sigma \int_0^{\frac{2}{\sigma}} e^{-y} y^3 dy + 2\sigma \int_0^{\frac{1}{\sigma}} e^{-y} y^3 dy \right]$$

$$= \frac{n_0'}{s_{\alpha 0}} \left[2\int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^2 dy - \sigma\left(\int_0^{\frac{2}{\sigma}} e^{-y} y^3 dy - 2\int_0^{\frac{1}{\sigma}} e^{-y} y^3 dy \right) \right] \qquad \frac{1}{\sigma} = y_1 \quad \frac{2}{\sigma} = y_2$$

$$2 - (2 + 2y_2 + y_2^2)e^{-y_2} \qquad 6 - (6 + 6y_2 + 3y_2^2 + y_2^3)e^{-y_2}$$

$$-2 + (2 + 2y_1 + y_1^2)e^{-y_1}$$

$$\gamma_1 + \gamma_2 = \frac{n_0'}{s_{20}}[\,]$$

$$[\,] = 2\left(2+\frac{2}{\sigma}+\frac{1}{\sigma^2}\right)e^{-\frac{1}{\sigma}} - 2\left(2+\frac{4}{\sigma}+\frac{4}{\sigma^2}\right)e^{-\frac{2}{\sigma}} - \sigma\left[6-\left(6+\frac{12}{\sigma}+\frac{12}{\sigma^2}+\frac{8}{\sigma^3}\right)e^{-\frac{2}{\sigma}} - 12 + 2\left(6+\frac{6}{\sigma}+\frac{3}{\sigma^2}+\frac{1}{\sigma^3}\right)e^{-\frac{1}{\sigma}}\right]$$

$$\sigma \ll 1,\ \frac{1}{\sigma} \gg 1,\ \gamma_1+\gamma_2 = \frac{n_0'}{s_{20}}\left[0+0-\sigma[6-0-12+0]\right] = \frac{n_0'}{s_{20}}\, 6\, \frac{s}{s_{20}}$$

$$\sigma \gg 1,\ e^{-\frac{1}{\sigma}} = 1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}+\frac{1}{24}\frac{1}{\sigma^4} \qquad e^{-\frac{2}{\sigma}} = 1-\frac{2}{\sigma}+\frac{1}{2}\frac{4}{\sigma^2}-\frac{1}{6}\frac{8}{\sigma^3}+\frac{1}{24}\frac{16}{\sigma^4}$$

$$2\left(2+\frac{2}{\sigma}+\frac{1}{\sigma^2}\right)\left(1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}\right)$$
$$-\frac{2}{\sigma}-\frac{2}{\sigma^2}-\frac{1}{\sigma^3}$$
$$+\frac{1}{\sigma^2}+\frac{1}{\sigma^3} \qquad 2\left(-\frac{1}{3}+\frac{8}{3}\right)\frac{1}{\sigma^3} = +\frac{14}{3}\frac{1}{\sigma^3}$$
$$-\frac{1}{3}\frac{1}{\sigma^3}$$

$$\left(2+\frac{4}{\sigma}+\frac{4}{\sigma^2}\right)\left(1-\frac{2}{\sigma}+\frac{2}{\sigma^2}-\frac{4}{3}\frac{1}{\sigma^3}\right)\ \frac{2}{3}\frac{1}{\sigma^4}$$
$$-\frac{4}{\sigma}-\frac{8}{\sigma^2}-\frac{8}{\sigma^3}$$
$$+\frac{4}{\sigma^2}+\frac{8}{\sigma^3}$$
$$-\frac{8}{3}\frac{1}{\sigma^3}$$

$$\left(6+\frac{12}{\sigma}+\frac{12}{\sigma^2}+\frac{8}{\sigma^3}\right)\left(1-\frac{2}{\sigma}+\frac{2}{\sigma^2}-\frac{4}{3}\frac{1}{\sigma^3}+\frac{2}{3}\frac{1}{\sigma^4}\right)$$
$$-\frac{12}{\sigma}-\frac{24}{\sigma^2}-\frac{24}{\sigma^3}-\frac{16}{\sigma^4}$$
$$+\frac{12}{\sigma^2}+\frac{24}{\sigma^3}+\frac{24}{\sigma^4}$$
$$-\frac{8}{\sigma^3}-\frac{16}{\sigma^4}$$
$$+\frac{4}{\sigma^4}$$
$$-\frac{4}{\sigma^4}$$

$$2\left(6+\frac{6}{\sigma}+\frac{3}{\sigma^2}+\frac{1}{\sigma^3}\right)\left(1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}+\frac{1}{24}\frac{1}{\sigma^4}\right)$$
$$-\frac{6}{\sigma}-\frac{6}{\sigma^2}-\frac{3}{\sigma^3}-\frac{1}{\sigma^4}$$
$$+\frac{3}{\sigma^2}+\frac{3}{\sigma^3}+\frac{3}{2}\frac{1}{\sigma^4}$$
$$-\frac{1}{\sigma^3}-\frac{1}{\sigma^4}$$
$$+\frac{1}{4}\frac{1}{\sigma^4}$$
$$-2\cdot\frac{1}{4}\frac{1}{\sigma^4} = -\frac{1}{2}\frac{1}{\sigma^4}$$

$$\frac{4}{\sigma^3}+\frac{1}{2}\frac{1}{\sigma^3} = \frac{9}{2}\frac{1}{\sigma^3} = 4\frac{3}{6}\frac{1}{\sigma^3}$$
$$+\frac{14}{3}\frac{1}{\sigma^3} = +4\frac{4}{6}\frac{1}{\sigma^3}$$
$$9\frac{1}{6}\frac{1}{\sigma^3}$$

$$\left(4+\cancel{\frac{4}{\sigma}}+\cancel{\frac{2}{\sigma^2}}-12\sigma-12-\cancel{\frac{6}{\sigma}}-\cancel{\frac{2}{\sigma^2}}\right)e^{-\frac{1}{\sigma}} + \left(6\sigma+12+\cancel{\frac{12}{\sigma}}+\cancel{\frac{8}{\sigma^2}}-4-\cancel{\frac{8}{\sigma}}-\cancel{\frac{8}{\sigma^2}}\right)e^{-\frac{2}{\sigma}}$$
$$-\left(\frac{2}{\sigma}+8+12\sigma\right)e^{-\frac{1}{\sigma}} + \left(\frac{4}{\sigma}+8+6\sigma\right)e^{-\frac{2}{\sigma}} + 6\sigma$$

$$\left(\frac{4}{\sigma}+8+6\sigma\right)\left(1-\frac{2}{\sigma}+\frac{2}{\sigma^2}-\frac{4}{3}\frac{1}{\sigma^3}+\frac{2}{3}\frac{1}{\sigma^4}\right) \qquad \left(\frac{2}{\sigma}+8+12\sigma\right)\left(1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}+\frac{1}{24}\frac{1}{\sigma^4}\right)$$

$$\frac{4}{\sigma}-\frac{8}{\sigma^2}+\frac{8}{\sigma^3}$$
$$8-\frac{16}{\sigma}+\frac{16}{\sigma^2}-\frac{32}{3}\frac{1}{\sigma^3}$$
$$+6\sigma-12+\frac{12}{\sigma}-\frac{8}{\sigma^2}+4\frac{1}{\sigma^3}$$

$$\frac{2}{\sigma}-\frac{2}{\sigma^2}+\frac{1}{\sigma^3}$$
$$8-\frac{8}{\sigma}+\frac{4}{\sigma^2}-\frac{4}{3}\frac{1}{\sigma^3}$$
$$12\sigma-12+\frac{6}{\sigma}-\frac{2}{\sigma^2}+\frac{1}{2}\frac{1}{\sigma^3}$$
$$12\sigma-4+\frac{1}{6}\frac{1}{\sigma^3}$$

$$\cancel{6\sigma}\,\cancel{-4}+\frac{4}{3}\frac{1}{\sigma^3}\ \cancel{-12\sigma}\,\cancel{+4}-\frac{1}{6}\frac{1}{\sigma^3}\ \cancel{+6\sigma} = \frac{7}{6}\frac{1}{\sigma^3}$$

$$J_1 = \frac{n_0'}{s_{\alpha 0}} \int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^2 [2-\sigma y]\, dy = \frac{n_0'}{s_{\alpha 0}} \left[2\int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^2 dy - \sigma \int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^3 dy \right]$$

$$= \frac{n_0'}{s_{\alpha 0}}\left[2\left(2+\frac{2}{\sigma}+\frac{1}{\sigma^2}\right)e^{-\frac{1}{\sigma}} - 2\left(2+\frac{4}{\sigma}+\frac{4}{\sigma^2}\right)e^{-\frac{2}{\sigma}} - \overbrace{\sigma\left(6+\frac{6}{\sigma}+\frac{3}{\sigma^2}+\frac{1}{\sigma^3}\right)}^{6\sigma+6+\frac{3}{\sigma}+\frac{1}{\sigma^2}}e^{-\frac{1}{\sigma}} + \overbrace{\sigma\left(6+\frac{12}{\sigma}+\frac{12}{\sigma^2}+\frac{8}{\sigma^3}\right)}^{6\sigma+12+\frac{12}{\sigma}+\frac{8}{\sigma^2}}e^{-\frac{2}{\sigma}}\right]$$

$$= \frac{n_0'}{s_{\alpha 0}}\left[\left(\frac{1}{\sigma^2}+\frac{1}{\sigma}-2-6\sigma\right)e^{-\frac{1}{\sigma}} + \left(\frac{4}{\sigma}+8+6\sigma\right)e^{-\frac{2}{\sigma}}\right]$$

$$\sigma \gg 1,\ \frac{1}{\sigma} \ll 1,\quad e^{-\frac{1}{\sigma}} = \left(1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}+\frac{1}{24}\frac{1}{\sigma^4}\right)\left(\frac{1}{\sigma^2}+\frac{1}{\sigma}-2-6\sigma\right)$$

$$\begin{array}{lllll}
-\frac{12}{12} & -\frac{1}{\sigma^3} & -\frac{1}{\sigma^2} & +\frac{2}{\sigma} & +6 \\
+\frac{6}{12} & +\frac{1}{2}\frac{1}{\sigma^3} & -\frac{1}{\sigma^2} & -\frac{3}{\sigma} & \\
+\frac{4}{12} & +\frac{1}{3}\frac{1}{\sigma^3} & +\frac{1}{\sigma^2} & & \\
-\frac{3}{12} & -\frac{1}{4}\frac{1}{\sigma^3} & & & \\
\hline
 & -\frac{5}{12}\frac{1}{\sigma^3} & +4 & -6\sigma &
\end{array}$$

$$J_1 = \frac{n_0}{s_{\alpha 0}}\frac{11}{12}\frac{1}{\sigma^3}$$

$$e^{-\frac{2}{\sigma}} = \left(1-\frac{2}{\sigma}+\frac{2}{\sigma^2}-\frac{4}{3}\frac{1}{\sigma^3}+\frac{2}{3}\frac{1}{\sigma^4}\right)\left(\frac{4}{\sigma}+8+6\sigma\right)$$

$$\begin{array}{llll}
 & -\frac{8}{\sigma^2} & -\frac{16}{\sigma} & -12 \\
+\frac{8}{\sigma^3} & +\frac{16}{\sigma^2} & +\frac{12}{\sigma} & \\
-\frac{32}{3}\frac{1}{\sigma^3} & -\frac{8}{\sigma^2} & & \\
+\frac{4}{\sigma^3} & & & \\
\hline
+\frac{4}{3}\frac{1}{\sigma^3} & -4 & +6\sigma &
\end{array}$$

$$\frac{4}{3}-\frac{5}{12} = \frac{16}{12}-\frac{5}{12} = \frac{11}{12}$$

$$\sigma \ll 1,\ \frac{1}{\sigma} \gg 1 \qquad J_1 = \frac{n_0'}{s_{\alpha 0}}\frac{1}{\sigma^2}e^{-\frac{1}{\sigma}}$$

$$J_2 = \frac{n_0'}{s_{\alpha 0}}\sigma\int_0^{\frac{1}{\sigma}} e^{-y}y^3 dy = \frac{n_0'}{s_{\alpha 0}}\sigma\left[6-\left(6+\frac{6}{\sigma}+\frac{3}{\sigma^2}+\frac{1}{\sigma^3}\right)e^{-\frac{1}{\sigma}}\right] = \frac{n_0'}{s_{\alpha 0}}\left[6\sigma-\left(\frac{1}{\sigma^2}+\frac{3}{\sigma}+6+6\sigma\right)e^{-\frac{1}{\sigma}}\right]$$

$$\sigma \gg 1,\ \frac{1}{\sigma} \ll 1 \qquad e^{-\frac{1}{\sigma}} = \left(1-\frac{1}{\sigma}+\frac{1}{2}\frac{1}{\sigma^2}-\frac{1}{6}\frac{1}{\sigma^3}+\frac{1}{24}\frac{1}{\sigma^4}\right)\left(\frac{1}{\sigma^2}+\frac{3}{\sigma}+6+6\sigma\right)$$

$$\begin{array}{lllll}
-\frac{1}{\sigma^3} & -\frac{3}{\sigma^2} & -\frac{6}{\sigma} & -6 & \\
+\frac{3}{2}\frac{1}{\sigma^3} & +\frac{3}{\sigma^2} & +\frac{3}{\sigma} & & \\
-\frac{1}{\sigma^3} & -\frac{1}{\sigma^2} & & & \\
+\frac{1}{4}\frac{1}{\sigma^3} & & & & \\
\hline
-\frac{1}{4}\frac{1}{\sigma^3} & & & & +6\sigma
\end{array}
\qquad = \frac{n_0'}{s_{\alpha 0}}\frac{1}{4}\frac{1}{\sigma^3}$$

$$\sigma \ll 1,\ \frac{1}{\sigma} \gg 1 \qquad J_2 = \frac{n_0'}{s_{\alpha 0}}6\sigma$$

$$dn = n_0 e^{-y} y\, dy\,,\quad y = \frac{s_\alpha}{s}\,,\quad dy = -\frac{s_\alpha}{s^2}ds = -\frac{1}{s_\alpha} y^2 ds\,,\quad \frac{dn}{ds} = \frac{n_0}{s_\alpha} e^{-y} y^3 = J$$

$dJ = \frac{dn_0}{s_\alpha} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^3}{s^3}$, dn_0 Zahl d. Mol. m. $\mu = \mu_0 + \mu_r^z$, μ_r^z zw. μ_r^z und $\mu_r^z + d\mu_r^z$

$$\frac{s_\alpha}{s} = y \ll 1\,,\quad dJ = \frac{dn_0}{s_\alpha}\frac{s_\alpha^3}{s^3} = dn_0 \frac{s_\alpha^2}{s^3}\,,\quad J = \frac{1}{s^3}\int_{\mu_r=0}^{\mu_r^z = \frac{2}{\sqrt{\pi}}\mu_\alpha = \mu_0} s_\alpha^2\, dn_0$$

$$dn_0 = n_0\, d\frac{\mu_r^z}{\mu_\alpha}\frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}\frac{\mu_r^z}{\mu_\alpha}\right) = n_0\, dx\, \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right)\,,\quad x = \frac{\mu_r^z}{\mu_\alpha}$$

$$\int_{x=0}^{x=\frac{2}{\sqrt{\pi}}} dn_0 = n_0 \frac{\sqrt{\pi}}{2}\left[\frac{2}{\sqrt{\pi}} - \frac{\sqrt{\pi}}{2}\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\right)^2\right] = n_0\left[1 - \frac{1}{2}\right] = n_0 \frac{1}{2}\,,\quad \frac{\mu_\alpha}{\mu_0} = \frac{\sqrt{\pi}}{2}$$

$$s_\alpha = s_{\alpha 0}\frac{\mu_0 + \mu_r^z}{\mu_0} = s_{\alpha 0}\left(1 + \frac{\mu_r^z}{\mu_\alpha}\frac{\mu_\alpha}{\mu_0}\right) = s_{\alpha 0}\left(1 + \frac{\sqrt{\pi}}{2}x\right)$$

$$J = \frac{s_{\alpha 0}^2}{s^3} n_0 \frac{\sqrt{\pi}}{2}\int_{x=0}^{x=\frac{2}{\sqrt{\pi}}} \left(1 - \frac{\sqrt{\pi}}{2}x\right)\left(1 + \frac{\sqrt{\pi}}{2}x\right)^2 dx = \frac{s_{\alpha 0}^2}{s^3} n_0 \frac{\sqrt{\pi}}{2}\int_{x=0}^{x=\frac{2}{\sqrt{\pi}}}\left[1 + \frac{\sqrt{\pi}}{2}x - \frac{\pi}{4}x^2 - \left(\frac{\pi}{4}\right)^{\frac{3}{2}} x^3\right] dx$$

$(1 - \frac{\pi}{4}x^2)(1 + \frac{\sqrt{\pi}}{2}x)$

18 − 7 = 11

$$J = \frac{s_{\alpha 0}^2}{s^3} n_0 \frac{\sqrt{\pi}}{2}\left[\frac{2}{\sqrt{\pi}} + \frac{\sqrt{\pi}}{2}\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\right)^2 - \frac{\pi}{4}\frac{1}{3}\left(\frac{2}{\sqrt{\pi}}\right)^3 - \left(\frac{\pi}{4}\right)^{\frac{3}{2}}\frac{1}{4}\left(\frac{2}{\sqrt{\pi}}\right)^4\right] = \frac{s_{\alpha 0}^2}{s^3} n_0\left[1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\right] = \frac{11}{12}\frac{s_{\alpha 0}^3}{s^3}\frac{n_0}{s_{\alpha 0}}$$

$$dJ = \frac{dn_0}{s_\alpha} e^{-\frac{s_\alpha}{s}}\frac{s_\alpha^3}{s^3} = \frac{1}{s^3} dn_0\, e^{-\frac{s_\alpha}{s}} s_\alpha^2\,,\quad s_\alpha = s_{\alpha 0}\left(1 + \frac{\sqrt{\pi}}{2}x\right)\,,\quad x = \frac{\mu_r^z}{\mu_\alpha}\,,\ x = 0 \text{ bis } x = \frac{2}{\sqrt{\pi}}$$

$$dn_0 = n_0\, dx \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right)$$

$$\frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(1 + \frac{\sqrt{\pi}}{2}x\right)\,,\quad dJ = \frac{1}{s^3} s_{\alpha 0}^2\left(1 + \frac{\sqrt{\pi}}{2}x\right)^2 e^{-\frac{s_{\alpha 0}}{s}\left(1 + \frac{\sqrt{\pi}}{2}x\right)} n_0\, dx \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2}x\right)$$

$$\frac{s_{\alpha 0}}{s}\left(1 + \frac{\sqrt{\pi}}{2}x\right) = y\,,\quad 1 + \frac{\sqrt{\pi}}{2}x = \frac{s}{s_{\alpha 0}}y\,,\quad \frac{\sqrt{\pi}}{2}x = \frac{s}{s_{\alpha 0}}y - 1\,,\quad 1 - \frac{\sqrt{\pi}}{2}x = 2 - \frac{s}{s_{\alpha 0}}y\,,\quad \frac{\sqrt{\pi}}{2}dx = \frac{s}{s_{\alpha 0}}dy$$

$$dJ = y^2 e^{-y} n_0 \frac{1}{s_{\alpha 0}} dy\left(2 - \frac{s}{s_{\alpha 0}}y\right) = \frac{n_0}{s_{\alpha 0}} e^{-y} y^2 (2 - \sigma y)\, dy \qquad \sigma = \frac{s}{s_{\alpha 0}}$$

$x = 0,\ y = \frac{1}{\sigma}$

$x = \frac{2}{\sqrt{\pi}},\ y = \frac{2}{\sigma}$

$$J = \frac{n_0}{s_{\alpha 0}}\int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} e^{-y} y^2 (2 - \sigma y)\, dy \qquad \int_{\frac{1}{\sigma}}^{\frac{2}{\sigma}} y^2(2 - \sigma y)\, dy = \frac{2}{3}\left(\frac{8}{\sigma^3} - \frac{1}{\sigma^3}\right) - \sigma\frac{1}{4}\left(\frac{16}{\sigma^4} - \frac{1}{\sigma^4}\right) = \frac{11}{12}\frac{1}{\sigma^3}$$

$$dJ = \frac{1}{s^3}\, dn_0\, e^{-\frac{s_\alpha}{s}} s_\alpha^2\,,\quad dn_0 = n_0\, dx\, \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right),\quad x = \frac{\mu_+^2}{\mu_\alpha}\,,\quad \frac{\mu_\alpha}{\mu_0} = \frac{\sqrt{\pi}}{2}$$

$$s_\alpha = s_{\alpha 0}\frac{\mu_0-\mu_+^2}{\mu_0} = s_{\alpha 0}\left(1-\frac{\mu_+^2}{\mu_0}\right) = s_{\alpha 0}\left(1-\frac{\mu_\alpha}{\mu_0}\frac{\mu_+^2}{\mu_\alpha}\right) = s_{\alpha 0}\left(1-\frac{\sqrt{\pi}}{2}x\right),\quad \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(1-\frac{\sqrt{\pi}}{2}x\right)$$

$$dJ_- = \frac{1}{s_3}\, n_0\, dx\, \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right) e^{-\frac{s_{\alpha 0}}{s}\left(1-\frac{\sqrt{\pi}}{2}x\right)} s_{\alpha 0}^2\left(1-\frac{\sqrt{\pi}}{2}x\right)^2$$

$$\frac{s_{\alpha 0}}{s}\left(1-\frac{\sqrt{\pi}}{2}x\right) = y\,,\quad 1-\frac{\sqrt{\pi}}{2}x = \frac{s}{s_{\alpha 0}}y\,,\quad -\frac{\sqrt{\pi}}{2}dx = \frac{s}{s_{\alpha 0}}dy \qquad \begin{array}{l} x=0,\ y=\frac{1}{\sigma} \quad \sigma = \frac{s}{s_{\alpha 0}} \\ x=\frac{2}{\sqrt{\pi}},\ y=0 \end{array}$$

$$dJ_- = -\frac{n_0}{s_{\alpha 0}}\, dy\, \frac{s}{s_{\alpha 0}}\, y^3 e^{-y}\,,\quad J_- = -\frac{n_0}{s_{\alpha 0}}\sigma\int_{\frac{1}{\sigma}}^{0} e^{-y}y^3\,dy = \frac{n_0}{s_{\alpha 0}}\sigma\int_0^{\frac{1}{\sigma}} e^{-y}y^3\,dy$$

$$dJ_- = \frac{1}{s^3}\, dn_0\, s_\alpha^2 = \frac{1}{s^3}\, n_0\, dx\, \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right) s_{\alpha 0}^2\left(1-\frac{\sqrt{\pi}}{2}x\right)^2$$

$$\int_0^{\frac{1}{\sigma}} y^3\,dy = \frac{1}{4}\frac{1}{\sigma^4}\,,\quad \sigma \gg 1 \quad J_- = \frac{n_0}{s_{\alpha 0}}\frac{3}{12}\frac{1}{\sigma^3} \qquad J_+ = \frac{n_0}{s_{\alpha 0}}\frac{11}{12}\frac{1}{\sigma^3} \qquad \sigma = \frac{s}{s_{\alpha 0}}\,,\ \frac{1}{\sigma} = \frac{s_{\alpha 0}}{s}$$

$$J_+ + J_- = \frac{n_0}{s_{\alpha 0}}\frac{7}{6}\left(\frac{s_{\alpha 0}}{s}\right)^3$$

$$J_+ = \frac{n_0'}{s_{\alpha 0}}\left[\left(6\sigma+8+\frac{4}{\sigma}\right)e^{-\frac{2}{\sigma}} - \left(6\sigma+2-\frac{1}{\sigma}-\frac{1}{\sigma^2}\right)e^{-\frac{1}{\sigma}}\right]$$

$$J_- = \frac{n_0'}{s_{\alpha 0}}\left[6\sigma - \left(6\sigma+6+\frac{3}{\sigma}+\frac{1}{\sigma^2}\right)e^{-\frac{1}{\sigma}}\right]$$

$\sigma \ll 1 \quad J_+ = \frac{n_0'}{s_{\alpha 0}}\frac{1}{\sigma^2}e^{-\frac{1}{\sigma}} \quad J_- = \frac{n_0'}{s_{\alpha 0}}6\sigma \quad J_+ + J_- = \frac{n_0'}{s_{\alpha 0}}6\sigma \quad \left(= \frac{n_0'}{s_{\alpha 0}}\frac{1}{\sigma^3}e^{-\frac{1}{\sigma}} \text{ für } \mu_\alpha = 0\right)$

$\sigma \gg 1 \quad J_+ = \frac{n_0'}{s_{\alpha 0}}\frac{11}{12}\frac{1}{\sigma^3} \quad J_- = \frac{n_0'}{s_{\alpha 0}}\frac{1}{4}\frac{1}{\sigma^3} \quad J_+ + J_- = \frac{n_0'}{s_{\alpha 0}}\frac{7}{6}\frac{1}{\sigma^3} \quad \left(= \frac{n_0'}{s_{\alpha 0}}\frac{1}{\sigma^3} \text{ für } \mu_\alpha = 0\right)$

$\sigma = 1 \quad J_+ = \frac{n_0'}{s_{\alpha 0}}0.228 \quad J_- = \frac{n_0'}{s_{\alpha 0}}0.114 \quad J_+ + J_- = \frac{n_0'}{s_{\alpha 0}}0.342 \quad \left(= \frac{n_0'}{s_{\alpha 0}}0.368 \text{ für } \mu_\alpha = 0\right)$

$$dJ = \frac{dn_0}{s_\alpha} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^3}{s^3}, \quad dn_0 = n_0 \, d\frac{\mu_r^z}{\mu_\alpha} \int_{\frac{\mu_r}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi, \quad \mu_r^z \text{ von } 0 \text{ bis } \infty$$

$$s_\alpha = s_{\alpha 0} \frac{\mu_0 + \mu_r^z}{\mu_0} = s_{\alpha 0}\left(1 + \frac{\mu_r^z}{\mu_0}\right) = s_{\alpha 0}\left(1 + \frac{\mu_\alpha}{\mu_0} x\right), \quad x = \frac{\mu_r^z}{\mu_\alpha}, \quad y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(1 + \frac{\mu_\alpha}{\mu_0} x\right)$$

$$dn_0 = n_0 \, dx \int_x^\infty e^{-\xi^2} d\xi, \quad 1 + \frac{\mu_\alpha}{\mu_0} x = \frac{s}{s_{\alpha 0}} y, \quad \frac{\mu_\alpha}{\mu_0} dx = \frac{s}{s_{\alpha 0}} dy, \quad x = \frac{\mu_0}{\mu_\alpha}\left(\frac{s}{s_{\alpha 0}} y - 1\right)$$

$$dn_0 = n_0 \frac{\mu_0}{\mu_\alpha} \frac{s}{s_{\alpha 0}} dy \int_{\frac{\mu_0}{\mu_\alpha}\left(\frac{s}{s_{\alpha 0}} y - 1\right)}^{\infty} e^{-\xi^2} d\xi, \quad s_\alpha = y s, \quad dJ = \frac{dn_0}{y s} e^{-y} y^3 = \frac{1}{s} e^{-y} y^2 \, dn_0$$

$$\mu_z = 0, \quad s_\alpha = s_{\alpha 0}, \quad y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}$$

$$J = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_{\frac{s_{\alpha 0}}{s}}^{\infty} e^{-y} y^2 \, dy \int_{\frac{\mu_0}{\mu_\alpha}\left(\frac{s}{s_{\alpha 0}} y - 1\right)}^{\infty} e^{-\xi^2} d\xi \qquad \mu_z = \infty, \quad s_\alpha = \infty, \quad y = \frac{s_\alpha}{s} = \infty \qquad \sigma = \frac{s}{s_{\alpha 0}}$$

$$J^+ = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 \, dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y - 1)}^{\infty} e^{-\xi^2} d\xi$$

$$\int_x^\infty e^{-\xi^2} d\xi \simeq \frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2} x\right), \quad \frac{\mu_0}{\mu_\alpha} = \frac{2}{\sqrt{\pi}}, \quad \int_x^\infty = \frac{\sqrt{\pi}}{2}\left[1 - \frac{\sqrt{\pi}}{2} \frac{\mu_0}{\mu_\alpha}(\sigma y - 1)\right] = \frac{\sqrt{\pi}}{2}(2 - \sigma y)$$

$$\mu_z = \mu_0 \quad y = \frac{s_\alpha}{s} = \frac{2 s_{\alpha 0}}{s} = \frac{2}{\sigma} \qquad J = \frac{n_0}{s_{\alpha 0}} \cancel{\frac{\mu_0}{\mu_\alpha}} \cancel{\frac{\sqrt{\pi}}{2}} \int_{\frac{1}{\sigma}} e^{-y} y^2 (2 - \sigma y) \, dy$$

$$dJ = \frac{dn_0}{s_\alpha} e^{-\frac{s_\alpha}{s}} \left(\frac{s_\alpha}{s}\right)^3 \quad dn_0 = n_0 \, d\frac{\mu_r^z}{\mu_\alpha} \int_{\frac{\mu_r^z}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi$$

$\mu = |\mu_0 - \mu_r^z|$ Zwei Gebiete: μ_r^z 1) von $0 \to \mu_0$ und 2) von $\mu_0 \to \infty$

$$1)\ s_\alpha = s_{\alpha_0} \frac{\mu_0 - \mu_r^z}{\mu_0} = s_{\alpha 0}\left(1 - \frac{\mu_r^z}{\mu_0}\right) = s_{\alpha_0}\left(1 - \frac{\mu_\alpha}{\mu_0}\frac{\mu_r^z}{\mu_\alpha}\right) = s_{\alpha_0}\left(1 - \frac{\mu_\alpha}{\mu_0} x\right),\ x = \frac{\mu_r^z}{\mu_\alpha},\ y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(1 - \frac{\mu_\alpha}{\mu_0} x\right)$$

$$d\frac{\mu_r^z}{\mu_r} = dx$$

$$1 - \frac{\mu_\alpha}{\mu_0} x = \frac{s}{s_{\alpha 0}} y,\ -\frac{\mu_\alpha}{\mu_0} dx = \frac{s}{s_{\alpha 0}} dy,\ x = \frac{\mu_0}{\mu_\alpha}\left(1 - \frac{s}{s_{\alpha 0}} y\right),\ \mu_z = 0,\ y = \frac{s_{\alpha 0}}{s},\ \mu_z = \mu_0,\ y = 0$$

$$dJ_1^- = \frac{dn_0}{s y} e^{-y} y^3 = \frac{1}{s} e^{-y} y^2 dn_0 = -\frac{1}{\cancel{s}} e^{-y} y^2 n_0 \frac{\mu_0}{\mu_\alpha} \frac{\cancel{s}}{s_{\alpha 0}} dy \int_{\frac{\mu_0}{\mu_\alpha}\left(1 - \frac{s}{s_{\alpha 0}} y\right)}^{\infty} e^{-\xi^2} d\xi \qquad \sigma = \frac{s}{s_{\alpha 0}}$$

$$J_1^- = -\frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_{\frac{1}{\sigma}}^{0} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(1 - \sigma y)}^{\infty} e^{-\xi^2} d\xi = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_0^{\frac{1}{\sigma}} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(1 - \sigma y)}^{\infty} e^{-\xi^2} d\xi$$

Näh.: $\int_{\frac{\mu_0}{\mu_\alpha}(1 - \sigma y)}^{\infty} e^{-\xi^2} d\xi \simeq \frac{\sqrt{\pi}}{2}\left[1 - \frac{\sqrt{\pi}}{2} \frac{\mu_0}{\mu_\alpha}(1 - \sigma y)\right] = \frac{\sqrt{\pi}}{2} \sigma y,\quad J = \frac{n_0 \sigma}{s_{\alpha 0}} \int_0^{\frac{1}{\sigma}} e^{-y} y^3 dy$

$$2)\ s_\alpha = s_{\alpha_0} \frac{\mu_r^z - \mu_\alpha}{\mu_0} = s_{\alpha_0}\left(\frac{\mu_r^z}{\mu_0} - 1\right) = s_{\alpha_0}\left(x \frac{\mu_\alpha}{\mu_0} - 1\right),\ x = \frac{\mu_z^r}{\mu_0},\ y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(x \frac{\mu_\alpha}{\mu_0} - 1\right)$$

$$\frac{\mu_z}{\mu_\alpha}$$

$$x \frac{\mu_\alpha}{\mu_0} - 1 = \frac{s}{s_{\alpha 0}} y,\ \frac{\mu_\alpha}{\mu_0} dx = \frac{s}{s_{\alpha 0}} dy,\ x = \frac{\mu_0}{\mu_\alpha}\left(\frac{s}{s_{\alpha 0}} y + 1\right),\ \mu_z = \mu_0,\ y = 0,\ \mu_z = \infty,\ y = \infty$$

$$dJ_2^- = \frac{1}{s} e^{-y} y^2 dn_0 = \frac{1}{\cancel{s}} e^{-y} y^2 n_0 \frac{\mu_0}{\mu_\alpha} \frac{\cancel{s}}{s_{\alpha 0}} dy \int_{\frac{\mu_0}{\mu_\alpha}\left(\frac{s}{s_{\alpha 0}} y + 1\right)}^{\infty} e^{-\xi^2} d\xi$$

$$J_2^- = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_0^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y + 1)}^{\infty} e^{-\xi^2} d\xi$$

$$J^+ + J^- = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha}\left[\int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y - 1)}^{\infty} e^{-\xi^2} d\xi + \int_0^{\frac{1}{\sigma}} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(1 - \sigma y)}^{\infty} e^{-\xi^2} d\xi + \int_0^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y + 1)}^{\infty} e^{-\xi^2} d\xi\right]$$

$$\int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y-1)}^{\infty} e^{-\xi^2} d\xi$$

$\underline{\underline{\sigma \ll 1}}$, $\frac{1}{\sigma} \gg 1$, d.h. $\int_{\frac{\mu_0}{\mu_\alpha}(\sigma y-1)}^{\infty} e^{-\xi^2} d\xi$ braucht nur für sehr kleine Werte von $\frac{\mu_0}{\mu_\alpha}(\sigma y-1)$ richtig zu sein

$= \frac{1}{\sigma^2} e^{-\frac{1}{\sigma}} \frac{\sqrt{\pi}}{2}$, $J^+ = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \frac{\sqrt{\pi}}{2} \frac{1}{\sigma^2} e^{-\frac{1}{\sigma}}$ gilt in Annäherung über für $\frac{\mu_0}{\mu_\alpha} = \frac{2}{\sqrt{\pi}}$

$$\int_{x_0}^{\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2} - \int_0^{x_0} e^{-\xi^2} d\xi \cong \frac{\sqrt{\pi}}{2} - x_0$$ für $x_0 \ll 1$, besser $\frac{\sqrt{\pi}}{2}\left(1 - \frac{\sqrt{\pi}}{2} x\right) = \frac{\sqrt{\pi}}{2} - \frac{\pi}{4} x$, damit Fehlergr. $\frac{1}{2}\mu_0$ gibt

$\frac{\mu_0}{\mu_\alpha} \ll 1$, x_0 sehr klein

Wenn $\mu_0 \ll \mu_\alpha$ spielen nur kleine $\frac{\mu_\xi}{\mu_\alpha}$ eine Rolle, daher ist die Annäherung $\frac{\sqrt{\pi}}{2} - x_0$ besser

$$J^+ = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \left[\frac{\sqrt{\pi}}{2} - \frac{\mu_0}{\mu_\alpha}(\sigma y - 1)\right] = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \left[\left(\frac{\sqrt{\pi}}{2} + \frac{\mu_0}{\mu_\alpha}\right) \int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy - \frac{\mu_0}{\mu_\alpha} \sigma \int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^3 dy\right]$$

$$J^+ + J^- = \frac{n_0}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \left[\int_{\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y - 1)}^{\infty} e^{-\xi^2} d\xi + \int_0^{\frac{1}{\sigma}} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(1-\sigma y)}^{\infty} e^{-\xi^2} d\xi + \int_0^{\infty} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha}(\sigma y + 1)}^{\infty} e^{-\xi^2} d\xi\right]$$

$\sigma = \frac{s}{s_{\alpha 0}}$, $\frac{s_{\alpha 0}}{s_{\alpha\alpha}} = \frac{\mu_0}{\mu_\alpha}$, $\frac{1}{s_{\alpha\alpha}} = \frac{1}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha}$, $\frac{1}{s_{\alpha 0}} = \frac{1}{s_{\alpha\alpha}} \frac{\mu_\alpha}{\mu_0}$, $\sigma = \frac{s}{s_{\alpha 0}} = \frac{s}{s_{\alpha\alpha}} \frac{\mu_\alpha}{\mu_0}$

$\frac{\mu_0}{\mu_\alpha}(\sigma y \pm 1) = \frac{\mu_0}{\mu_\alpha} \sigma y \pm \frac{\mu_0}{\mu_\alpha} = \frac{s}{s_{\alpha\alpha}} y \pm \frac{\mu_0}{\mu_\alpha}$, $\frac{1}{\sigma} = \frac{s_{\alpha\alpha}}{s} \frac{\mu_0}{\mu_\alpha}$, $\frac{s}{s_{\alpha\alpha}} = \sigma'$, $\frac{\mu_0}{\mu_\alpha}(1 - \sigma y) = \frac{\mu_0}{\mu_\alpha} - \frac{s}{s_{\alpha\alpha}} y$

$$J^+ + J^- = \frac{n_0}{s_{\alpha\alpha}} \left[\int_{\frac{1}{\sigma'}\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-y} y^2 dy \int_{\sigma' y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi + \int_0^{\frac{1}{\sigma'}\frac{\mu_0}{\mu_\alpha}} e^{-y} y^2 dy \int_{\frac{\mu_0}{\mu_\alpha} - \sigma' y}^{\infty} e^{-\xi^2} d\xi + \int_0^{\infty} e^{-y} y^2 dy \int_{\sigma' y + \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi\right]$$

$\mu_0 = 0$ $$J^+ + J^- = \frac{n_0}{s_{\alpha\alpha}} \left[2 \int_0^{\infty} e^{-y} y^2 dy \int_{\sigma' y}^{\infty} e^{-\xi^2} d\xi\right]$$

$$dJ = \frac{dn_0}{s^5} e^{-\frac{s_\alpha}{s}} s_\alpha^2$$

$$dn_0' = n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi$$ Zahl d. Mol mit μ_r^z zw. μ_r^z und $\mu_r^z + d\mu_r^z$

dn_0 " " μ_z zw. μ_z und $\mu_z + d\mu_z$

wobei $\mu_z = \mu_0 + \mu_r^z$ ist. Derselbe Wert von μ^z entspricht 3 Werten von μ_r^z:

1) $\mu_0 + \mu_r^z$ 2) $\mu_z = \mu_r^{z'} - \mu_0$, d.h. $\mu_r^{z'} = \mu_z + \mu_0$, von $\mu_z = 0$ bis $\mu_z = \infty$

von $\mu_z = \mu_0$ bis $\mu_z = \infty$ 3) $\mu_z = \mu_0 - \mu_r^{z''}$, d.h. $\mu_r^{z''} = \mu_0 - \mu_z$, von $\mu_z = 0$ bis $\mu_z = \mu_0$

$0 < \mu_z < \mu_0$

$$dn_0 = n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \int_{\frac{\mu_z+\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi + n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \int_{\frac{\mu_0-\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi = n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \left[\int_{\frac{\mu_0+\mu_z}{\mu_\alpha}}^{\infty} + \int_{\frac{\mu_0-\mu_z}{\mu_\alpha}}^{\infty} \right]$$

$$= n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \left[\sqrt{\pi} - \left(\int_0^{\frac{\mu_0+\mu_z}{\mu_\alpha}} + \int_0^{\frac{\mu_0-\mu_z}{\mu_\alpha}} \right) \right] \approx n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \left[\sqrt{\pi} - \left(\frac{\mu_0+\mu_z}{\mu_\alpha} + \frac{\mu_0-\mu_z}{\mu_\alpha} \right) \right] \text{ für } \mu_0 \ll \mu_\alpha$$

$$= n_0' \, d\frac{\mu_z}{\mu_\alpha} \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha} \right] \quad \underline{\underline{d\mu_z}}!$$

$\mu_0 < \mu_z < \infty$

$$dn_0 = n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \left[\int_{\frac{\mu_z-\mu_0}{\mu_\alpha}}^{\infty} + \int_{\frac{\mu_z+\mu_0}{\mu_\alpha}}^{\infty} \right] = n_0' \, d\frac{\mu_r^z}{\mu_\alpha} \left[2\int_{\frac{\mu_z+\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi + \int_{\frac{\mu_z-\mu_0}{\mu_\alpha}}^{\frac{\mu_z+\mu_0}{\mu_\alpha}} e^{-\xi^2} d\xi \right]$$

$0 < \mu_z < \mu_0$

$$dJ = \frac{dn_0}{s} e^{-\frac{s_\alpha}{s}} \left(\frac{s_\alpha}{s}\right)^2, \quad dn_0 \approx n_0' \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha}\right] d\frac{\mu_z}{\mu_\alpha}, \quad y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}\mu_z}{s\,\mu_0}, \quad dy = \frac{s_{\alpha 0}\mu_\alpha}{s\,\mu_0} d\frac{\mu_z}{\mu_\alpha}$$

$$dJ = \frac{1}{s} e^{-y} y^2 n_0' \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha}\right] \frac{s}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} dy = \frac{n_0'}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha}\right] e^{-y} y^2 dy$$

$$J = \frac{n_0'}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha}\right] \int_0^{\frac{s_{\alpha 0}}{s}} e^{-y} y^2 dy = \frac{n_0'}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} \left[\sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha}\right] \left[2 - \left(2 + 2\frac{s_{\alpha 0}}{s} + \frac{s_{\alpha 0}^2}{s^2}\right) e^{-\frac{s_{\alpha 0}}{s}}\right]$$

$$\frac{s_{\alpha\alpha}}{s_{\alpha 0}} = \frac{\mu_\alpha}{\mu_0}; \quad \frac{n_0'}{s_{\alpha 0}} \frac{\mu_0}{\mu_\alpha} = \frac{n_0'}{s_{\alpha\alpha}}$$

$$s_{\alpha\alpha} = s_{\alpha 0} \frac{\mu_\alpha}{\mu_0}$$

I) $\mu_z < \mu_0$ II) $\mu_z > \mu_0$

$$dn_0 = n_0' \, d\frac{\mu_z}{\mu_\alpha}\left[\int_{\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi + \int_{\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi\right], \quad dn_0 = n_0' \, d\frac{\mu_z}{\mu_\alpha}\left[\int_{\frac{\mu_z}{\mu_\alpha}+\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi + \int_{\frac{\mu_z}{\mu_\alpha}-\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi\right]$$

I) $$\frac{d\,[\;]}{d\frac{\mu_z}{\mu_\alpha}} = -e^{-\left(\frac{\mu_0+\mu_z}{\mu_\alpha}\right)^2} + e^{-\left(\frac{\mu_0-\mu_z}{\mu_\alpha}\right)^2} = e^{-\left(\frac{\mu_0}{\mu_\alpha}\right)^2} e^{+\frac{2\mu_0\mu_z}{\mu_\alpha^2}} e^{-\left(\frac{\mu_z}{\mu_\alpha}\right)^2} - e^{-\left(\frac{\mu_0}{\mu_\alpha}\right)^2} e^{-\frac{2\mu_0\mu_z}{\mu_\alpha^2}} e^{-\left(\frac{\mu_z}{\mu_\alpha}\right)^2}$$

$$= e^{-\left[\left(\frac{\mu_0}{\mu_\alpha}\right)^2+\left(\frac{\mu_z}{\mu_\alpha}\right)^2\right]}\left[e^{\frac{2\mu_0\mu_z}{\mu_\alpha^2}} - e^{-\frac{2\mu_0\mu_z}{\mu_\alpha^2}}\right] \simeq \left[\frac{4\mu_0\mu_z}{\mu_\alpha^2} + \frac{8}{3}\left(\frac{\mu_0\mu_z}{\mu_\alpha^2}\right)^3 + \cdots\right] e^{-\left[\left(\frac{\mu_0}{\mu_\alpha}\right)^2+\left(\frac{\mu_z}{\mu_\alpha}\right)^2\right]}$$

$$e^x = 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 \qquad = 1 - \left(\frac{\mu_0-\mu_z}{\mu_\alpha}\right)^2 - 1 + \left(\frac{\mu_0+\mu_z}{\mu_\alpha}\right)^2 = \frac{4\mu_0\mu_z}{\mu_\alpha^2}$$

$$e^{-x} = 1 - x + \tfrac{1}{2}x^2 - \tfrac{1}{6}x^3$$

$$e^x - e^{-x} = 2x + \tfrac{1}{3}x^3 \qquad = 4\frac{\mu_0}{\mu_\alpha}\frac{\mu_z}{\mu_\alpha}\left[1 - \frac{\mu_0^2}{\mu_\alpha^2} - \frac{\mu_z^2}{\mu_\alpha^2}\right]$$

$$e^{-(x_1-x_2)^2} = 1 - (x_1-x_2)^2 + \tfrac{1}{2}(x_1-x_2)^4$$

$$e^{-(x_1+x_2)^2} = 1 - (x_1+x_2)^2 + \tfrac{1}{2}(x_1+x_2)^4$$

$$(x_1+x_2)^2 - (x_1-x_2)^2 - \tfrac{1}{2}\left[(x_1+x_2)^4 - (x_1-x_2)^4\right]$$

$$4x_1x_2 + \tfrac{1}{2}\left[8x_1^3x_2 + 8x_1x_2^3\right]$$

$$= 4x_1x_2\left[1 + (x_1^2 + x_2^2)\right]$$

$$F\left(\frac{\mu_z}{\mu_\alpha}\right) = \int_{\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}}^{\infty} + \int_{\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}}^{\infty} = \sqrt{\pi} - \left[\int_0^{\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}} e^{-\xi^2}d\xi + \int_0^{\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}} e^{-\xi^2}d\xi\right] \simeq \sqrt{\pi} - \left[\frac{\mu_0}{\mu_\alpha} + \frac{\mu_z}{\mu_\alpha} - \left(\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}\right)^3\frac{1}{3} + \left(\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}\right)^5\frac{1}{10}\right.$$
$$\left. + \frac{\mu_0}{\mu_\alpha} - \frac{\mu_z}{\mu_\alpha} - \left(\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}\right)^3\frac{1}{3} + \left(\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}\right)^5\frac{1}{10}\right]$$

$$= \sqrt{\pi} - 2\frac{\mu_0}{\mu_\alpha} + \frac{2}{3}\left(\frac{\mu_0}{\mu_\alpha}\right)^3 + 2\frac{\mu_0}{\mu_\alpha}\frac{\mu_z^2}{\mu_\alpha^2} - \frac{1}{5}\left(\frac{\mu_0}{\mu_\alpha}\right)^5 - 2\left(\frac{\mu_0}{\mu_\alpha}\right)^3\left(\frac{\mu_z}{\mu_\alpha}\right)^2 - \frac{\mu_0}{\mu_\alpha}\left(\frac{\mu_z}{\mu_\alpha}\right)^4$$

$$\frac{dF}{d\frac{\mu_z}{\mu_\alpha}} = 0 + 4\frac{\mu_0}{\mu_\alpha}\frac{\mu_z}{\mu_\alpha} + 0 - 4\left(\frac{\mu_0}{\mu_\alpha}\right)^3\frac{\mu_z}{\mu_\alpha} - 4\frac{\mu_0}{\mu_\alpha}\left(\frac{\mu_z}{\mu_\alpha}\right)^3 = 4\frac{\mu_0}{\mu_\alpha}\frac{\mu_z}{\mu_\alpha}\left[1 - \left(\frac{\mu_0}{\mu_\alpha}\right)^2 - \left(\frac{\mu_z}{\mu_\alpha}\right)^2\right]$$

$$F\left(\frac{\mu_z}{\mu_\alpha}\right) = \sqrt{\pi}\left\{1 - \frac{2}{\sqrt{\pi}}\frac{\mu_0}{\mu_\alpha}\left[1 - \frac{1}{3}\left(\frac{\mu_0}{\mu_\alpha}\right)^2 - \left(\frac{\mu_z}{\mu_\alpha}\right)^2 + \frac{1}{10}\left(\frac{\mu_0}{\mu_\alpha}\right)^4 + \left(\frac{\mu_0}{\mu_\alpha}\right)^2\left(\frac{\mu_z}{\mu_\alpha}\right)^2 + \frac{1}{2}\left(\frac{\mu_z}{\mu_\alpha}\right)^4\right]\right.$$

$$\mu_z = 0 \quad F\left(\frac{\mu_z}{\mu_\alpha}\right) = \sqrt{\pi}\left\{1 - \frac{2}{\sqrt{\pi}}\frac{\mu_0}{\mu_\alpha}\left[1 - \frac{1}{3}\left(\frac{\mu_0}{\mu_\alpha}\right)^2 + \frac{1}{10}\left(\frac{\mu_0}{\mu_\alpha}\right)^4\right]\right.$$

$$\mu_z = \mu_0 \quad F\left(\frac{\mu_z}{\mu_\alpha}\right) = \sqrt{\pi}\left\{1 - \frac{2}{\sqrt{\pi}}\frac{\mu_0}{\mu_\alpha}\left[1 - \left(1+\frac{1}{3}\right)\left(\frac{\mu_0}{\mu_\alpha}\right)^2 + \left(\frac{3}{2}+\frac{1}{10}\right)\left(\frac{\mu_0}{\mu_\alpha}\right)^4\right]\right.$$

$$\frac{\mu_0}{\mu_\alpha}=\frac{1}{4}\,,\quad dn_0 = n_0'\, d\frac{\mu_z}{\mu_\alpha}\,\mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right),\quad \mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=\int\limits_{\frac{\mu_0}{\mu_\alpha}+\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2}d\xi+\int\limits_{\frac{\mu_0}{\mu_\alpha}-\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-\xi^2}d\xi$$

$$\mu_z=0\quad \mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=2\int\limits_{\frac14}^{\infty}e^{-\xi^2}d\xi=2\left[\frac{\sqrt{\pi}}{2}-\int\limits_0^{\frac14}e^{-\xi^2}d\xi\right]=\sqrt{\pi}\left[1-\frac{2}{\sqrt{\pi}}\int\limits_0^{\frac14}e^{-\xi^2}d\xi\right]=\sqrt{\pi}(1-0.2763)$$

$$\mu_z=\mu_0\quad \mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=\int\limits_{\frac12}^{\infty}e^{-\xi^2}d\xi+\int\limits_0^{\infty}e^{-\xi^2}d\xi=\sqrt{\pi}-\int\limits_0^{\frac12}e^{-\xi^2}d\xi=\sqrt{\pi}\left[1-\frac12\frac{2}{\sqrt{\pi}}\int\limits_0^{\frac12}\right]=\sqrt{\pi}(1-0.2603^-)$$

Näh'g $\mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=\sqrt{\pi}-2\frac{\mu_0}{\mu_\alpha}+\frac23\left(\frac{\mu_0}{\mu_\alpha}\right)^3=\sqrt{\pi}-0.5+\frac23\,\frac{1}{64}=\sqrt{\pi}-0.5+\frac{1}{96}=\sqrt{\pi}\left(1-\frac{\overset{0,2762}{0.50000-0,01042}}{1,772454}\right)$

$$\begin{array}{r} 0.2820948 \\ -0.00589 \\ \hline 0.27620 \end{array}\qquad \left(1-\frac{0.5}{1.77\ldots}+\frac{0.01042}{\ldots}\right.$$

$$\mu_z=0\quad \mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=2\int\limits_{\frac{\mu_0}{\mu_\alpha}}^{\infty}e^{-\xi^2}d\xi \qquad \mu_z=\mu_0\quad \mathfrak{F}\!\left(\frac{\mu_z}{\mu_\alpha}\right)=\int\limits_{2\frac{\mu_0}{\mu_\alpha}}^{\infty}e^{-\xi^2}d\xi+\int\limits_0^{\infty}e^{-\xi^2}d\xi$$

$$=\sqrt{\pi}-2\int\limits_0^{\frac{\mu_0}{\mu_\alpha}}e^{-\xi^2}d\xi \qquad\qquad =\sqrt{\pi}-\int\limits_0^{2\frac{\mu_0}{\mu_\alpha}}e^{-\xi^2}d\xi$$

$$=\sqrt{\pi}\Bigg(1-\underbrace{\frac{2}{\sqrt{\pi}}\int\limits_0^{\frac{\mu_0}{\mu_\alpha}}}\,e^{-\xi^2}d\xi\Bigg) \qquad =\sqrt{\pi}\Bigg(1-\frac12\underbrace{\frac{2}{\sqrt{\pi}}\int\limits_0^{2\frac{\mu_0}{\mu_\alpha}}e^{-\xi^2}d\xi}\Bigg)$$

$$\frac{2}{\sqrt{\pi}}\frac{\mu_0}{\mu_\alpha}\left[1-\frac13\left(\frac{\mu_0}{\mu_\alpha}\right)^2\right] \qquad \frac{2}{\sqrt{\pi}}\frac{\mu_0}{\mu_\alpha}\left[1-\frac43\left(\frac{\mu_0}{\mu_\alpha}\right)^2\right]$$

$\frac{\mu_0}{\mu_\alpha}=0.1$	0.1125	$[\,]=1-\frac{1}{300}$; $0.11284-0.00037=0.1125$	$[\,]=1-\frac{4}{300}$; $0.11284-0.00150=0.1113$	0.1114^-
0.25	0.2763	$[\,]=1-\frac{1}{48}$ = $0.2821-0.0059=0.2762$	$[\,]=1-\frac{1}{12}$ = $0.2821-0.0235=0.2586$	0.2603^-
0.5	0.5205	$[\,]=1-\frac{1}{12}$ = $0.5642-0.0470=0.5172$; $+\frac{1}{60}$ 0.0035 ; 0.5207	$[\,]=1-\frac13$ = $0.5642-0.1881=0.3761$; $+\frac{1}{10}$ 0.0564 ; 4325	0.4214^-
0.75	0.7112			0.4831^-
1.0	0.8427	$[\,]=1.1-\frac13$; 1.1284, 1128, 1.2412, -0.3333, 0.9079	$[\,]=1.6-\frac13$; 0.9079	0.49766

$\frac{2}{\sqrt{\pi}}=1.12838$

$$d\mathfrak{I} = \frac{dn_0}{s} e^{-\frac{s_\alpha}{s}} \left(\frac{s_\alpha}{s}\right)^2$$

$dn_0' = n_0' \, d\frac{\mu_z^r}{\mu_\alpha} \int\limits_{\frac{\mu_z^r}{\mu_\alpha}}^{\infty} e^{-\xi^2} d\xi$, $\frac{dn_0'}{n_0'} = dn_0$, μ_α Einheit $dn_0 = d\mu_z^r \int\limits_{\mu_z^r}^{\infty} e^{-\xi^2} d\xi$

$\mu_z = \mu_0 + \mu_z^r$ und $\mu_z = \mu_z^r - \mu_0$ von $\mu_z = \mu_0$ bis $\mu_z = \infty$ $\quad d\mu_z = d\mu_z^r$

$\mu_z = \mu_0 - \mu_z^r$ und $\mu_z = \mu_z^r - \mu_0$ von $\mu_z = 0$ bis $\mu_z = \mu_0$ $\quad d\mu_z = -d\mu_z^r, \, d\mu_z = d\mu_z^r$

$$\mu_z > \mu_0$$

$$dn_0 = d\mu_z \left[\int\limits_{\mu_z - \mu_0}^{\infty} e^{-\xi^2} d\xi + \int\limits_{\mu_z + \mu_0}^{\infty} e^{-\xi^2} d\xi \right] \qquad dn_0 = d\mu_z \cdot \mathfrak{F}(\mu_z)$$

$$0 < \mu_z < \mu_0$$

$$dn_0 = d\mu_z \left[\int\limits_{\mu_0 - \mu_z}^{\infty} e^{-\xi^2} d\xi + \int\limits_{\mu_z + \mu_0}^{\infty} e^{-\xi^2} d\xi \right] ,$$

$\frac{s_\alpha}{s} = y$, $\frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_z}{\mu_\alpha} = \mu_z$, $y = \frac{s_\alpha}{s_{\alpha\alpha}} \frac{s_{\alpha\alpha}}{s} = \mu_z \frac{1}{\sigma}$, $\sigma = \frac{s}{s_{\alpha\alpha}}$, $\sigma y = \mu_z$, $d\mu_z = \frac{s}{s_{\alpha\alpha}} dy$

$$d\mathfrak{I} = \frac{1}{s} e^{-y} y^2 dn_0 = \frac{1}{s} e^{-y} y^2 \mathfrak{F}(\mu_z) \, d\mu_z = \frac{1}{s} e^{-y} y^2 \mathfrak{F}(\sigma y) \frac{s}{s_{\alpha\alpha}} dy = \frac{1}{s_{\alpha\alpha}} e^{-y} y^2 \mathfrak{F}(\sigma y) \, dy$$

$\mathfrak{I} = \frac{1}{s_{\alpha\alpha}} \int\limits_0^{\infty} e^{-y} y^2 \mathfrak{F}(\sigma y) \, dy \qquad \mu_z = \sigma y$, $y = \frac{1}{\sigma}\mu_z$, $\mu_z = 0$, $\mu_z = \mu_0$; $\mu_z = \infty$

$$y = 0 \, , \; y = \tfrac{1}{\sigma}\mu_0 \, , \; y = \infty$$

$$\mathfrak{I} = \frac{1}{s_{\alpha\alpha}} \left\{ \int\limits_0^{\frac{1}{\sigma}\mu_0} e^{-y} y^2 \left[\int\limits_{\mu_0 - \sigma y}^{\infty} e^{-\xi^2} d\xi + \int\limits_{\mu_0 + \sigma y}^{\infty} e^{-\xi^2} d\xi \right] dy + \int\limits_{\frac{1}{\sigma}\mu_0}^{\infty} e^{-y} y^2 \left[\int\limits_{\sigma y - \mu_0}^{\infty} e^{-\xi^2} d\xi + \int\limits_{\sigma y + \mu_0}^{\infty} e^{-\xi^2} d\xi \right] dy \right\}$$

$$0 < \mu_z < \mu_0 \; , \; \mu_0 \ll 1 \, , \, \mu_z \ll 1$$

$$\mathfrak{F} = \int\limits_{\mu_0 - \mu_z}^{\infty} + \int\limits_{\mu_0 + \mu_z}^{\infty} = \sqrt{\pi} - \int\limits_0^{\mu_0 - \mu_z} - \int\limits_0^{\mu_0 + \mu_z} = \sqrt{\pi} - (\mu_0 - \mu_z) - (\mu_0 + \mu_z) = \sqrt{\pi} - 2\mu_0$$

$\mu_z = 0 \quad \mathfrak{F} = \sqrt{\pi} - 2\mu_0 \left[1 - \frac{1}{3}\mu_0^2 + \frac{1}{10}\mu_0^4 \right] \qquad \mathfrak{F}_{\mu_0} - \mathfrak{F}_0 = 2\mu_0 \left[\mu_0^2 - \frac{3}{2}\mu_0^4 \right]$

$\mu_z = \mu_0 \quad \mathfrak{F} = \sqrt{\pi} - 2\mu_0 \left[1 - \left(1 + \frac{1}{3}\right)\mu_0^2 + \left(\frac{3}{2} + \frac{1}{10}\right)\mu_0^4 \right] \qquad = \mu_0^3 \left[2 - 3\mu_0^2 \right]$

$$\frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi} x} \simeq \int_x^\infty e^{-\xi^2} d\xi$$

$$\int_x^\infty e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi\right) = \frac{\sqrt{\pi}}{2}(1-\Phi),\ x \ll 1,\ \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}} x\right)$$

$$x \gg 1,\ \frac{1}{2x} e^{-x^2}$$

$$\frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi} x},\ x \ll 1,\ \frac{\sqrt{\pi}}{2}(1-\sqrt{\pi} x),\ x \gg 1,\ \frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi} x}$$

Normalisierung: $\int_0^\infty dx \int_x^\infty e^{-\xi^2} d\xi = \frac{1}{2}$

$$\int_0^\infty dx \int_x^\infty e^{-\xi^2} d\xi = \int_0^\infty d\xi\, e^{-\xi^2} \int_0^\xi dx = \int_0^\infty d\xi\, e^{-\xi^2} \xi = \frac{1}{2} \int_0^\infty e^{-\xi^2} d\xi^2 = \frac{1}{2} \left| -e^{-\xi^2} \right|_0^\infty = \frac{1}{2}$$

$$\int_0^\infty dx \frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi} x} = \frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-\sqrt{\pi} x} d\sqrt{\pi} x = \frac{1}{2} \left| -e^{-\sqrt{\pi} x} \right|_0^\infty = \frac{1}{2}$$

$$0 < \mu_z < \mu_0,\ \mu_0 \ll 1,\ \mu_z \ll 1$$

$$F = \int_{\mu_0-\mu_z}^\infty e^{-\xi^2} d\xi + \int_{\mu_0+\mu_z}^\infty e^{-\xi^2} d\xi \simeq \frac{\sqrt{\pi}}{2}\left[e^{-\sqrt{\pi}(\mu_0-\mu_z)} + e^{-\sqrt{\pi}(\mu_0+\mu_z)}\right]$$

$$= \frac{\sqrt{\pi}}{2}\left[1 - \sqrt{\pi}(\mu_0-\mu_z) + 1 - \sqrt{\pi}(\mu_0+\mu_z)\right] = \frac{\sqrt{\pi}}{2}\left[2 - \sqrt{\pi}(\mu_0 - \mu_z + \mu_0 + \mu_z)\right]$$

$$= \sqrt{\pi} - \pi \mu_0 \text{ statt } \sqrt{\pi} - 2\mu_0$$

$$e^{-\sqrt{\pi}\mu_0}\left(e^{\sqrt{\pi}\mu_z} + e^{-\sqrt{\pi}\mu_z}\right) = e^{-\sqrt{\pi}\mu_0}\left(2 + \pi\mu_z^2 + \frac{1}{12}\pi^2\mu_z^4\right).$$

$$e^x = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \qquad \mu_z = 0,\ F = \frac{\sqrt{\pi}}{2} 2 e^{-\sqrt{\pi}\mu_0} = \sqrt{\pi} e^{-\sqrt{\pi}\mu_0}$$

$$e^{-x} = \underline{1 - \frac{x}{1} + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}} \qquad \mu_z = \mu_0,\ F = \sqrt{\pi} e^{-\sqrt{\pi}\mu_0}\left(1 + \frac{\pi}{2}\mu_0^2 + \frac{\pi^2}{24}\mu_0^4\right)$$

$$2 + x^2 + \frac{x^4}{12} \qquad F_{\mu_0} - F_0 = \sqrt{\pi} e^{-\sqrt{\pi}\mu_0}\left(\frac{\pi}{2}\mu_0^2 + \frac{\pi^2}{24}\mu_0^4\right)$$

$$F_0 = \sqrt{\pi} e^{-\sqrt{\pi}\mu_0} = \sqrt{\pi}\left(1 - \sqrt{\pi}\mu_0 + \frac{\pi}{2}\mu_0^2 - \frac{\pi^{\frac{3}{2}}}{6}\mu_0^3 + \right.$$

$$\mathcal{J} = \frac{1}{s_{\alpha\alpha}}\left\{\int_0^{\frac{1}{\sigma}\mu_0} e^{-y} y^2 \left[\frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi}(\mu_0-\sigma y)} + \frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi}(\mu_0+\sigma y)}\right] dy + \int_{\frac{1}{\sigma}\mu_0}^{\infty} e^{-y} y^2 \left[\frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi}(\sigma y-\mu_0)} + \frac{\sqrt{\pi}}{2} e^{-\sqrt{\pi}(\sigma y+\mu_0)}\right] dy\right\}$$

$$= \frac{1}{s_{\alpha\alpha}}\frac{\sqrt{\pi}}{2}\left\{ e^{-\sqrt{\pi}\mu_0}\int_0^{\frac{1}{\sigma}\mu_0} e^{-y} y^2 \left[e^{\sqrt{\pi}\sigma y} + e^{-\sqrt{\pi}\sigma y}\right] dy + e^{\sqrt{\pi}\mu_0}\int_{\frac{1}{\sigma}\mu_0}^{\infty} e^{-y} y^2 e^{-\sqrt{\pi}\sigma y} dy + e^{-\sqrt{\pi}\mu_0}\int_{\frac{1}{\sigma}\mu_0}^{\infty} e^{-y} y^2 e^{-\sqrt{\pi}\sigma y} dy\right.$$

$$\int_0^{\frac{1}{\sigma}\mu_0} e^{-y} y^2 e^{\sqrt{\pi}\sigma y} dy = \int_0^{\frac{1}{\sigma}\mu_0} e^{-y(1-\sqrt{\pi}\sigma)} y^2 dy, \quad y(1-\sqrt{\pi}\sigma) = x,\ y=0,\ x=0;\ y = \frac{\mu_0}{\sigma},\ x = \mu_0\left(\frac{1}{\sigma}-\sqrt{\pi}\right)$$

$$\int_0^{\frac{1}{\sigma}\mu_0} e^{-y(1-\sqrt{\pi}\sigma)} y^2 dy = (1-\sqrt{\pi}\sigma)^{-\frac{3}{2}} \int_0^{\mu_0(\frac{1}{\sigma}-\sqrt{\pi})} e^{-x} x^2 dx = (1-\sigma\sqrt{\pi})^{-\frac{3}{2}}\left[2 - \left(2 + 2\mu_0\left(\tfrac{1}{\sigma}-\sqrt{\pi}\right) + \mu_0(\ )^2\right) e^{-\mu_0(\frac{1}{\sigma}-\sqrt{\pi})}\right]$$

$$d\mathcal{J} = \frac{1}{s} e^{-\frac{s_\alpha}{s}}\left(\frac{s_\alpha}{s}\right)^2 dn_0 = \frac{1}{s} e^{-\frac{s_\alpha}{s}}\left(\frac{s_\alpha}{s}\right)^2 d\mu_z\, F(\mu_z), \quad s_\alpha = s_{\alpha\alpha}\mu_z, \quad d\mu_z = \frac{1}{s_{\alpha\alpha}} ds_\alpha$$

$$d\mathcal{J} = \frac{1}{s} e^{-\frac{s_\alpha}{s}}\left(\frac{s_\alpha}{s}\right)^2 \frac{1}{s_{\alpha\alpha}} ds_\alpha\, F\left(\frac{s_\alpha}{s_{\alpha\alpha}}\right) = \frac{1}{s_{\alpha\alpha}} e^{-\frac{s_\alpha}{s}}\left(\frac{s_\alpha}{s}\right)^2 F\left(\frac{s_\alpha}{s_{\alpha\alpha}}\right) d\frac{s_\alpha}{s}$$

$$\int_x^\infty e^{-\xi^2} d\xi = \int_0^\infty e^{-\xi^2} d\xi - \int_0^x e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}\left(1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi\right) = \frac{\sqrt{\pi}}{2}(1-\Phi)$$

$$x \ll 1, \quad \int_x^\infty e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}\left[1 - \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{1!\,3} + \frac{x^5}{2!\,5} - + \cdots\right)\right] = \frac{\sqrt{\pi}}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{10} - + \cdots\right)$$

$$x \gg 1, \quad \int_x^\infty e^{-\xi^2} d\xi = \frac{1}{2x} e^{-x^2}\left(1 - \frac{1}{2x^2} + \frac{1.3}{(2x^2)^2} - \frac{1.3.5}{(2x^2)^3} + - \cdots\right)$$

$0.9475-1$
$+0.6808-1$
$0.6283-1$
0.8862×0.4795

$x = \frac{1}{2}$ $(\ ) = \frac{1}{2} - \frac{1}{24} + \frac{1}{320} - +$ $= \frac{0.50313 \\ -0.0416}{0.4615}$ $\frac{\sqrt{\pi}}{2} - 0.4615$ $\frac{0.8862 \\ -0.4615}{0.4247}$ 0.4247 soll 0.4249

0.8862×0.1573

$x = 1$ $(\ ) = 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} - \frac{1}{1320} + \frac{1}{9360}$: 0.6667, 0.0461, 0.0039, 0.7467; 0.8862 − 0.7467 = 0.1395; 0.1395 " 0.1394

$x = 1$ $\frac{1}{2x} e^{-x^2} = \frac{1}{2} e^{-1} = 0.1840$ $1 - \frac{1}{2} + \frac{3}{4} - \frac{3.5}{8} + \frac{105}{16} -$

$x = 2$ $(\ ) = 2 - \frac{8}{3} + \frac{32}{10} - \frac{128}{42} + \frac{512}{216} - \frac{2048}{1320} + \frac{8192}{9360} - \frac{32768}{75600}$

0.8862×0.00468

$x = 2$ $\frac{1}{4} e^{-4} = 0.00458$ $1 - \frac{1}{8} + \frac{3}{64} - \frac{3.5}{8 \cdot 64}$ $\frac{0.875 \\ +0.018}{0.893}$ 0 $\quad$ 0.00410 soll 0.00415

$\frac{10.5}{4096}\left(1 - \frac{9}{8}\right)$ $\frac{3}{64}\left(1 - \frac{5}{8}\right)$ $\frac{9}{512}$

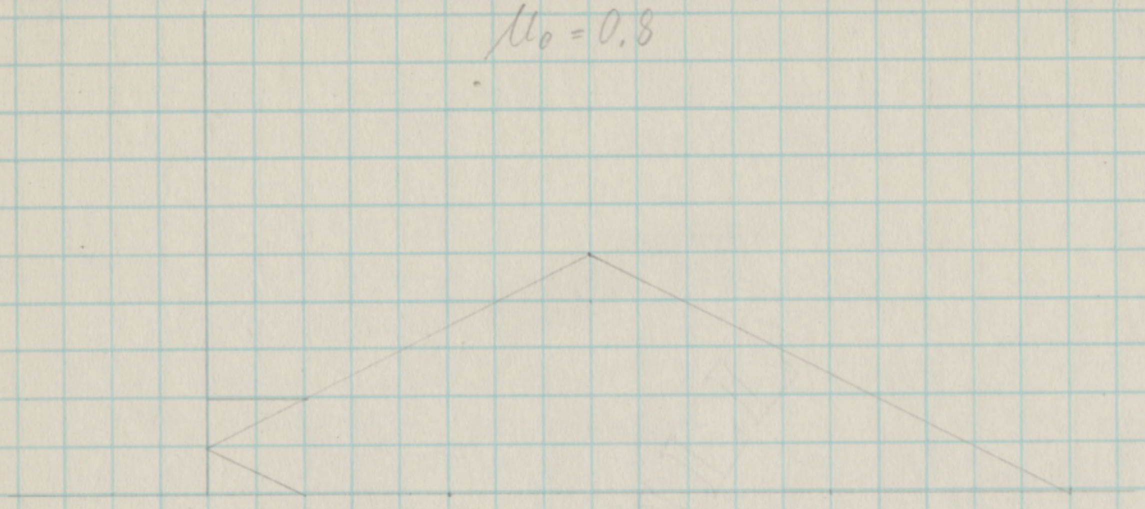
0.886 − 0.853 = 0.033 $\quad$ 0.375, 0.351, 0.112, 0.015, 853

0.8862×0.0339

$x = 1.5$ $(\ ) = 1.5 - 1.125 + 3.375 \cdot 0.225 - 3.375 \frac{1.5^4}{6 \cdot 7} +$ $\quad$ 0.033 soll 0.030

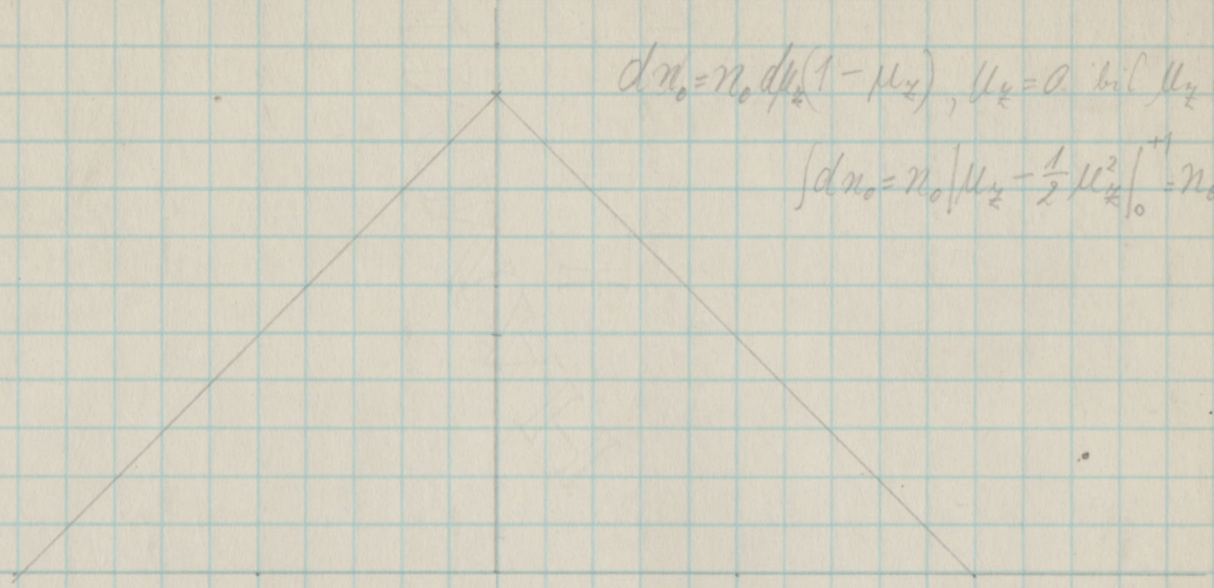
$3.375\left(0.225 - \frac{8.37}{28}\right)$

$\mu_0 = 0.8$



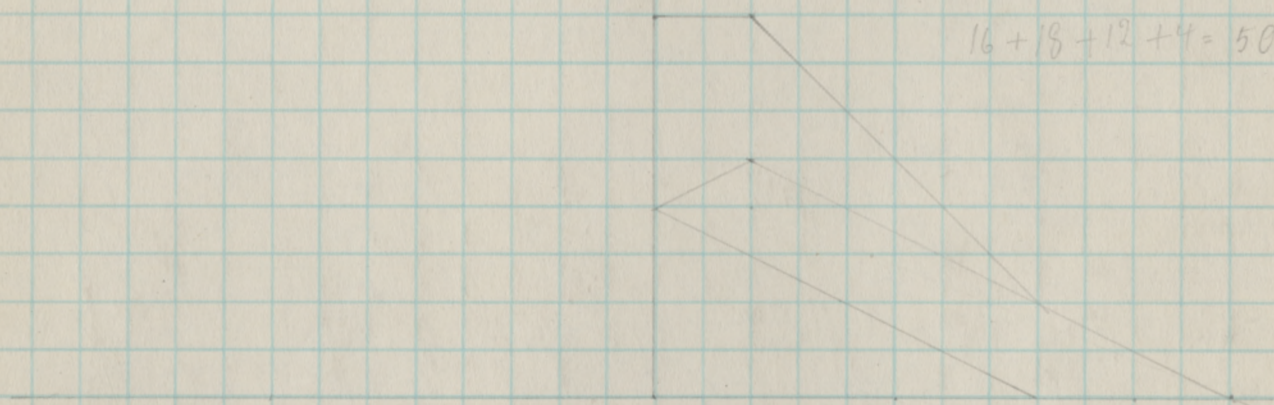
$$d n_0 = n_0 \, d\mu_z (1-\mu_z), \quad \mu_z = 0 \text{ bis } \mu_z = \pm 1$$

$$\int d n_0 = n_0 \left| \mu_z - \frac{1}{2}\mu_z^2 \right|_0^{+1} = n_0 \frac{1}{2}$$

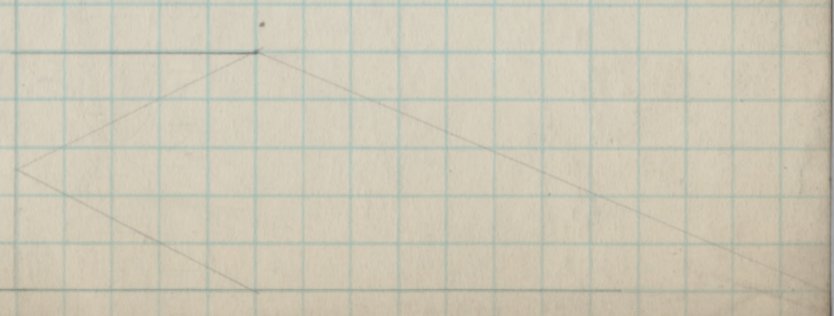


$\mu_0 = 0.2$

16 + 18 + 12 + 4 = 50



$\mu_0 = 0.5$

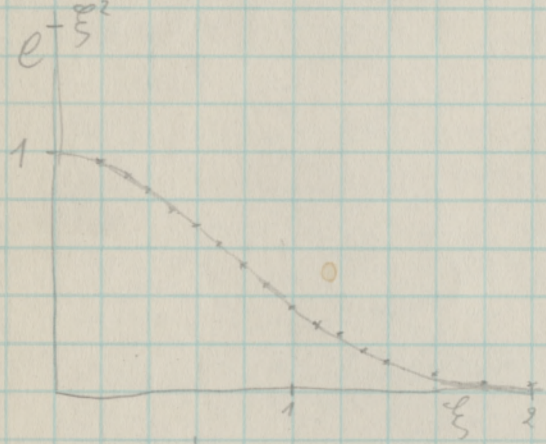


$$y_s = \frac{dn_0}{s_\alpha}\int_{\frac{s}{s_\alpha}}^{\infty} e^{-x^2}dx \simeq \frac{dn_0}{s_\alpha}\left[\frac{\sqrt{\pi}}{2} - \frac{s}{s_\alpha} + \frac{1}{3}\left(\frac{s}{s_\alpha}\right)^3\right] = \frac{dn_0}{s_\alpha}\left[\frac{\sqrt{\pi}}{2} - \frac{s}{s_\alpha}\left(1-\frac{1}{3}\left(\frac{s}{s_\alpha}\right)^2\right)\right]$$

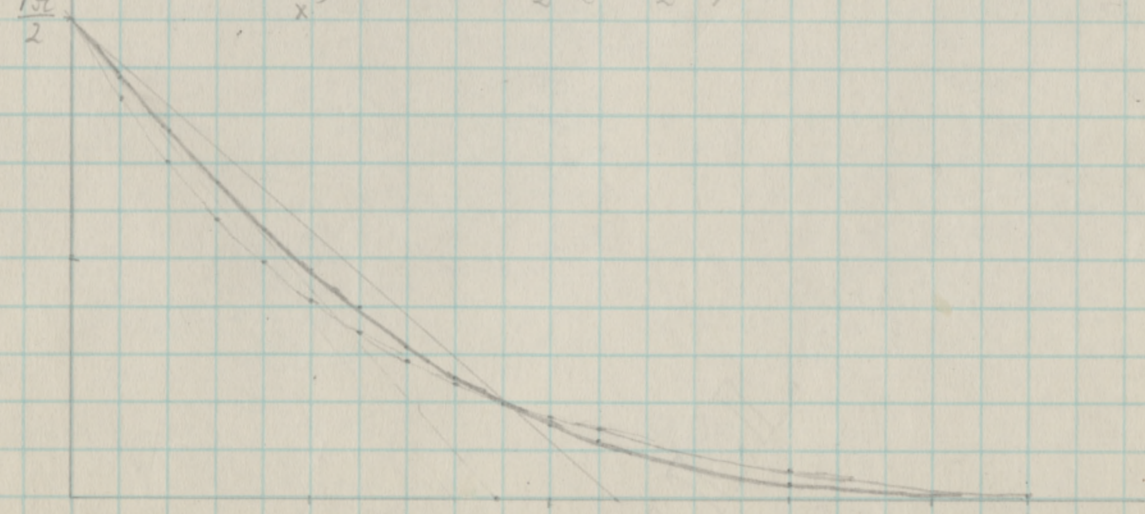
$\frac{s}{s_\alpha} = \frac{1}{2}$ $\quad \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{10}$

$1-\frac{1}{3}\left(\frac{s}{s_\alpha}\right)^2 = 1-\frac{1}{12} \quad 1-\frac{1}{27} \quad 1-\frac{1}{75} \quad 1-\frac{1}{300}$

$$s = u\frac{\mathcal{E}}{y} = u_0\frac{\mathcal{E}}{y} + u_\alpha x\frac{\mathcal{E}}{y}$$



$$\int_x^{\infty} e^{-\xi^2}d\xi \simeq \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right)$$



$$\frac{\mu_0}{\mu_\alpha}=1 \quad J=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left[2-\left(2+\frac{2}{\sigma}+\frac{1}{\sigma^2}\right)e^{-\frac{1}{\sigma}}\right] \qquad \sigma=\frac{s}{s_{\alpha\alpha}}$$

$$\sigma=0,\ J=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\cdot 2,\quad \sigma=\infty,\ J=0$$

$$\frac{dJ}{d\sigma}=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left[\left(\frac{2}{\sigma^2}+\frac{1}{\sigma^3}\right)e^{-\frac{1}{\sigma}}-\left(2+\frac{2}{\sigma}+\frac{1}{\sigma^2}\right)\frac{1}{\sigma^2}e^{-\frac{1}{\sigma}}\right]$$

$$=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}e^{-\frac{1}{\sigma}}\left[\cancel{\frac{2}{\sigma^2}}+\frac{1}{\sigma^3}-\cancel{\frac{2}{\sigma^2}}-\frac{2}{\sigma^3}-\frac{1}{\sigma^4}\right]=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left(\frac{1}{\sigma^3}+\frac{1}{\sigma^4}\right)e^{-\frac{1}{\sigma}}$$

$$\sigma=0\ \frac{dJ}{d\sigma}=0,\quad \sigma=\infty\ \frac{dJ}{d\sigma}=0$$

$$\sigma=1,\ J=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left[2-\overset{1.84}{5\times 0.368}\right]=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\ 0.16$$

$$\sigma=\frac{1}{2}\quad J=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left[2-\overset{10\times\ =1.3534}{(2+4+4)\ 0.13534}\right]=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\times 0.6466$$

$$\sigma=\frac{1}{4}\quad J=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\left[2-\overset{26\times\ =0.476}{(2+8+16)\ 0.01832}\right]=\frac{n_0'}{s_{\alpha\alpha}}\mathcal{C}\times 1.524$$

$$\mu_\alpha=1$$

$$\int_{\mu_z+\mu_0}^{\infty}+\int_{\mu_z-\mu_0}^{\infty}=\int_{\mu_z}^{\infty}-\int_{\mu_z}^{\mu_z+\mu_0}+\int_{\mu_z}^{\infty}+\int_{\mu_z-\mu_0}^{\mu_z}=2\int_{\mu_z}^{\infty}+\int_{\mu_z-\mu_0}^{\mu_z}-\int_{\mu_z}^{\mu_z+\mu_0}$$

$\sqrt{\pi}=1.772454$

886 23
2.658 7

$$\int_x^{\infty}e^{-\xi^2}d\xi=\int_0^{\infty}e^{-\xi^2}d\xi-\int_0^{x}e^{-\xi^2}d\xi=\frac{\sqrt{\pi}}{2}\left(1-\frac{2}{\sqrt{\pi}}\int_0^{x}e^{-\xi^2}d\xi\right)=\frac{\sqrt{\pi}}{2}(1-\Phi)$$

$$\simeq\frac{\sqrt{\pi}}{2}e^{-\sqrt{\pi}x}\ \text{or}\ \frac{\sqrt{\pi}}{2}\left(1-\frac{\sqrt{\pi}}{2}x\right),\ x\ll 1:\ \frac{\sqrt{\pi}}{2}\left(1-\overset{1.772}{\sqrt{\pi}}x\right)\ \text{or}\ \frac{\sqrt{\pi}}{2}\left(1-\overset{0.886}{\frac{\sqrt{\pi}}{2}}x\right),\ \text{act.}\ \frac{\sqrt{\pi}}{2}\left(1-\overset{1.128}{\frac{2}{\sqrt{\pi}}}x\right)$$

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.5	2
$\sqrt{\pi}x$	0.17725	0.35450	0.53174	0.70898	0.88623	1.06347	1.24072	1.4180	1.5952	1.7725	1.950	2.66	3.5449
$e^{-\sqrt{\pi}x}$	0.8376	0.7016	0.5876	0.4921	0.4123	0.3453	0.2892	0.2422	0.2029	0.1699	0.14227	0.07	0.02884
$1-\Phi$	0.8875	0.7773	0.6714	0.5716	0.4795	0.3961	0.3222	0.2579	0.2031	0.1573	0.1198	0.033	0.00468
$1-\frac{\sqrt{\pi}}{2}x$	0.9114	0.8228	0.7341	0.6455	0.5569	0.4683	0.3796	0.2910	0.2024	0.1134	0.025		

$$\mathcal{J} = \frac{n_0'}{s_\alpha} e^{-y} y^3 , \quad y = \frac{s_\alpha}{s} = \frac{1}{\sigma} , \quad \mathcal{J} = \frac{n_0'}{s_\alpha} e^{-\frac{1}{\sigma}} \frac{1}{\sigma^3}$$

$$s_\alpha = s_{\alpha 0}(1+\varepsilon), \quad y = \frac{s_{\alpha 0}(1+\varepsilon)}{s} = \frac{s_{\alpha 0}}{s} + \frac{\varepsilon s_{\alpha 0}}{s} = \frac{1}{\sigma} + \delta , \quad \delta = \varepsilon \frac{s_{\alpha 0}}{s} = \frac{\varepsilon}{\sigma}$$

$$\mathcal{J} = \frac{n_0'}{s_{\alpha 0}} \frac{1}{1+\varepsilon} e^{-\frac{1}{\sigma}} e^{-\delta} \left(\frac{1}{\sigma} + \delta\right)^3 = \frac{n_0'}{s_{\alpha 0}} e^{-\frac{1}{\sigma}} (1-\delta\sigma)(1-\delta) \frac{1}{\sigma^3} (1+3\delta\sigma) = \frac{n_0'}{s_{\alpha 0}} e^{-\frac{1}{\sigma}} \frac{1}{\sigma^3} (1+2\delta\sigma-\delta)$$

$$\overline{\mu_z} = \frac{2}{3\sqrt{\pi}} \mu_\alpha , \quad \mu_\alpha = \mu_0 \frac{\sqrt{\pi}}{2} , \quad \overline{\mu_z} = \frac{2}{3\sqrt{\pi}} \frac{\sqrt{\pi}}{2} \mu_0 = \frac{1}{3}\mu_0 \qquad \frac{s_\alpha}{s_{\alpha 0}} = 1+\varepsilon = \frac{\mu_0 + \frac{1}{3}\mu_0}{\mu_0} = 1 + \frac{1}{3} , \quad \varepsilon = \frac{1}{3}$$

$$\mathcal{J} = \frac{n_0'}{s_{\alpha 0}} e^{-y} y^3 , \quad y = \frac{s_{\alpha 0}}{s} = \frac{1}{5} , \quad s = 5 s_{\alpha 0} , \quad \mathcal{J} = \frac{n_0'}{s_{\alpha 0}} \frac{0.8187}{125} = \frac{n_0'}{s_{\alpha 0}} 0.0065496$$

$$\mathcal{J} = \frac{n_0'}{s_\alpha} e^{-y} y^3 , \quad s_\alpha = s_{\alpha 0}\left(1+\frac{1}{3}\right) , \quad y = \frac{s_\alpha}{s} = \frac{s_{\alpha 0}}{s}\left(1+\frac{1}{3}\right) = 0.2 \times \frac{4}{3} = 0.26666 \quad y^3 = \frac{1}{125}\left(\frac{4}{3}\right)^3$$

$$\mathcal{J} = \frac{n_0'}{s_{\alpha 0}} \frac{3}{4} e^{-0.2} e^{-0.06666} \frac{1}{125} \left(\frac{4}{3}\right)^3 = \frac{n_0'}{s_{\alpha 0}} e^{-0.2} \frac{1}{125} \underbrace{\times \frac{3}{4} \times 0.9387 \frac{64}{27}}_{0.9387 \cdot \frac{16}{9} = 1.67}$$

$$d\mathcal{J} = \frac{1}{s} e^{-y} y^2 \, dn_0 , \quad y = \frac{s_\alpha}{s} , \quad \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_z}{\mu_\alpha} , \quad s_\alpha = s_{\alpha\alpha} \frac{\mu_z}{\mu_\alpha} , \quad dn_0 = n_0' \, d\frac{\mu_z}{\mu_\alpha} \, \mathfrak{F}\left(\frac{\mu_z}{\mu_\alpha}\right)$$

$$\frac{\mu_z}{\mu_\alpha} = \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{s}{s_{\alpha\alpha}} y = \sigma y , \quad d\frac{\mu_z}{\mu_\alpha} = \sigma \, dy , \quad dn_0 = n_0' \sigma \, dy \, \mathfrak{F}(\sigma y)$$

$$d\mathcal{J} = \frac{1}{s} n_0' \sigma e^{-y} y^2 \mathfrak{F}(\sigma y) \, dy , \quad \mathcal{J} = \frac{n_0'}{s_{\alpha\alpha}} \int_{y=0}^{y=\infty} e^{-y} y^2 \mathfrak{F}(\sigma y) \, dy$$

$\mathfrak{F}(\sigma y) = \mathcal{C}$ von $\mu_z = 0$ bis $\mu_z = \mu_0$ d.h. $\sigma y = 0$ bis $\sigma y = \frac{\mu_0}{\mu_\alpha}$, $y = \frac{1}{\sigma}\frac{\mu_0}{\mu_\alpha}$

$$\mathcal{J} = \frac{n_0'}{s_{\alpha\alpha}} \mathcal{C} \int_{y=0}^{y=\frac{1}{\sigma}\frac{\mu_0}{\mu_\alpha}} e^{-y} y^2 \, dy = \frac{n_0'}{s_{\alpha\alpha}} \mathcal{C} \left[2 - \left(2 + 2\frac{1}{\sigma}\frac{\mu_0}{\mu_\alpha} + \frac{1}{\sigma^2}\frac{\mu_0}{\mu_\alpha}\right) e^{-\frac{1}{\sigma}\frac{\mu_0}{\mu_\alpha}} \right]$$

$$\mu_0 - \sigma y = 0,\ \sigma y = \mu_0,\ \frac{s}{s_{\alpha\alpha}} = \frac{\mu_0}{y} = \frac{\mu_0}{s_\alpha} s,\ y = \frac{\mu_0}{\sigma} = \frac{\mu_0 s_{\alpha\alpha}}{s} = \frac{s_\alpha}{s}$$

$$\mu_0 + \sigma y = 2\mu_0 \qquad x \ll 1$$

$$\int_x^\infty e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2} - \left(x - \frac{x^3}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + \frac{x^9}{4!9} - + \dots + \frac{x^{2n+1}}{n!(2n+1)}(-1)^n\right)$$

$$\int_{\mu_0-\mu_z}^\infty + \int_{\mu_0+\mu_z}^\infty = \sqrt{\pi} - \left[\begin{matrix} \mu_0 - \mu_z \\ + \mu_0 + \mu_z \end{matrix} - \frac{1}{3}\begin{pmatrix} \mu_0^3 - 3\mu_0^2\mu_z + 3\mu_0\mu_z^2 - \mu_z^3 \\ + \mu_0^3 + 3\mu_0^2\mu_z + 3\mu_0\mu_z^2 + \mu_z^3 \end{pmatrix}\right.$$

$$2\mu_0 - \frac{2}{3}(\mu_0^3 + 3\mu_0\mu_z^2)$$

$$= \sqrt{\pi} - 2\mu_0 + \frac{2}{3}\mu_0(\mu_0^2 + 3\mu_z^2) = \sqrt{\pi} - 2\mu_0\left[1 - \frac{1}{3}(\mu_0^2 + 3\mu_z^2) + \right]$$

$$\int_{y_0}^\infty e^{-y} dy = -e^{-y}\Big|_{y_0}^\infty = e^{-y_0}$$

$$\int_{y_0}^\infty e^{-y} y\, dy = -\int_{y_0}^\infty y\, de^{-y} = -ye^{-y}\Big|_{y_0}^\infty + \int_{y_0}^\infty e^{-y} dy = y_0 e^{-y_0} + e^{y_0} = (1+y_0)e^{-y_0} = y_0 e^{-y_0} + \int$$

$$\int_{y_0}^\infty e^{-y} y^2 dy = -\int_{y_0}^\infty y^2 de^{-y} = -y^2 e^{-y}\Big|_{y_0}^\infty + 2\int_{y_0}^\infty e^{-y} y\, dy = y_0^2 e^{-y_0} + 2\int_{y_0}^\infty e^{-y} y\, dy$$

$$\int_{y_0}^\infty e^{-y} y^3 dy = -\int_{y_0}^\infty y^3 de^{-y} = -y^3 e^{-y}\Big|_{y_0}^\infty + 3\int_{y_0}^\infty e^{-y} y^2 dy = y_0^3 e^{-y_0} + 3\int_{y_0}^\infty e^{-y} y^2 dy$$

$$\int_{y_0}^\infty e^{-y} y^n dy = -\int_{y_0}^\infty y^n de^{-y} = -y^n e^{-y}\Big|_{y_0}^\infty + n\int e^{-y} y^{(n-1)} dy = y_0^n e^{-y_0} + n\int_{y_0}^\infty e^{-y} y^{(n-1)} dy$$

$$F_1 = (y_0+1)e^{-y_0},\ F_2 = (y_0^2 + 2y_0 + 2)e^{-y_0}\ F_3 = (y_0^3 + 3y_0^2 + 6y_0 + 6)e^{-y_0}$$

$$F_4 = (y_0^4 + 4y_0^3 + 12y_0^2 + 24y_0 + 24)e^{-y_0},\ F_5 = (y_0^5 + 5y_0^4 + 20y_0^3 + 60y_0^2 + 120y_0 + 120)e^{-y_0}$$

$$F_n = \left[y_0^n + n y_0^{n-1} + n(n-1)y_0^{n-2} + n(n-1)(n-2)y_0^{n-3} + \dots + n!\, y_0 + n!\right]e^{-y_0}$$

$$\int_{y_1}^{y_2} e^{-y}\,dy = e^{-y_1} - e^{-y_2}$$

$$\int_{0}^{y_2} e^{-y}\,dy = 1 - e^{-y_2} \qquad \int_0^\infty = 1$$

$$\int_{y_1}^{\infty} e^{-y}\,dy = e^{-y_1}$$

$$\int_{y_1}^{y_2} e^{-y} y\,dy = (1+y_1)e^{-y_1} - (1+y_2)e^{-y_2}$$

$$\int_{0}^{y_2} \quad = \quad 1 - (1+y_2)e^{-y_2} \qquad \int_0^\infty = 1$$

$$\int_{y_1}^{\infty} \quad = (1+y_1)e^{-y_1}$$

$$\int_{y_1}^{y_2} e^{-y} y^2\,dy = (2+2y_1+y_1^2)\,e^{-y_1} - (2+2y_2+y_2^2)\,e^{-y_2}$$

$$\int_{0}^{y_2} \quad = \quad 2 \quad - (2+2y_2+y_2^2)\,e^{-y_2} \qquad \int_0^\infty = 2$$

$$\int_{y_1}^{\infty} \quad = (2+2y_1+y_1^2)\,e^{-y_1}$$

$$\int_{y_1}^{y_2} e^{-y} y^3\,dy = (6+6y_1+3y_1^2+y_1^3)\,e^{-y_1} - (6+6y_2+3y_2^2+y_2^3)\,e^{-y_2}$$

$$\int_{0}^{y_2} \quad = \quad 6 \quad - (6+6y_2+3y_2^2+y_2^3)\,e^{-y_2} \qquad \int_0^\infty = 6$$

$$\int_{y_1}^{\infty} \quad = (6+6y_1+3y_1^2+y_1^3)\,e^{-y_1}$$

$\sqrt{\pi} = 1.772\ 454 \qquad \frac{\sqrt{\pi}}{2} = 0.886\ 227$

$\frac{1}{\sqrt{\pi}} = 0.564\ 190 \qquad \frac{2}{\sqrt{\pi}} = 1.128\ 379$

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt, \quad 1 = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}dt$$

$$\int_0^x e^{-t^2}dt = \frac{\sqrt{\pi}}{2}\Phi(x), \quad \int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$$

$$\int_x^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}\left[1-\Phi(x)\right]$$

$$\begin{array}{r} 6039 \\ \underline{9523} \\ 1.5562 \\ 7781 \end{array} \qquad \begin{array}{r} \cancel{5021} \\ 5205 \\ \underline{9661} \\ 14866 \\ 7433 \end{array} \qquad \begin{array}{r} 4284 \\ \underline{9763} \\ 14047 \\ 7023_5 \end{array}$$

$$\begin{array}{r} 2227 \\ \underline{9891} \\ 12118 \\ 6059 \\ \\ 4977 \end{array} \qquad \begin{array}{r} 2227 \\ \underline{9981} \\ 12208 \\ 6104 \end{array} \qquad \begin{array}{r} 2579 \\ 2580 \end{array}$$

$$\begin{array}{r} 3286 \\ \underline{9838} \\ 13124 \\ 6562 \end{array} \qquad \begin{array}{r} 1125 \\ \underline{9926} \\ 11053 \\ 5526_5 \end{array} \qquad \begin{array}{r} 1125 \\ \underline{9970_2} \\ 11096 \\ 5548 \end{array}$$

$$\begin{array}{r} 3286 \\ \underline{9988} \\ 13274 \\ 6637 \end{array} \qquad \begin{array}{r} 0.5716 \\ \underline{6} \\ 5722 \\ 2861 \end{array} \qquad \begin{array}{r} 4795 \\ 5 \\ 4800 \end{array} \qquad \begin{array}{r} 3961 \\ \underline{3} \\ 3964 \\ 3224 \end{array}$$

	y	y^2	e^{-y}	y^2e^{-y}
2032	3.5	12.25	0.03020	0.3700
1573	4.5	20.25	0.01111	0.2250
1198	5.5	30.25	0.00409	0.1237
0897				
0660				
0477				
0339				
238				
162				
109				

0.3675
25

2025
2025
2025
20

2250

1210
27

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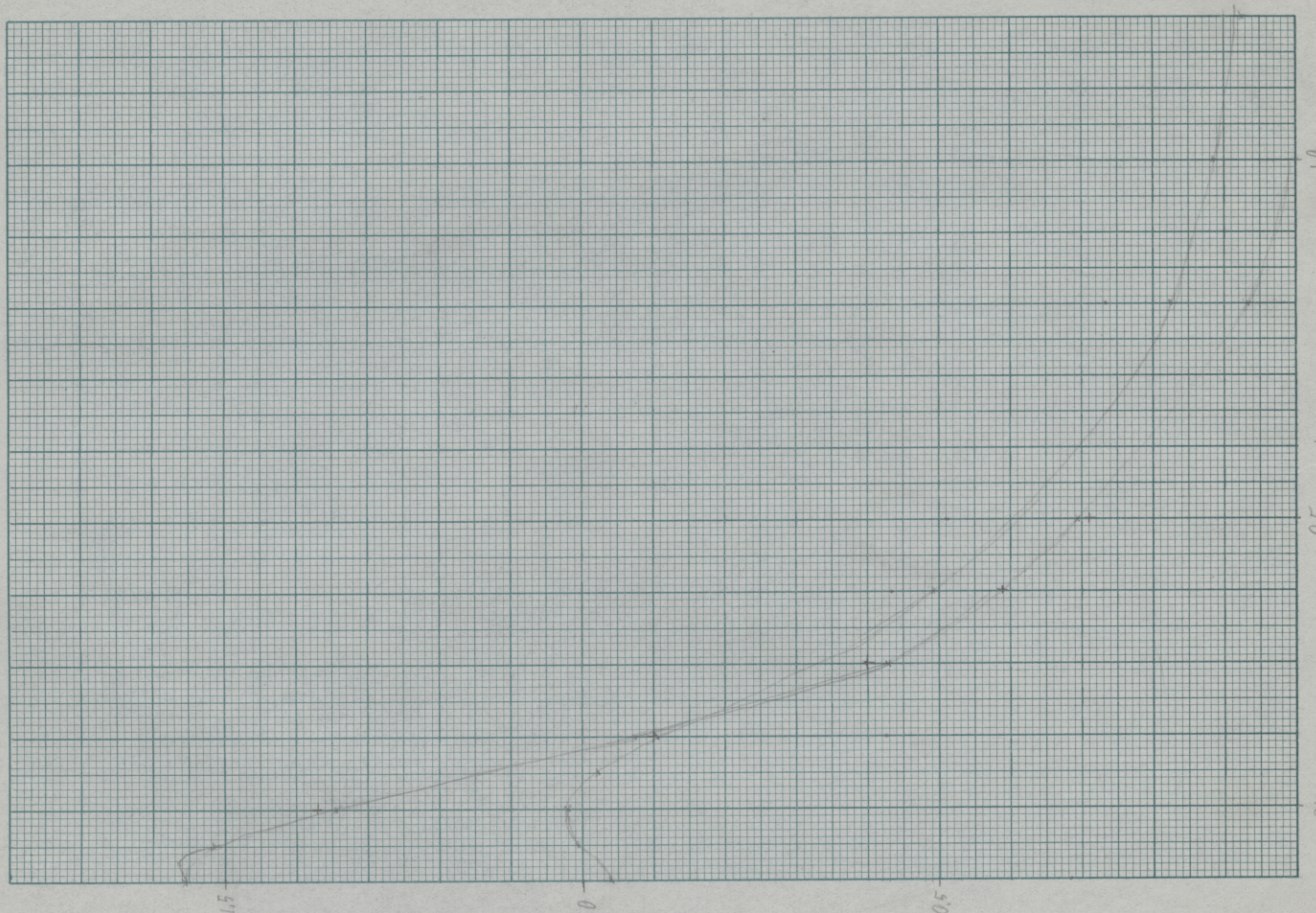

I

Kernspin $\frac{1}{2}$, Molecül $+1, 0, -1, 0$

$\frac{1}{2} n_0 : \mu_0 = 0$, $\frac{1}{4} n_0 : +\mu_0$, $\frac{1}{4} n_0 : -\mu_0$, μ_0 2mal Kernmoment

$$\underline{\mu_0 = 0}$$

$$dn = \frac{1}{2} n_0 \, d\frac{\mu_z}{\mu_\alpha} \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2} dx, \quad \int dn = \frac{1}{2} n_0 \int_0^\infty dy \int_y^\infty e^{-x^2} dx = \frac{1}{2} n_0 \int_0^\infty dx \int_0^x e^{-x^2} dy$$

$$= \frac{1}{2} n_0 \int_0^\infty e^{-x^2} x \, dx = \frac{1}{2} n_0 \frac{1}{2} = \frac{1}{4} n_0$$

$$dJ = \frac{dn}{s} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^2}{s^2}, \quad \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu_z}{\mu_\alpha}, \quad \frac{s_\alpha}{s} = y, \quad d\frac{\mu_z}{\mu_\alpha} = d\frac{s_\alpha}{s_{\alpha\alpha}}, \quad \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{s\,y}{s_{\alpha\alpha}} = \sigma y$$

$$dn = \frac{1}{2} n_0 \, d\frac{\mu_z}{\mu_\alpha} \int_{\frac{\mu_z}{\mu_\alpha}}^{\infty} e^{-x^2} dx = \frac{1}{2} n_0 \frac{1}{s_{\alpha\alpha}} ds_\alpha \int_{\frac{s_\alpha}{s_{\alpha\alpha}}}^{\infty} e^{-x^2} dx$$

$$dJ = \frac{1}{2} \frac{n_0}{s_{\alpha\alpha}} d\frac{s_\alpha}{s} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^2}{s^2} \int_{\frac{s_\alpha}{s_{\alpha\alpha}}}^{\infty} e^{-x^2} dx, \quad J = \frac{1}{2} \frac{n_0}{s_{\alpha\alpha}} \int_0^\infty e^{-y} y^2 dy \int_{\sigma y}^{\infty} e^{-x^2} dx$$

$$\underline{\mu_0 = \mu_0}$$

$$\mu = \mu_z \pm \mu_0, \quad \mu > \mu_0 \quad dn = \frac{1}{4} n_0 \, d\frac{\mu_z}{\mu_\alpha} \left[\int_{\frac{\mu - \mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx + \int_{\frac{\mu + \mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx \right]$$

$$\mu < \mu_0 \quad dn = \frac{1}{4} n_0 \, d\frac{\mu_z}{\mu_\alpha} \left[\int_{\frac{\mu_0 - \mu}{\mu_\alpha}} e^{-x^2} dx + \int_{\frac{\mu_0 + \mu}{\mu_\alpha}} e^{-x^2} dx \right]$$

$$dJ = \frac{dn}{s} e^{-\frac{s_\alpha}{s}} \frac{s_\alpha^2}{s^2}, \quad \frac{s_\alpha}{s_{\alpha\alpha}} = \frac{\mu}{\mu_\alpha}, \quad \frac{s_\alpha}{s} = y, \quad d\mu_z = d\mu, \quad d\frac{\mu_z}{\mu_\alpha} = d\frac{\mu}{\mu_\alpha} = \frac{1}{s_{\alpha\alpha}} ds_\alpha$$

$$\mu > \mu_0 \quad dn = \frac{1}{4} \frac{n_0}{s_{\alpha\alpha}} ds_\alpha \left[\int_{\sigma y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx + \int_{\sigma y + \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx \right] \qquad \frac{\mu}{\mu_\alpha} = \frac{s}{s_{\alpha\alpha}} y = \sigma y,$$

$$\mu < \mu_0 \quad dn = \frac{1}{4} \frac{n_0}{s_{\alpha\alpha}} ds_\alpha \left[\int_{\frac{\mu_0}{\mu_\alpha} - \sigma y}^{\infty} e^{-x^2} dx + \int_{\frac{\mu_0}{\mu_\alpha} + \sigma y}^{\infty} e^{-x^2} dx \right]$$

II

$$\mu > \mu_0 : dJ = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}} dy\, e^{-y} y^2 \left[\int_{\sigma y - \frac{\mu_0}{\mu_\alpha}} + \int_{\sigma y + \frac{\mu_0}{\mu_\alpha}} \right]$$

$$y = \frac{s_\alpha}{s} = \frac{\mu}{\mu_\alpha}\frac{s_{\alpha\alpha}}{s} = \frac{\mu}{\mu_\alpha}\frac{1}{\sigma}$$

$$\mu < \mu_0 : dJ = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}} dy\, e^{-y} y^2 \left[\int_{\frac{\mu_0}{\mu_\alpha} - \sigma y} + \int_{\frac{\mu_0}{\mu_\alpha} + \sigma y} \right]$$

$\mu = 0 \quad y = 0$

$\mu = \mu_0 \quad y = \frac{\mu_0}{\mu_\alpha}\frac{1}{\sigma}$

$\mu = \infty \quad y = \infty$

$$J_\sigma = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}} \int_{\frac{\mu_0}{\mu_\alpha}\frac{1}{\sigma}}^{\infty} e^{-y} y^2 dy \left[\int_{\sigma y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx + \int_{\sigma y + \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx \right] + \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}} \int_0^{\frac{\mu_0}{\mu_\alpha}\frac{1}{\sigma}} e^{-y} y^2 dy \left[\int_{\frac{\mu_0}{\mu_\alpha} - \sigma y}^{\infty} e^{-x^2} dx + \int_{\frac{\mu_0}{\mu_\alpha} + \sigma y}^{\infty} e^{-x^2} dx \right]$$

$$J_\sigma = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}} \int_0^\infty e^{-y} y^2 F(\sigma y)\, dy$$

$$0 < y < \frac{\mu_0}{\mu_\alpha}\frac{1}{\sigma} : F(\sigma y) = \int_{\frac{\mu_0}{\mu_\alpha} - \sigma y}^{\infty} e^{-x^2} dx + \int_{\frac{\mu_0}{\mu_\alpha} + \sigma y}^{\infty} e^{-x^2} dx$$

$$\frac{\mu_0}{\mu_\alpha}\frac{1}{\sigma} < y < \infty : F(\sigma y) = \int_{\sigma y - \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx + \int_{\sigma y + \frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx$$

$\sigma y = \frac{\mu}{\mu_\alpha} \quad \mu_\alpha = 1 \quad \sigma y = \mu$

$$\sigma = 0,\ \sigma y = 0,\ F(\sigma y) = 2\int_{\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx,\quad J_{\sigma=0} = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}}\, 2 \times 2 \int_{\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx = \frac{n_0}{s_{\alpha\alpha}} \int_{\frac{\mu_0}{\mu_\alpha}}^{\infty} e^{-x^2} dx$$

$$\int_{x_0}^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}\left[1 - \Phi(x_0)\right],\quad F(\sigma y) = \sqrt{\pi}\left\{1 - \frac{1}{2}\left[\Phi\left(\frac{\mu_0}{\mu_\alpha} - \sigma y\right) + \Phi\left(\frac{\mu_0}{\mu_\alpha} + \sigma y\right)\right]\right\} = \sqrt{\pi}\, F$$

$$F(0) = \sqrt{\pi}\left\{1 - \Phi\left(\frac{\mu_0}{\mu_\alpha}\right)\right\}$$

$$\sigma y < \frac{\mu_0}{\mu_\alpha} : F = 1 - \frac{1}{2}\left[\Phi\left(\frac{\mu_0}{\mu_\alpha} - \sigma y\right) + \Phi\left(\frac{\mu_0}{\mu_\alpha} + \sigma y\right)\right]$$

$$\sigma y > \frac{\mu_0}{\mu_\alpha} : F = 1 - \frac{1}{2}\left[\Phi\left(\sigma y - \frac{\mu_0}{\mu_\alpha}\right) + \Phi\left(\sigma y + \frac{\mu_0}{\mu_\alpha}\right)\right]$$

$$\mu_0 = 0 : F = 1 - \Phi(\sigma y)$$

$$J_\sigma = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}}\sqrt{\pi}\int_0^\infty e^{-y} y^2 F\, dy = \frac{1}{4}\frac{n_0}{s_{\alpha\alpha}}\sqrt{\pi}\int_0^\infty P\, dy$$

$$P = e^{-y} y^2 F$$

$$\frac{\mu_0}{\mu_d} = 1$$

$\sigma y < 1$

$$F = 1 - \frac{1}{2}\left[\Phi(1-\sigma y) + \Phi(1+\sigma y)\right]$$

$\sigma y > 1$

$$F = 1 - \frac{1}{2}\left[\Phi(\sigma y - 1) + \Phi(\sigma y + 1)\right]$$

$\mu = \sigma y$	F		
0	0.1573		
0.1	0.1615	2.1	0.0599
0.2	0.1738	2.2	0.0449
0.3	0.1941	2.3	0.0330
0.4	0.2219	2.4	0.0239
0.5	0.2567	2.5	0.0170
0.6	0.2976_5	2.6	0.0119
0.7	0.3438	2.7	0.0081
0.8	0.3941	2.8	0.0055
0.9	0.4473_5	2.9	0.0036
1.0	0.5023	3.0	0.0023
1.1	0.4452	3.1	0.0015
1.2	0.3896	3.2	0.0009
1.3	0.3363		
1.4	0.2861		
1.5	0.2400		
1.6	0.1982		
1.7	0.1612		
1.8	0.1290		
1.9	0.1016		
2.0	0.0787		

IV

$$\frac{\mu_0}{\mu_a} = 1$$

$\sigma = 0.2$

y	σy	F		P				y	$y^2 e^{-y}$
0	0	0.1573		0				0	0
0.5	0.1	0.1615		0.0245	0.0061			0.5	0.1516_3
1.0	0.2	0.1738	885	0.0640	0.0221			1.0	0.3679
1.5	0.3	0.1941	1614	0.0974	0.0403_5			1.5	0.5020
2.0	0.4	0.2219	2174	0.1200	0.0543_5			2.0	0.5414
2.5	0.5	0.2567	2516	0.1316	0.0629			2.5	0.5129
3.0	0.6	0.2977	2648	0.1332	0.0662			3.0	0.4480
3.5	0.7	0.3438	2602	0.1270		0.0650_5 0.0606 12 56			
4.0	0.8	0.3941	2424	0.1154	0.1243	0.1256	+0.0013	4.0	0.2931
4.5	0.9	0.4473_5	2158	0.1004		0.0540 0.0462			
5.0	1.0	0.5023	1850	0.0846	0.1000	0.1002	+0.0002	5.0	0.1684_5
5.5	1.1	0.4452	1398	0.0552		0.0350 0.0225			
6.0	1.2	0.3896	900	0.0348	0.0597	0.0575	-0.0022	6.0	0.0892_4
7.0	1.4	0.2861	446	0.0128	0.0238			7.0	0.0447
8.0	1.6	0.1982	170	0.0042	0.0085			8.0	0.0215
9.0	1.8	0.1290	55	0.0013	0.0028			9.0	0.0100
10.0	2.0	0.0787	14	0.0004	0.0008			10.0	0.0045
					356				
					0.572 : 4 − 0.0007			12	0.0009
					0.143				

$$\frac{\mu_0}{\mu_2} = 1$$

$\sigma = 0.3$ $\sigma = 0.4$

y	σy	F	P		σy	F	P	
0	0	0.1573	0		0	0.1573	0	
0.5	0.15	0.1677	0.0254	0.0063_5	0.2	0.1738	0.0264	0.0066
1.0	0.3	0.1941	978 0.0724	0.0244_5	0.4	0.2219	1080 0.0816	0.0270
1.5	0.45	0.2393	1926 0.1202	0.0481_5	0.6	0.2976_5	2310 0.1494	0.0577_5
2.0	0.6	0.2976_5	2812 0.1610	0.0703	0.8	0.3941	3628 0.2134	0.0907
2.5	0.7_5	0.3690	3500 0.1890	0.0875	1.0	0.5023	4710 0.2546	0.1177_5
3.0	0.9	0.4473_5	3892 0.2002	0.0973	1.2	0.3896	4318 0.1742	0.1080
4.0	1.2	0.3896	3142 0.1140	0.1571	1.6	0.1982	2326 0.0584	0.1163
5.0	1.5	0.2400	1544 0.0404_3	0.0772	2.0	0.0787	716 0.0132_7	0.0358
6.0	1.8	0.1290	519 0.0115	0.0260	2.4	0.0239	154 0.0021_3	0.0077
7.0	2.1	0.0599	1418 0.0026_8	0.0071	2.8	0.0055	24 0.0002_5	0.0012
8.0	2.4	0.0239	32 0.0005_2	0.0016	3.2	0.0009		1 2541
9.0	2.7	0.0081	60 0.0000_8	0.0003				5689
				563 0.6033				0.569

VI

$\frac{\mu_0}{\mu_\alpha} = 1$

$\sigma = 0.5$ $\sigma = 0.8$

y	σy	$\mathfrak{F}$	P		σy	$\mathfrak{F}$	P		
0	0	0.1573	0		0	0.1573	0		
0.5	0.25	0.1840	0.0279	0.0070	0.4	0.2219	0.0336	0.0084	
1.0	0.5	0.2567	1224 0.0945	0.0306	0.8	0.3941	1784 0.1448	0.0446	
1.5	0.75	0.3690	27974 0.18524	0.0700	1.2	0.3896	3404 0.1956	0.0851	(0.0971)
2.0	1.0	0.5023	45718 0.27194	0.1143	1.6	0.1982	3028 0.1072	0.0757	
2.5	1.25	0.3630	45814 0.1862	0.1145	2.0	0.0787	1475 0.0403	0.0369	
3.0	1.5	0.2400	29372 0.10752	0.0734	2.4	0.0239	510 0.0107	0.0128	
4.0	2.0	0.0787	13056 0.02304	0.0653	3.2	0.0009	110 0.00026	0.0028	
5.0	2.5	0.0170	259 0.00286	0.0130				341 0.2664	
6.0	3.0	0.0023	306 0.00020	0.0015				12 0.2676	
				2221 0.4897				0.267	
				0.490					

$\sigma = 0.1$ $\quad$ $\underline{\underline{\mu_0 = 0.2}}$ $\quad$ $\sigma = 0.2$

y	$e^{-y}y^2$	σy	$\mathfrak{F}$	$P = e^{-y}y^2\mathfrak{F}$		σy	$\mathfrak{F}$	P	
0	0	0	0.777_3	0		0.	0.777_3	0	
0.5	0.15116_3	0.05	0.778^{78}_3	$0.118_{(79)}$	0.029_5	0.1	0.779_4	0.1180	0.029_5
1.0	0.3679	0.1	0.779_4	$\overset{405}{0.287_0}$	0.101	0.2	0.785_8	$\overset{4070}{0.289_0}$	0.101_8
1.5	0.5020	0.15	0.782_6	$\overset{6792}{0.392_2}$	0.170	0.3	0.683_5	$\overset{6318}{0.342_8}$	0.158_0
2.0	0.5414	0.2	0.7858	$\overset{8175}{0.425_3}$	0.204_4	0.4	0.5867	$\overset{6608}{0.318_0}$	0.165_2
2.5	0.5129	0.2_5	0.7344	$\overset{8015}{0.376_2}$	0.200_4	0.5	0.4968	$\overset{5726}{0.254_6}$	0.143_1
3.0	0.4480	0.3	0.6835	$\overset{6822}{0.3060}$	0.170_6	0.6	0.4147	$\overset{4402}{0.185_6}$	0.110_0
4.0	0.2931	0.4	0.5867	$\overset{478}{0.1720}$	0.239	0.8	0.2767	$\overset{2666}{0.081_0}$	0.133_3
5.0	0.1684_5	0.5	0.4968	$\overset{2557}{0.0837}$	0.127_8	1.0	0.1738	$\overset{1112}{0.030_2}$	0.055_6
6.0	0.0892_4	0.6	0.4147	$\overset{1204}{0.037_0}$	0.060_3	1.2	0.1025	$\overset{394}{0.009_2}$	0.019_4
7.0	0.0447	0.7	0.3413	$\overset{522}{0.015_2}$	0.026_1	1.4	0.0567	$\overset{117}{0.002_5}$	0.0059
8.0	0.0215	0.8	0.2767	$\overset{212}{0.0059_5}$	0.010_6	1.6	0.0293	$\overset{31}{0.000_6}$	0.001_6
9.0	0.0100	0.9	0.2210	$\overset{816}{0.0022_1}$	0.004_1	1.8	0.0142		
10.0	0.0045	1.0	0.1738	0.0008	$\underline{0.001_5}$ 344	2.0	0.0064		$\underline{\quad}$ 355
					1.345				0.924

$$\mathcal{Y}' = \frac{\Delta_{\alpha\alpha}}{\sqrt{\pi}}\mathcal{Y} = \int_0^\infty e^{-y}y^2\,\mathfrak{F}(\sigma y)\,dy$$

$\mu_0 = 0 \quad \mathfrak{F}(0) = 1 \quad \mathcal{Y}' = 2$

$$\sigma = 0,\ \sigma y = 0,\ \mathcal{Y}' = \mathfrak{F}(0)\overbrace{\int_0^\infty e^{-y}y^2\,dy}^{2},\ \mu_0 = 0.2,\ \mathfrak{F}(0) = 0.7773,\ \mathcal{Y}' = 1.5546$$

$\mathfrak{F}(0) = 1 - \Phi(\mu_0)$