

$$\theta = 1 - e^{-Cx_m^2} \left[1 - 2C \frac{x_m^2}{3-2x_m^2} \right]$$

$$X_m^2 = X_{m_0}^2 + \varepsilon = 1.5 + \varepsilon, 3 - 2X_m^2 = 3 - 2X_{m_0}^2 - 2\varepsilon + \frac{X_m^2}{3-2X_m^2} = \frac{1.5 + \varepsilon}{-2\varepsilon}$$

$$C = e^{-Cx_m^2} \left[1 + C \frac{1.5 + \varepsilon}{\varepsilon} \right] = e^{-Cx_m^2} (1 - C\varepsilon) \left(1 + C \frac{1.5 + \varepsilon}{\varepsilon} \right) = e^{-1.5C} \left(1 - C\varepsilon + C \frac{1.5 + \varepsilon}{\varepsilon} - C \left(1.5 + \varepsilon \right) \right)$$

$$e^{1.5C} - 1 = -C^2(1.5 + \varepsilon) - C \left(\varepsilon + \frac{1.5 + \varepsilon}{\varepsilon} \right) = C^2(1.5 + \varepsilon) - C \left(\frac{1.5 + \varepsilon + \varepsilon^2}{\varepsilon} \right)$$

$$C = 4, 402.43 = 16(1.5 + \varepsilon) - 4 \left(1 + \varepsilon + \frac{1.5}{\varepsilon} \right) \stackrel{\varepsilon \approx 0}{=} 20 - \frac{6}{\varepsilon}$$

$$y_m = 1.85 \quad \frac{1.85}{0.35} = \frac{34}{7} \quad \frac{34.4}{60} = 5.2854 \quad 6.2854 \quad e^{1.85} = 6.3598 \quad \frac{653.8}{314}$$

$$y_m = 1.84 \quad 6.412 \quad 6.30 \quad \frac{633.4}{253.2} \quad \frac{598}{345}$$

$$y_m = 1.845 \quad \frac{1.845}{0.345} = \frac{184.5}{345} = 5.3449 \quad 6.3449 \quad 6.3281$$

$$y_m = 1.846 \quad \frac{1.846}{0.346} = \frac{1846}{346} = 5.3353 \quad 6.3353 \quad 6.3345$$

$$1.5 + \frac{1.5}{3.9814} = 1.5 \left(1 + \frac{1}{4-0.0183} \right) = 1.5 \left[1 + \frac{1}{4(1-0.0046)} \right] = 1.5 \left[1 + \frac{1}{4} (1+0.0046) \right]$$

$$= 1.5 + \frac{1.5}{4} + \frac{1.5}{4} 0.0046 = \frac{1.5000}{1.8464} \quad \text{found } 1.846$$

$$Y_0 = Y_0 \left(1 - \frac{1}{(1+C)^2}\right), dW = e^{-y} y (1 - e^{-cy}) dy, y = \frac{C^2}{\alpha^2}$$

$$\underline{C=1}: \frac{1}{(1+C)^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, 25\% \text{ overall weakening}$$

$$y=1, C=\alpha, Cy=1, e^{-1}=0.368, 36.8\% \text{ missing}$$

$$y=2, C=\alpha \sqrt{2}, Cy=2, e^{-2}=0.135, 13.5\% "$$

$$y=\frac{1}{2}, C=\alpha \frac{1}{\sqrt{2}}, Cy=0.5, e^{-0.5}=0.604, 60.4\% "$$

$$y=\frac{1}{4}, C=\alpha \frac{1}{2}, Cy=0.25, e^{-0.25}=0.449, 44.9\% "$$

$$\underline{C=2}: \frac{1}{(1+C)^2} = \frac{1}{3^2} = \frac{1}{9}, 11.1\% \text{ overall weakening}$$

$$y=1, C=\alpha, Cy=2, e^{-2}=0.135 13.5\% \text{ missing}$$

$$y=2, C=\alpha \sqrt{2}, Cy=4, e^{-4}=0.018 1.8\% "$$

$$y=\frac{1}{2}, C=\alpha \frac{1}{\sqrt{2}}, Cy=1, e^{-1}=0.368 36.8\% "$$

$$y=\frac{1}{4}, C=\alpha \frac{1}{2}, Cy=0.5, e^{-0.5}=0.604 60.4\% "$$

$$e^{-x^2} x^3 : X = \frac{c}{\alpha} = \sqrt{y}, \frac{d e^{-x^2} x^3}{dx} = 3x^2 e^{-x^2} - 2x^4 e^{-x^2}, 3 - 2X_m^2 = 0, X_m^2 = \frac{3}{2}, X_m = \frac{C_m}{\alpha} = \sqrt{1.5} = 1.225$$

$$e^{-x^2} x^2 : = 2x e^{-x^2} - 2x^3 e^{-x^2}, 1 - X_m^2 = 0, X_m = \frac{C_m}{\alpha} = 1$$

$$e^{-x^2} x^3 (1 - e^{-cx^2}) : 3x^2 (1 - e^{-cx^2}) e^{-x^2} - 2x^4 e^{-x^2} (1 - e^{-cx^2}) + 2cx^4 e^{-x^2} e^{-cx^2} -$$

$$= 3x^2 e^{-x^2} - 3x^2 e^{-x^2} e^{-cx^2} - 2x^4 e^{-x^2} + 2x^4 e^{-x^2} e^{-cx^2} + 2cx^4 e^{-x^2} e^{-cx^2}$$

$$f = 3 - 3e^{-cx_m^2} - 2X_m^2 + 2X_m^2 e^{-cx_m^2} + 2cX_m^2 e^{-cx_m^2}$$

$$= (3 - 2X_m^2) - e^{-cx_m^2} [3 - 2X_m^2 - 2cX_m^2]$$

$$\text{Approx: } 3 - 2X_m^2 = [] e^{-cx_m^2}, 2X_m^2 - 3 = [2X_m^2 - 3 + 2cX_m^2] e^{-cx_m^2}$$

$$C = \infty, 2X_m^2 - 3 = 0, X_m^2 = \frac{3}{2}$$

$$1. \text{ Appr., } X_m = X_{m_0} + \varepsilon, X_m^2 = X_{m_0}^2 + 2\varepsilon X_{m_0}, 2X_m^2 - 3 = 4\varepsilon X_{m_0} = 4\sqrt{1.5} \times \varepsilon$$

$$4\sqrt{1.5} \times \varepsilon = [4\sqrt{1.5} \varepsilon - 3c + 4c\varepsilon\sqrt{1.5}] e^{-c\frac{3}{2}} = [4\sqrt{1.5} \varepsilon (1+c) - 3c] e^{-c\frac{3}{2}}$$

$$e^y y^{\frac{3}{2}} \frac{d(y e^{-y})}{dy} = \frac{3}{2} y^{\frac{1}{2}} e^{-y} - y^{\frac{3}{2}} e^{-y}, \frac{3}{2} - y_m = 0, y_m = \frac{3}{2}$$

$$\frac{d y^{\frac{3}{2}} e^{-y} (1 - e^{-cy})}{dy} = (\frac{3}{2} y^{\frac{1}{2}} e^{-y} - y^{\frac{3}{2}} e^{-y})(1 - e^{-cy}) + y^{\frac{3}{2}} e^{-cy} c e^{-cy}$$

$$3 y^{\frac{1}{2}} (\frac{3}{2} - y_m)(1 - e^{-cy_m}) + y_m c e^{-cy_m} = 0$$

$$y_{m\infty} = \frac{3}{2} \quad y_m = y_{m\infty} + \varepsilon, e^{-cy_m} = e^{-cy_{m\infty}} e^{-c\varepsilon} = e^{-cy_{m\infty}} - c\varepsilon e^{-cy_{m\infty}}$$

$$-(\frac{3}{2} - y_m) e^{-cy_m} + y_m c e^{-cy_m} = y_m - \frac{3}{2} = [\bar{y}_m(1+c) - \frac{3}{2}] e^{-cy_m}$$

$$\varepsilon = [\bar{y}_m - \frac{3}{2} + y_m c] e^{-cy_{m\infty}} (1 - c\varepsilon) = (\varepsilon + c \frac{3}{2} + \varepsilon c)(1 - c\varepsilon) e^{-\frac{3}{2}c}$$

$$(\varepsilon + \frac{3}{2}c + \varepsilon c - \varepsilon^2 c - \frac{3}{2}\varepsilon c^2 - \varepsilon^2 c^2) e^{-\frac{3}{2}c}$$

$$\varepsilon \ll 1, \varepsilon = [\frac{3}{2}c + \varepsilon(1+c - \frac{3}{2}c^2)] e^{-\frac{3}{2}c} = \frac{3}{2}c e^{-\frac{3}{2}c} + \varepsilon(1+c - \frac{3}{2}c^2) e^{-\frac{3}{2}c}$$

$$\varepsilon [1 - (1+c - \frac{3}{2}c^2) e^{-\frac{3}{2}c}] = \frac{3}{2}c e^{-\frac{3}{2}c}, \varepsilon = \frac{\frac{3}{2}c e^{-\frac{3}{2}c}}{1 - (1+c - \frac{3}{2}c^2) e^{-\frac{3}{2}c}}$$

$$\varepsilon = \frac{\frac{3}{2}c}{e^{\frac{3}{2}c} - 1 - c + \frac{3}{2}c^2}$$

$$c=1, \varepsilon = \frac{\frac{3}{2}}{e^{\frac{3}{2}} - 1 - 1 + \frac{3}{2}} = \frac{\frac{3}{2}}{4.48 - 2 + 1.5} = \frac{1.5}{3.98}, y_m = y_{m\infty} (1 + \frac{\varepsilon}{y_{m\infty}}) = 1.5 (1 + \frac{1}{3.98})$$

$$c=2, \varepsilon = \frac{\frac{3}{2}}{e^{\frac{3}{2}} - 1 - 2 + 6} = \frac{1.5}{\frac{1}{2}e^{\frac{3}{2}} + \frac{3}{2}} = \frac{1.5}{20.1 + 3} = \frac{1.5}{11.5}, y_m = 1.5 (1 + \frac{1}{11.5})$$

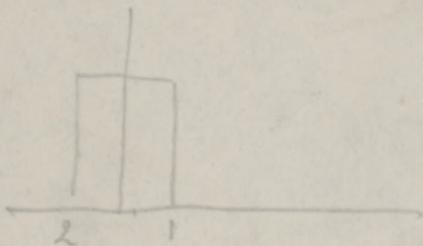
$$c=4, \varepsilon = \frac{\frac{3}{2}}{e^{\frac{6}{2}} - 1 - 4 + 24} = \frac{\frac{3}{2}}{403.4 + 19} = \frac{\frac{3}{2}}{\frac{1}{4}22.4} = \frac{3}{2} \frac{1}{105.6}, y_m = 1.5 (1 + \frac{1}{105.6})$$

$$1 = [1 + c \frac{y_m}{y_{m\infty} - \frac{3}{2}}] e^{-cy_m}, e^{cy_m} = 1 + c \frac{y_m}{y_{m\infty} - \frac{3}{2}}$$

$\frac{184}{140} : 34 = 5.412$
 $\frac{140}{136}$
 $\frac{136}{40}$
 $\frac{40}{6}$

$$c=1, y_m = 1.80, 1 + \frac{1.8}{0.3} = 4, e^{1.8} = 6.05, y_m = 1.84, 1 + \frac{1.84}{0.34} = 1 + 5.412 = 6.412, e^{1.84} = 6.30$$

$$100:45 = 20:9 = 2.2$$



$$dY = \frac{y}{\beta_\alpha} \frac{ds_\alpha}{\beta_\alpha} e^{-\frac{\beta_\alpha}{\beta-\beta_0}} \left(\frac{s_\alpha}{\beta-\beta_0} \right)^3$$

$$y = \frac{s_\alpha}{\beta-\beta_0}, \frac{\beta-\beta_0}{\beta_\alpha} = \frac{1}{y}, \frac{\beta_0}{\beta_\alpha} = \frac{1}{\beta_\alpha} - \frac{1}{y}$$

$$d\left(\frac{s_0}{\beta_\alpha}\right) = -d\left(\frac{1}{y}\right) = \frac{dy}{y^2}$$

$$d\frac{y}{y} = \frac{dy}{y^2} e^{-y} y^3 = e^{-y} y dy$$

$$\frac{y}{y_0} = \int_0^y e^{-y} y dy, y = \frac{\beta_\alpha}{\beta-\beta_0}$$

$$\int_{y_1}^{y_2} e^{-y} y dy = \int_{y_1}^{y_2} y de^{-y} = (1+y)e^{-y} \Big|_{y_1}^{y_2}$$

4992 - 252
1598:9 - 178
178

$$\frac{\frac{44}{1545} \beta^2}{1 + \frac{4}{45} \beta^2} = 1 + \left(\frac{44}{1545} - \frac{4}{45} \right) \beta^2 = 1 - \frac{4 \times 35 - 44}{35 \times 45} \beta^2 = 1 - \frac{96}{35 \times 45} \beta^2 = 1 - \frac{32}{525} \beta^2$$

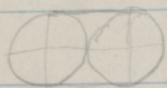
$$\frac{44 \times 45 - 4 \times 1545}{45 \times 1545} = \frac{1545 : 45 = 35}{\frac{135}{225}} = \frac{350}{145} = \frac{32}{525}$$

$$\frac{8}{105} 0.273^2 = 0.0445 + \frac{8}{105} = 0.57\%$$

$$\frac{5}{16} \frac{\pi+4}{\pi+3} \frac{1}{2} (3 \cos^2 \eta - 1) = \frac{5}{16} \frac{\pi+4}{\pi+3} \frac{3}{2} \left(\cos^2 \eta - \frac{1}{3} \right)$$

$$\beta = \frac{15}{32} \frac{\pi+4}{\pi+3} = \frac{15}{32} \frac{1 + \frac{4}{\pi}}{1 + \frac{3}{\pi}} = 0.469 \times \frac{2.273}{1.955} = 0.469 \times 1.161 = 0.545$$

0.273



$$\pi r^2 + 4r^2 \quad \pi 4r^2$$

$$\frac{5}{16} \frac{4.14}{6.14} =$$

θ

r

$r \cos \theta$

$$\eta = 0 \quad \cos \eta = 1 \quad E(\sin \eta) = \frac{\pi}{2} \quad A = 2\pi r^2 + 4r^2 \frac{\pi}{2} = 4\pi r^2 - 4\pi r^2 + 2.566$$

$$\eta = \frac{\pi}{2} \quad \cos \eta = 0 \quad E(\sin \frac{\pi}{2}) = 1 \quad A = \pi r^2 + 4r^2 (r + 4)^2 \quad \pi r^2 + 4 = \frac{4.14}{16} = 1.493$$

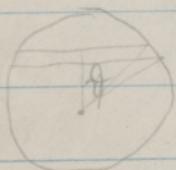
$$\alpha \left\{ 1 + \beta \left(\cos^2 \eta - \frac{1}{3} \right) \right\}, \quad \beta = 0.243, \quad \frac{1 + \frac{2}{3} \cdot 0.243}{1 - \frac{1}{3} \cdot 0.243} = \frac{1.182}{0.909} = 1.301$$

$$\frac{1}{3} n c \lambda \frac{dQ}{dx} \quad \text{visc: } Q = \mu u \quad \frac{1}{3} n c \lambda m \frac{du}{dz} = \underbrace{\frac{1}{3} n c \lambda}_{\frac{1}{3} g c \lambda} \frac{du}{dz}$$

$$\text{heat cond: } Q = mcT \quad \underbrace{\frac{1}{3} g c \lambda}_{\frac{1}{3} k} \frac{dT}{dz}$$

$$\omega = \frac{H\mu}{P} \quad \mu = \frac{1}{2} \frac{e}{mc} \frac{h}{2\pi} \quad h\nu = H\mu = P_2 \pi r = P\omega$$

$$\omega = \frac{H\mu}{P}, \quad h\nu = H\mu, \quad \frac{h}{2\pi} \omega = P\omega = H\mu$$



$$\frac{2\pi r \sin \theta r d\theta}{2\pi r^2} = \sin \theta d\theta - d \cos \theta$$

$$\int_0^{\pi} \cos^2 \theta d\cos \theta = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3}$$

$$K \times \frac{dT}{dx} \times a$$
$$Q = 0.0021 \times \frac{1}{3 \times 10^2} \times a = 0.07 \times a \frac{\text{cal}}{\text{sec}}$$

$$\textcircled{O}^{-1 \times 10^{-1}} \quad b = 2 \times 10^{-1} \quad 0.0021 \times \frac{dT}{dx} \times 10^{-1} = 0.07 \times a$$

$$X \times \left(\frac{dT}{dx} \right)_b \times b = X \times \left(\frac{dT}{dx} \right)_a \times a, \left(\frac{dT}{dx} \right)_b = \frac{a}{b} \left(\frac{dT}{dx} \right)_a \sim 10 \times 33 = 330$$

$$b = 2\pi r + d \quad a = 2\pi r + l$$

$$4 \times 10^{-4}$$

$$\frac{r^2}{T} = 4\pi^2 r^2 t \quad 4\pi r v_r v_s \quad \frac{4\pi^2 r^2 r}{4\pi r v_s} = \frac{\pi v_r}{v_s} = \frac{1}{2} \frac{v_r}{v_s}$$

$$m \frac{d\mathcal{H}}{dt} = mg, g = \frac{m}{M} \frac{d\mathcal{H}}{dt} = \frac{M}{M} \frac{d\mathcal{H}}{dt}$$

$$S_2 = g \frac{l^2}{\alpha^2} = \frac{M}{N} \frac{\partial \mathcal{H}}{\partial r} \frac{l^2}{\alpha^2} = \frac{M}{2RT} \frac{\partial \mathcal{H}}{\partial r} l^2 = \frac{3}{16,8 \times 10^7 \times 60} \times 10^5 \times 10^3 \text{ cm} = \frac{3}{\frac{5}{3} \times 10^8 \times 60} 10^8 \text{ cm} = \frac{3}{100} \text{ cm} = 0,3 \text{ mm}$$

$$\frac{1}{2} M \alpha^2 = \frac{3}{2} RT \quad \alpha = \sqrt{3 \frac{RT}{M}} = \sqrt{3 R} \sqrt{\frac{T}{M}} = 1,58 \times 10^4 \sqrt{\frac{T}{M}} \frac{\text{cm}}{\text{nm}} = 158 \sqrt{\frac{T}{M}} \frac{\text{m}}{\text{nm}} = 158 \sqrt{\frac{60}{2}} = 158 \times 5,98 = 865 \frac{\text{m}}{\text{nm}}$$

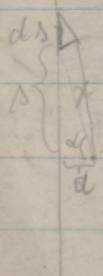
$$\frac{\partial \mathcal{H}}{\partial r} = \frac{1}{3} \times 10^5, S_2 = 0,1 \text{ mm} \quad \frac{g}{f} = \frac{1}{12} \frac{6}{5} e^{-\frac{f}{5}} \left(\frac{S_2}{5} \right)^2, \frac{f}{5} = \frac{2 \times 10^{-2}}{20 \times 10^2 \text{ nm}} = \frac{1}{10}, \frac{S_2}{5} = \frac{1}{2}, \frac{4}{12} = \frac{1}{3} \frac{1}{10} e^{-0.5} \frac{1}{4} = 1,26 \times 10^{-3}$$

$$\frac{g}{f} = \frac{1}{6} \frac{1}{10} e^{-1} = \frac{0,368}{60} = 6,1 \times 10^{-3}$$

$$\mu_0 2\pi r \quad \textcircled{O} \quad \mathcal{H} = \frac{g}{r} \quad 2\pi r, \mu_0 = \mathcal{H} \quad \mathcal{H} = \frac{g}{r} \quad \frac{\partial \mathcal{H}}{\partial r} = -\frac{g}{r^2}$$

$$\cancel{\left(\frac{\partial \mathcal{H}}{\partial r} \right)} \cdot \frac{1}{2} \int_{\theta=0}^{\theta=R-\alpha} \frac{2\mu_0 g d\theta}{r} \cos \alpha \quad \left) \cdot \mathcal{H} = \right. \mathcal{H}_0 = \frac{g}{r_0} = 2 \times 10^4, g = 2 \times 10^4 \times r_0$$

$$\left(\frac{\partial \mathcal{H}}{\partial r} \right)_0 = -\frac{2 \times 10^4}{r_0^2} r_0 = -\frac{2 \times 10^4}{r_0}$$

$$\frac{2\mu_0 ds}{r} \frac{a}{r} \quad \frac{r d\alpha}{ds} = \frac{a}{s} \frac{ads}{r} = s dd \quad 2\mu_0 \frac{s d\alpha}{r} = 2\mu_0 s n \alpha dd$$


v u m

$$p = n \frac{RT}{V_a}, \frac{dp}{dt} = \frac{RT}{V_a} \frac{dn}{dt} = -\frac{RT}{V_a} \frac{p V_p}{RT} = -p \frac{V_p}{V_a}$$

$$\frac{dn}{dt} = -\frac{p V_p}{RT}, \frac{d \ln p}{dt} = -\frac{V_p}{V_a}, \ln p = \frac{V_p}{V_a} t + \text{konst}$$

$$\ln \frac{p_2}{p_1} = \frac{V_p}{V_a} (t_2 - t_1), \frac{p_1}{p_2} = e^{\frac{V_p}{V_a} (t_2 - t_1)}, \frac{p_2}{p_1} = e^{-\frac{V_p}{V_a} (t_2 - t_1)}$$

$$\frac{p_2}{p_1} = 0.65 = e^{-0.43}, 0.43 = \frac{V_p}{1000} \cdot 6000 = V_p \times 0.60, V_p = \frac{0.43}{0.60} = 0.71$$

$$\begin{array}{c} x \\ 0 \end{array} \quad \begin{array}{c} y \\ 4 \end{array}$$

$$\begin{array}{c} 1 \\ 3 \end{array}$$

$$\begin{array}{c} 2 \\ 2 \end{array}$$

$$\begin{array}{c} 3 \\ 1 \end{array}$$

$$\begin{array}{c} 4 \\ 0 \end{array}$$

2

$$\frac{d^2s}{dt^2} = \frac{ds}{dt} = g(t), s = \int_{t_0}^{t_1} g(t) dt + s_0 = s_0 + \frac{1}{2} \int_{t_0}^{t_1} g(t) dt$$

$$\frac{ds}{dt} = s(t), s = s_0 + \frac{1}{2} \int_{t_0}^{t_1} s(t) dt = s_0 + \frac{1}{2} s_0 (t_1 - t_0) + \frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^t g(\tau) d\tau dt$$

$$g(l) = g_0, s_0 = s_0 + \frac{1}{2} g_0 l_0, s_0 = 0 + \frac{1}{2} s_0 l_0 + \frac{1}{2} g_0 \frac{l_0^2}{2} = 0, s_0 = -\frac{1}{2} g_0 l_0$$

$$s_{l_0} = \frac{1}{2} \frac{g_0 l_0}{2}$$

$$\dot{s}_t = s_0 + \frac{l}{\vartheta} g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{g_0}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{l_0}{\vartheta} \left[\frac{a_0^2 l + a_0 (l)}{l_0} \right] \varepsilon + \frac{1}{3} \left(\frac{l}{l_0} \right)^3 \varepsilon^2$$

$$s_{l_0} = 0 + s_0 \frac{l_0}{\vartheta} + \frac{1}{2} \frac{l_0^2}{\vartheta^2} g_0 \frac{\Delta I}{I_0} + \frac{3}{2} \frac{g_0}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{l_0^2}{\vartheta^2} \left[\frac{a_0^2 l_0}{2} + a_0 \varepsilon \frac{1}{3} + \frac{1}{3} \varepsilon^2 \frac{1}{4} \right] = 0$$

$$s_0 = -\frac{1}{2} g_0 \frac{l_0}{\vartheta} \frac{\Delta I}{I_0} + \frac{3}{2} g_0 \frac{l_0}{\vartheta} \left(1 + \frac{\Delta I}{I_0}\right) \frac{\frac{1}{2} a_0^2 + \frac{1}{3} \varepsilon a_0 + \frac{1}{12} \varepsilon^2}{\vartheta^2} \quad \varepsilon = a_{l_0} - a_0$$

$$\dot{s}_{l_0} = s_0 + g_0 \frac{l_0}{\vartheta} \frac{\Delta I}{I_0} - \frac{3}{2} g_0 \frac{l_0}{\vartheta} \left(1 + \frac{\Delta I}{I_0}\right) \frac{\frac{1}{2} a_0^2 + \varepsilon a_0 + \frac{1}{3} \varepsilon^2}{l_0^2}$$

$$s_0 = \frac{1}{2} g_0 \frac{l_0}{\vartheta} \frac{\Delta I}{I_0} - \frac{3}{2} g_0 \frac{l_0}{\vartheta} \left(1 + \frac{\Delta I}{I_0}\right) \frac{\frac{1}{2} a_0^2 + \frac{2}{3} a_0 \varepsilon + \frac{1}{4} \varepsilon^2}{\vartheta^2}$$

$$l_0 < l < 2l_0$$

$$\dot{s}_l = s_{l_0} + \frac{1}{\vartheta} \int_{l_0}^l g(l) dl = s_{l_0} + \frac{g_0}{\vartheta} \frac{\Delta I}{I_0} (l - l_0) - \frac{3}{2} \frac{g_0}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{l_0}{\vartheta} \left[\frac{a_0^2}{l_0} \frac{l - l_0}{l_0} + a_0 \varepsilon \frac{l^2 - l_0^2}{l_0^2} + \frac{1}{3} \frac{(l^2 - l_0^2)^2}{l_0^3} \right]$$

$$s_{l_0}$$

$$\int_{l_0}^{2l_0} s_{l_0} dl = s_{l_0} \frac{l_0}{\vartheta} + \frac{g_0}{\vartheta^2} \frac{\Delta I}{I_0} \left[\frac{3}{2} l_0^2 - l_0^2 \right] - \frac{3}{2} \frac{g_0}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{l_0^2}{\vartheta^2} \left[\frac{g_0^2}{l_0^2} \frac{\frac{3}{2} l_0^2 - l_0^2}{l_0^2} + a_0 \varepsilon \frac{\frac{7}{5} l_0^3 - l_0^3}{l_0^3} + \frac{1}{3} \varepsilon^2 \frac{\frac{15}{4} l_0^4 - l_0^4}{l_0^4} \right]$$

$$= \frac{1}{2} g_0 \frac{l_0^2}{\vartheta^2} \frac{\Delta I}{I_0} - \frac{3}{2} g_0 \frac{l_0^2}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{\frac{1}{2} a_0^2 + \frac{2}{3} a_0 \varepsilon + \frac{1}{4} \varepsilon^2}{\vartheta^2} + \frac{1}{2} g_0 \frac{l_0}{\vartheta^2} \frac{\Delta I}{I_0} - \frac{3}{2} g_0 \frac{l_0^2}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{\frac{1}{2} a_0^2 + \frac{4}{3} a_0 \varepsilon + \frac{11}{12} \varepsilon^2}{\vartheta^2}$$

$$s_{2l_0} = g_0 \frac{l_0^2}{\vartheta^2} \frac{\Delta I}{I_0} - \frac{3}{2} g_0 \frac{l_0^2}{\vartheta^2} \left(1 + \frac{\Delta I}{I_0}\right) \frac{a_0^2 + 2a_0 \varepsilon + \frac{7}{6} \varepsilon^2}{\vartheta^2}$$

$$\varepsilon = a_{l_0} - a_0, a_{l_0} = a_0 + \varepsilon, a_{l_0}^2 = a_0^2 + 2a_0 \varepsilon + \varepsilon^2$$

$$\frac{a_{l_0}^2 + \frac{1}{6} (a_{l_0} - a_0)^2}{\vartheta^2}$$

$$a_0 \left\{ \begin{array}{l} \\ \end{array} \right\} a_0$$

$$a = a_0 + \frac{l}{l_0} (a_{l_0} - a_0), l=0, a=a_0, l=l_0, a=a_0+a_0-a_0=a_{l_0}$$

$$l=2l_0, a = a_0 + 2(a_{l_0} - a_0) - a_0 + 2a_{l_0} - 2a_0 = 2a_{l_0} - a_0$$

$$a_{l_0} - a_0 = \varepsilon, a_{l_0} = a_0 + \varepsilon, a_{l_0}^2 = a_0^2 + 2a_0\varepsilon + \varepsilon^2$$

W

$$b = 4.5 \quad s_a = 1.8 \quad S = 2$$

0.8104

0.562

$$\frac{Y}{Y_0} = \frac{1}{16} \left[0.44\bar{9} e^{-\frac{1.8}{2}} - \frac{2-1.8}{4.5} - \frac{1}{2} \frac{1.8^2}{4.5^2} \left\{ \left[\frac{1}{1+\frac{4}{9}} + \frac{1}{2+\frac{4}{9}} - \frac{1}{3+\frac{4}{9}} \right] - \frac{1.8}{3 \cdot 4.5} \left[\frac{1}{(1+\frac{4}{9})^2} + \frac{1}{(2+\frac{4}{9})^2} - \frac{1}{(3+\frac{4}{9})^2} \right] \right\} \right]$$

$$\frac{1}{1+\frac{4}{9}} = \frac{1}{\frac{13}{9}} = \frac{9}{13} = 0.6920 \quad 0.44789$$

$$\frac{1}{2+\frac{4}{9}} = \frac{1}{\frac{22}{9}} = \frac{9}{22} = \frac{0.4090}{1.1010} \quad \begin{array}{r} 0.1673 \\ 0.6462 \\ -0.0843 \end{array}$$

$$\frac{1}{3+\frac{4}{9}} = \frac{1}{\frac{31}{9}} = \frac{9}{31} = \frac{0.2903}{0.8104} \quad 0.562$$

$$\frac{Y}{Y_0} = \frac{1}{16} \left[\frac{4}{9} 0.40654 - 0.04444 - 0.08 \left\{ \frac{0.8104 - 0.1\frac{1}{3} \times 0.562}{\frac{0.0448}{0.1807} - \frac{0.1363}{0.0589}} \right\} \right] = 4.84\% \quad (\text{und } 4.54\% \text{ für } b=5)$$

$$0.4444 : 16 = 0.00484$$

$$\text{I} \quad Y_1 = \frac{s_a}{s+b} \quad \frac{b}{s-b} \quad \frac{18}{29+4.5} \quad \frac{18}{29-4.5} - \frac{0.8988}{0.8320} \quad \frac{18}{49+4.5} \quad \frac{18}{49-4.5} \quad \frac{0.9546}{0.9343}$$

$$0.534 \quad 0.435 \quad 6.63 \quad 0.336_2 \quad 0.404_2 \quad 1.73$$

$$J = \frac{Y}{Y_0} = \frac{1}{b} e^{-\frac{bx}{s}} - \frac{s-b}{b} e^{-\frac{bx}{s}} - \dots, \quad J \frac{b}{s_a} = \frac{1}{s_a} e^{-\frac{bx}{s}} - \frac{s-b}{s_a} e^{-\frac{bx}{s}} \quad f(x) = x e^{-\frac{1}{x}}$$

$$J \frac{b}{s_a} = 4\left(\frac{1}{s_a}\right) - 4\left(\frac{1}{s_a} - \frac{b}{s_a}\right) - 4\left(\frac{1}{s_a} - 2\frac{b}{s_a}\right) + 4\left(\frac{1}{s_a} - 3\frac{b}{s_a}\right)$$

$$b = 4.5 \quad s_a = 1.8, \quad \frac{b}{s_a} = \frac{4.5}{1.8} = \frac{1}{4} = 0.25 \quad \frac{1}{s_a} = 0.05 \quad s = 1.8 \times 0.05 = 0.9$$

$$g = g_0 - \frac{\mu}{m} \frac{2I}{r^2} = g_0 + \frac{\mu}{m} \frac{2I}{r_0^2} \left(1 - \frac{3}{2} \frac{a^2}{r_0^2}\right) = -g_0 + g_0 \left(1 + \frac{\Delta I}{I_0}\right) \left(1 - \frac{3}{2} \frac{a^2}{r_0^2}\right)$$

$$\frac{\mu}{m} \frac{2I}{r^2} = \frac{\mu}{m} \frac{2(I_0 + \Delta I)}{r_0^2} = \frac{\mu}{m} \frac{2I_0}{r_0^2} + \frac{\mu}{m} \frac{2I_0}{r_0^2} \frac{\Delta I}{I_0} = g_0 + g_0 \frac{\Delta I}{I_0}$$

$$g = -g_0 + g_0 + g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{a^2}{r_0^2} g_0 \left(1 + \frac{\Delta I}{I_0}\right)$$

$$a = a_0 + \frac{l}{l_0} (a_{l_0} - a_0) \quad l=0 \quad a = a_0 \quad l=l_0 \quad a = a_0 + a_{l_0} - a_0 = a_{l_0}$$

$$a_{l_0} - a_0 = 0, \quad a = a_0 \quad g = g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{a_0^2}{r_0^2} g_0 \left(1 + \frac{\Delta I}{I_0}\right) = 0, \quad \frac{\Delta I}{I_0} = -\frac{3}{2} \frac{a_0^2}{r_0^2} + \frac{3}{2} \frac{a_0^2}{r_0^2} \frac{\Delta I}{I_0}$$

$$\frac{\Delta I}{I_0} = \frac{\frac{3}{2} \frac{a_0^2}{r_0^2}}{1 - \frac{3}{2} \frac{a_0^2}{r_0^2}} \quad \frac{a_0}{l_0} = 10^{-1} \quad \frac{\Delta I}{I_0} = \frac{\frac{3}{2} 10^{-2}}{1 - \frac{3}{2} 10^{-2}} \approx \frac{3}{2} 10^{-2}$$

$$a = a_0 + \frac{l}{l_0} \varepsilon \quad a^2 = a_0^2 + 2a_0 \frac{l}{l_0} \varepsilon + \frac{l^2}{l_0^2} \varepsilon^2, \quad \varepsilon = a_{l_0} - a_0$$

$$g = g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{g_0}{r_0^2} \left(1 + \frac{\Delta I}{I_0}\right) \left(a_0^2 + 2a_0 \frac{l}{l_0} \varepsilon + \frac{l^2}{l_0^2} \varepsilon^2\right)$$

$\ell = 0$

$$\begin{aligned} & \frac{a_0^2 + 2a_0(a_{l_0} - a_0) + a_{l_0}^2 - 2a_0 a_0 + a_0^2}{a_0^2 + 2a_0 a_{l_0} - 2a_0^2 + a_{l_0}^2 - 2a_0 a_0 + a_0^2} \\ & \frac{a_0^2 + 4a_0 a_{l_0} - 4a_0^2 + 4a_{l_0}^2 - 8a_0 a_0 + 4a_0^2}{a_0^2 - 4a_0 a_0 + 4a_0^2} = (2a_{l_0} - a_0)^2 \end{aligned}$$

$$s = 0, \quad s = s_0$$

$$s = 0, \quad s = s_0 + \int_0^{t_0} g dt = s_0 + \frac{1}{2} \int_0^{l_0} g dl \quad g = g_0 \quad s_{l_0} = s_0 + \frac{l_0}{2} g_0, \quad s_{l_0} = s_0 \frac{l_0}{2} + \frac{1}{2} g_0 \frac{l_0^2}{2} = 0$$

$$s_{l_0} = \frac{1}{2} \frac{l_0}{2} g_0 \quad s_0 = -\frac{1}{2} g_0 \frac{l_0}{2}$$

$$s_{l_0} = s_0 + \frac{l_0}{2} g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{g_0}{r_0^2} \left(1 + \frac{\Delta I}{I_0}\right) \left(a_0^2 \frac{l_0}{2} + 2a_0 \frac{\frac{1}{2} l_0^2}{2 l_0} \varepsilon + \frac{\frac{1}{2} l_0^3}{2 l_0^2} \varepsilon^2\right)$$

$$s_{l_0} = s_0 + \frac{l_0}{2} g_0 \frac{\Delta I}{I_0} - \frac{l_0}{2} \frac{3}{2} \frac{g_0}{r_0^2} \left(1 + \frac{\Delta I}{I_0}\right) \left(a_0^2 + a_0 \varepsilon + \frac{1}{3} \varepsilon^2\right)$$

$$s_{l_0} = s_0 + \frac{l}{2} g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{g_0}{r_0^2} \left(1 + \frac{\Delta I}{I_0}\right) \left(a_0^2 \frac{l}{2} + 2a_0 \frac{\frac{1}{2} l^2}{2 l_0} \varepsilon + \frac{\frac{1}{2} l^3}{2 l_0^2} \varepsilon^2\right)$$

$$g = g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{\alpha^2}{\gamma^2} g_0 \left(1 + \frac{\Delta I}{I_0}\right)$$

$$a = a_0 - \frac{l}{l_0} (a_0 - a_{l_0}) a_0 \quad l = l_0 \quad a = a_0 - a_0 + a_l = a_{l_0}, \quad l = 2l_0, \quad a = a_0 - 2a_0 + 2a_{l_0}$$

$$l_0 = 1, \quad a_0 - a_{l_0} = \varepsilon, \quad a = a_0 - l\varepsilon$$

$$g = g_0 \frac{\Delta I}{I_0} - \frac{3}{2} \frac{g_0}{\gamma^2} \left(1 + \frac{\Delta I}{I_0}\right) (a_0^2 - 2a_0 l \varepsilon + \varepsilon^2 l^2) - \alpha - \beta (a_0^2 + 2a_0 \varepsilon l)$$

$$S = g \quad s = s_0 + \frac{1}{2} \int_{l_1}^{l_2} g dl, \quad \theta = 1, \quad s_{l_2} = s_{l_1} + \int_{l_1}^{l_2} g dl, \quad s_{l_2} = s_{l_1} + \int_{l_1}^{l_2} s_c dl = s_{l_1} + s_{l_1} (l_2 - l_1) + \int_{l_1}^{l_2} dl \int g dl$$

$$s_{l_1} = 0 + s_0 \times 1 + \int_0^{l_1} dl \int g dl = s_0 +$$

095838

0.869565

<u>405429</u>	<u>405429</u>	<u>482590</u>	<u>482590</u>	<u>535.1</u>
<u>404599</u>	<u>316</u>	<u>481554</u>	<u>585</u>	<u>483</u>
<u>0830</u>	<u>405113</u>	<u>1036</u>	<u>482005</u>	<u>45</u>
<u>1900</u>				<u>414</u>
<u>453</u>	<u>0.90909</u>	<u>0.46923</u>		<u>852</u>
<u>1149.9</u>				<u>413</u>
<u>382</u>	<u>443264</u>	<u>602400</u>	<u>0.602400</u>	<u>38</u>
<u>1900</u>	<u>442334</u>	<u>601316</u>	<u>318</u>	<u>362</u>
<u>392</u>	<u>930</u>	<u>1384</u>	<u>602382</u>	<u>362</u>
<u>7518</u>		<u>0.689655</u>		<u>315</u>
				<u>854</u>
<u>644449</u>	<u>644449</u>	<u>428405</u>	<u>0.428405</u>	<u>292</u>
<u>643235</u>	<u>1121</u>	<u>426922</u>	<u>1168</u>	<u>263</u>
<u>1514</u>	<u>643628</u>	<u>1483</u>	<u>424534</u>	<u>242</u>
				<u>240</u>
<u>0.666667</u>		<u>0.64516</u>		<u>89</u>
<u>471411</u>	<u>769484</u>	<u>813430</u>	<u>813430</u>	<u>213</u>
<u>469484</u>	<u>642</u>	<u>811359</u>	<u>332</u>	<u>211</u>
<u>1924</u>	<u>770826</u>	<u>2041</u>	<u>813098</u>	<u>20</u>
<u>642</u>				<u>4294</u>
				<u>854</u>
<u>0.60606</u>		<u>0.588235</u>		<u>0.541429</u>

$$\begin{array}{r} 0.900212 \\ \underline{- 894831} \\ \hline 2381 \end{array} \quad \begin{array}{r} 0.900212 \\ \underline{- 0.900069} \\ \hline 143 \end{array} \quad \begin{array}{r} 944621 \\ \underline{- 942045} \\ \hline 2546 \end{array} \quad \begin{array}{r} 0.944621 \\ \underline{- 599} \\ \hline 944022 \end{array} \quad \begin{array}{r} 0.989422 \\ \underline{- 986705} \\ \hline 2714 \end{array} \quad \begin{array}{r} 989422 \\ \underline{- 1162} \\ \hline 98826 \end{array}$$

0.54054

0.526316

0.512821

$$\begin{array}{r} 1.049163 \\ \underline{- 96092} \\ \hline 3071 \end{array} \quad \begin{array}{r} 1.049163 \\ \underline{- 1410} \\ \hline 1.047445 \end{array} \quad \begin{array}{r} 1.123506 \\ \underline{- 1.120253} \\ \hline 3253 \end{array} \quad \begin{array}{r} 1.123506 \\ \underline{- 1089} \\ \hline 1.122444 \end{array} \quad \begin{array}{r} 1.140508 \\ \underline{- 67050} \\ \hline 3450 \end{array} \quad \begin{array}{r} 1.140508 \\ \underline{- 614} \\ \hline 1.164664 \end{array}$$

0.555556

0.540541

$$\begin{array}{r} 1.0443644 \\ \underline{- 314721} \\ \hline 2892 \end{array} \quad \begin{array}{r} 1.034364 \\ \underline{- 1609} \\ \hline 1.03246 \end{array} \quad \begin{array}{r} 1.049163 \\ \underline{- 076092} \\ \hline 3071 \end{array} \quad \begin{array}{r} 1.049163 \\ \underline{- 1660} \\ \hline 1.047450 \end{array}$$

0.44044*

$$\begin{array}{r} 644449 \\ \underline{- 643235} \\ \hline 1514 \end{array} \quad \begin{array}{r} 643235 \\ \underline{- 394} \\ \hline 629 \end{array}$$

$$\begin{array}{r} 22.4 \\ 9.1 \\ \hline 13.3 \end{array} \quad \begin{array}{r} 33.3 \\ 23.2 \\ \hline 10.1 \end{array}$$

$$a\varphi + b\psi \quad , x\varphi + y\psi$$

$$ax + ay\alpha + bx\alpha + by = \lambda x + \lambda y$$

$$(aP_y + bP_x)f = \lambda f, f = x\varphi + y\psi$$

$$aP_y(x\varphi + y\psi) + bP_x(x\varphi + y\psi) = \lambda x\varphi + \lambda y\psi$$

$$a x \varphi + a y \int g^* y dx \cdot \varphi + b x \int y^* g dx \cdot \psi + b y \psi = \lambda x \varphi + \lambda y \psi$$

$$P_y(g) = g \quad P_g(\varphi) = g \cdot \int g^* y dx \quad P_g(\psi) = \psi \quad P_\psi(\psi) = \psi$$

$$(a - \lambda)x + a(\bar{g}\bar{\psi})y = 0, b(\bar{g}\bar{\psi})x + (b - \lambda)y = 0$$

$$\begin{vmatrix} a - \lambda & a(\bar{g}\bar{\psi}) \\ b(\bar{g}\bar{\psi}) & b - \lambda \end{vmatrix} = 0$$

$$g = c_1 u_1 + c_2 u_2 \quad \int g^* \psi dx = c_2^* c_3$$

$$\psi = c_3 u_2 \quad \int \psi^* g dx = c_3^* c_2$$

$$\frac{b}{a+b} \frac{b}{\lambda_a} = \left(\frac{b}{a+b} - \frac{c}{\lambda_a} \right) e^{-\frac{b}{a+b}} + \frac{c}{\lambda_a} e^{-\frac{b}{a}}$$

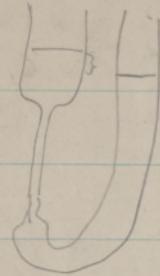
$$\begin{array}{r} 64 \\ 32 \\ 9 \\ \hline 69,4 \end{array} \quad \begin{array}{r} 229,4 : 64,4 = 3,403 \\ 202,2 \\ 24,2 \\ 24,0 \\ 2,00 \\ \hline 2,02 \end{array} \quad \begin{array}{r} 216,16 \\ 54,4 \\ 24,2 \end{array}$$

$$\text{Cu: } T_s = 1356, \sigma = 41 \times 63.6 = 2610 \text{ cal} \quad \frac{\sigma}{T_s} = 1.93$$

$$\text{Zn: } T_s = 692, \sigma = 23.0 \times 65.4 = 1500 \text{ cal} \quad \frac{\sigma}{T_s} = 2.17$$

$$\text{Ni: } T_s = 1723, \sigma = 65 \times 58.7 = 3820 \text{ cal} \quad \frac{\sigma}{T_s} = 2.22$$

1mm



$$h A \frac{1}{2} g = \frac{1}{2} g_0 l v^2$$

$$\frac{A h^2 g}{g_0 l} = v^2, v = h \sqrt{\frac{A g}{g_0 l}} = 10^{-1} \sqrt{\frac{1.25 \times 10^{-1}}{1.25 \times 10^{-4}} \frac{10^3}{16}} = \frac{10^3}{4} \times 10^{-1} = 25 \text{ cm/sec}$$

$$0.1 \times 1.25 \times 10^{-1} \times 0.15 \times \frac{0.1}{2} \times 10^3$$

2 cm

$$\frac{1.348}{0.2389} = 5.44$$

$$\sigma = 5.44 \times 10^{-12} (T_1^4 - T_2^4)$$

$$= 5.44 \times 10^{-12} \times 10^8 (5^4 - 3^4) = 0.314 \text{ Wattsec} \times 22$$

$$\frac{6.25}{5.44} \quad \frac{6.28}{6.87} \quad 1 \text{ Watt}$$

$$\lambda = 5 \times 10^{-3} \times 2 = 10^{-2} \text{ cm}^2$$

$$d \cos \alpha_0 - d \cos \alpha = \lambda$$

$$\cos \alpha = 1 - \frac{\alpha^2}{2} \quad 1 - \frac{\alpha_0^2}{2} - 1 + \frac{\alpha^2}{2} = \frac{\lambda}{d} \quad \alpha^2 - \alpha_0^2 = 2 \frac{\lambda}{d} \quad \lambda = 10^{-8} \text{ cm} \quad d = 2 \times 10^{-3} \text{ cm}, \frac{\lambda}{d} = \frac{2 \times 10^{-8}}{2 \times 10^{-3}} = 10^{-5}$$

$$\alpha^2 - \alpha_0^2 = 10^{-5} \quad \alpha_0 = 10^{-2} \quad \alpha^2 = 10^{-4} + 10^{-5} = 10^{-4} \times 1.1$$

$$\alpha_0 \ll 1 \quad \alpha \ll 1 \quad \alpha$$

$$(\alpha + \alpha_0)(\alpha - \alpha_0) = 2 \frac{\lambda}{d} \quad \alpha = \alpha_0 + \epsilon, \alpha - \alpha_0 = \epsilon, \alpha + \alpha_0 = 2\alpha_0 + \epsilon$$

$$(2\alpha_0 + \epsilon)\epsilon = 2 \frac{\lambda}{d}, \epsilon = \frac{2}{2\alpha_0(1 + \frac{\epsilon}{2\alpha_0})} \frac{\lambda}{d} = \frac{1}{\alpha_0} \frac{\lambda}{d} \frac{1}{1 + \frac{\epsilon}{2\alpha_0}}, \frac{\lambda}{d} = \frac{10^{-8}}{10^{-3}} = 10^{-5}$$

$$\alpha_0 = 5 \times 10^{-3} \quad \frac{\lambda}{d} = 10^{-5} \quad \epsilon = 2 \times 10^2 \times 10^{-5} = 2 \times 10^{-3} \quad \frac{\epsilon}{2\alpha_0} = \frac{2 \times 10^3}{10^{-2}} = 0.2 \quad \epsilon = 2.899$$

$$\epsilon = \frac{2}{1.2} \times 10^{-3} = 1.667 \times 10^{-3} \quad \frac{\epsilon}{2\alpha_0} = \frac{2 \times 10^{-3}}{10^{-2}} = \frac{5}{30}$$

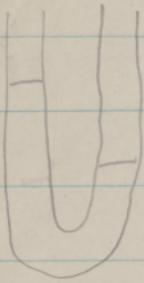
$$\epsilon = \frac{2}{\frac{35}{30}} \times 10^{-3} = 1.7 \times 10^{-3}$$

	Mag	5.000	+4.95%
2 "	2.222	-23.5%	
3 "	3.214	+10.9%	
4 "	2.942	-4.8%	
5 "	2.983	+1.86%	

$$\alpha^2 = \alpha_0^2 + 2 \frac{\lambda}{d}, \frac{\lambda}{d} = 10^{-5}, \alpha_0 = 2 \times 10^{-3} \quad \alpha^2 = 4 \times 10^{-6} + 2 \times 10^{-5} = 24 \times 10^{-6} \quad \alpha = 4.9 \times 10^{-3}, \epsilon = \alpha - \alpha_0 = 2.9 \times 10^{-3}$$

$$\epsilon = \frac{1}{2 \times 10^{-3}} \times 10^{-5} = \frac{1}{2} \times 10^{-2} = 5 \times 10^{-3} \quad \frac{1}{1 + \frac{5}{4}} = \frac{1}{\frac{9}{4}} = \frac{4}{9} \quad 5 \times \frac{4}{9} = \frac{20}{9} = 2.22 \quad \frac{1}{1 + 20} = \frac{1}{21} = \frac{1}{14} = \frac{9}{14}, \frac{45}{14} = 3.2$$

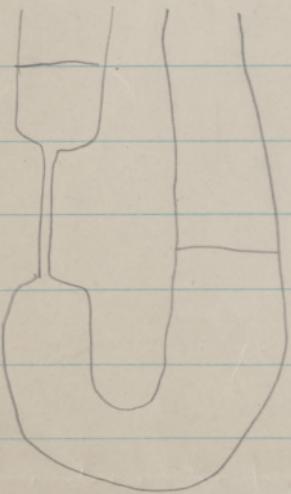
$$\frac{1}{1 + \frac{45}{56}} = \frac{1}{1 + \frac{45}{56}} = \frac{1}{\frac{101}{56}} = \frac{56}{101} = 9.80 \approx 9.81 \quad \frac{1}{1 + \frac{45}{101}} = \frac{1}{1 + \frac{45}{101}} = \frac{1}{101} = 0.953 \quad \frac{1}{101} = 0.953 \quad \frac{0.953}{0.984} = 1.067$$



$$h A \cancel{g} \frac{h}{2}, \cancel{\frac{1}{2}} A \cancel{g} (h_0^2 - h^2) = \cancel{\frac{1}{2}} A L \cancel{g} h^2, h = h_0 \cos \frac{2\pi}{T} t$$

$$h = -h_0 \frac{2\pi}{T} \sin \frac{2\pi}{T} t$$

$$h^2 = \frac{g}{2} (h_0^2 - h^2), T = 2\pi \sqrt{\frac{L}{g}}$$



$$\cancel{\frac{1}{2}} A \cancel{g} (h_0^2 - h^2) = \frac{1}{2} g_0 l g v^2 = \cancel{\frac{1}{2}} g_0 l g \frac{A^2}{g_0^2} h^2 \left(1 + \frac{g_0 L}{A^2}\right)$$

$$\frac{v}{h} = \frac{A}{g_0}, v = \frac{A}{g_0} h, h^2 = \frac{g g_0}{L A} (h_0^2 - h^2), T = 2\pi \sqrt{\frac{A L}{g g_0}}$$

$$V = \frac{1}{\eta} \frac{\pi}{8} \frac{r^4}{l} p, R = \frac{v_0 r}{\eta} < 1000, \nu = \frac{V}{L}$$

$$V = C' p^n, \nu = C p^n, \ln V = n \ln p + \ln C, \frac{d \ln V}{d \ln p} = n = \begin{cases} 1 & \text{laminar} \\ \frac{1}{2} & \text{turbulent} \end{cases}$$

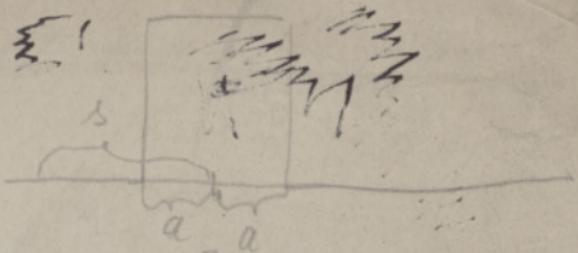
$$\eta = 0.01 \frac{gr}{cm \cdot sec} \text{ H}_2\text{O at } 20.2^\circ \quad \eta = 1.89 \times 10^{-4} \text{ He gas at } 0^\circ \quad \eta = 2.7 \times 10^{-4} \text{ He}_{\text{liq}} \text{ at } 2.3^\circ \text{ K}$$

$$\delta = 0.15 \quad "$$

λ -point: 2.186°K

$$\mu V = \mu g_0 v = \frac{1}{2} g_0 v g v^2, v = \sqrt{\frac{2p}{g}} = 14.15 \frac{cm}{sec} \text{ for } p = 15 \frac{dyn}{cm^2}, v = 46.2 \frac{cm}{sec} \text{ for } p = 160 \frac{dyn}{cm^2}$$

$$\frac{I}{I_0} = \frac{1}{2} \left[e^{-y} (y+1) \right] \frac{\frac{dy}{s+a}}{\frac{s\alpha}{s-a}}$$



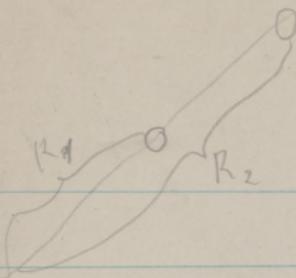
$$a \gg s-a, s-a=\sigma, s \ll s-a$$

$$\frac{I}{I_0} = \frac{1}{2} \left[1 - \left(1 + \frac{s\alpha}{\sigma} \right) e^{-\frac{s\alpha}{\sigma}} \right] \quad \frac{s\alpha}{\sigma} \ll 1, e^{-\frac{s\alpha}{\sigma}} = 1 - \frac{s\alpha}{\sigma} + \frac{1}{2} \left(\frac{s\alpha}{\sigma} \right)^2$$

$$\left(1 + \frac{s\alpha}{\sigma} \right) e^{-\frac{s\alpha}{\sigma}} = 1 - \frac{s\alpha}{\sigma} + \frac{1}{2} \left(\frac{s\alpha}{\sigma} \right)^2 = 1 - \frac{1}{2} \left(\frac{s\alpha}{\sigma} \right)^2$$

$$\frac{I}{I_0} = \frac{1}{2} \left[1 - 1 + \frac{1}{2} \left(\frac{s\alpha}{\sigma} \right)^2 \right] = \frac{1}{4} \left(\frac{s\alpha}{\sigma} \right)^2, s\alpha = s_2, \frac{\Delta I}{I_0} = \frac{s_2}{\sigma}$$

$$\frac{I}{I_0} = \frac{1}{4} \left(\frac{s_{\alpha_0}}{\sigma} \right)^2 \left(\frac{\Delta I}{I_0} \right)^2, \frac{s_{\alpha_0}}{\sigma} = 4, \frac{I}{I_0} = 4 \left(\frac{\Delta I}{I_0} \right)^2, \frac{s_{\alpha_0}}{\sigma} = 10, \frac{I}{I_0} = 25 \left(\frac{\Delta I}{I_0} \right)^2$$



$$R_2 - R_1 = v - v_0$$

$$\alpha R = v_0, R = \frac{v_0}{\alpha}$$

$$R = v_0 t, \alpha = \frac{1}{t}, \alpha R = v_0$$

$$\frac{g}{g_0} = \frac{l^3}{l^3}$$

$\alpha =$

$$m \frac{\frac{4\pi}{3} R^3 g}{R} g = \frac{1}{2} m v^2, \frac{g g (\frac{4\pi}{3} R^3)}{R} > \frac{v^2}{2} > \frac{\alpha^2 R^2}{2}, g > \frac{3\alpha^2}{8\pi g} = g_0$$

$$R_0, g_0 = \frac{g}{R_0^3}, v = \alpha R, \alpha = \frac{v}{R}, \alpha_0 = \frac{v}{R_0}, g = \frac{g}{R^3}, g_0 = g \frac{R^3}{R_0^3}$$

$$\frac{3v^2}{8\pi g R_0^2} = g \frac{R^3}{R_0^3}, \frac{3\alpha^2 R^2}{8\pi g R_0^2} = g \frac{R^3}{R_0^3}, \frac{g_0}{g} = \frac{R}{R_0}$$

$$v = \alpha R = \frac{1.8 \times 10^{-14}}{10^8} \times \frac{1.6 \times 10^{24}}{0.78} = 3 \times 10^7 \text{ cm/sec} = 300 \text{ km/sec}$$

$$\alpha = \frac{d(\log e^{\frac{1}{2}gt})}{dt}, \frac{d}{dt}(e^{\frac{1}{2}gt}) = \pm \left(\frac{8\alpha g}{3} e^{\frac{1}{2}gt} - \frac{c^2}{R^2} \right)^{\frac{1}{2}}$$

$$R = \infty, \frac{d}{dt}(e^{\frac{1}{2}gt}) = \left(\frac{8\alpha g}{3} e^{\frac{1}{2}gt} \right)^{\frac{1}{2}}, \frac{d \ln e^{\frac{1}{2}gt}}{dt} = \alpha = \left(\frac{8\alpha g}{3} \right)^{\frac{1}{2}}$$

$$\left(\frac{8\alpha g}{3} e^{\frac{1}{2}gt} \right)^{\frac{1}{2}} = \sqrt{8 \times 6.4 \times 10^{-8} \times 10^{-30}} = \sqrt{53.5 \times 10^{-38}} = 7.3 \times 10^{-19}$$

$$g_0 = g \frac{R^3}{R_0^5}, \alpha = \frac{v}{R}, \alpha_0 = \frac{v}{R_0}, \alpha = \alpha_0 \frac{R_0}{R}, \frac{3\alpha^2}{8\pi g} = g_0, \frac{3\alpha_0 R_0^2}{8\pi g R^2} = g_0$$

$$\frac{3\alpha_0^2}{8\pi g} = \frac{3\alpha_0^2 R^2}{8\pi g R^2}$$

$$\cos\alpha = \frac{dx}{d\gamma}$$

$$i \frac{dx'}{\gamma^2} = i \frac{dx}{d^2 + x^2} \cos\alpha$$

$$\cos\alpha = \frac{d}{\gamma}, \frac{1}{\gamma} = \frac{\cos\alpha}{d} \quad i \quad \tan\alpha = \frac{x}{d} \quad dx = d dx' = d \frac{d\alpha}{\cos^2\alpha}$$

$$\frac{dx}{\gamma^2} = \frac{\cos^2\alpha}{d^2} d \frac{d\alpha}{\cos^2\alpha} = \frac{dx}{d}$$

$$H = i \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\gamma^2} \cos\alpha = i \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d \cos^2\alpha}{\cos^2\alpha d^2} \cos\alpha d\alpha - \frac{i}{d} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d(\sin\alpha) = \frac{2i}{d}$$

$$i \frac{\Delta x}{\gamma^2} \cos\alpha, \cos\alpha = \frac{d}{\Delta x}, i \frac{\Delta h}{\gamma^2}$$

$$\int d(\sin\alpha) \quad \text{Diagram: } \begin{array}{c} d\sqrt{2} \\ \diagdown \quad \diagup \\ d \quad d \end{array} \quad \sin\alpha = \frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414$$

$$\begin{array}{c} d\sqrt{5} \\ \diagdown \quad \diagup \\ d \quad d \end{array} \quad \sin\alpha = \frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}} = \frac{4}{2.236} = 1.489$$

$$\begin{array}{c} d\sqrt{10} \\ \diagdown \quad \diagup \\ d \quad d \end{array} \quad \sin\alpha = \frac{3}{\sqrt{10}}, \frac{6}{\sqrt{10}} = \frac{6}{3.162} = 1.897$$

$$\begin{array}{c} d\sqrt{82} \\ \diagdown \quad \diagup \\ d \quad d \end{array} \quad \sin\alpha = \frac{9}{\sqrt{82}}, \frac{18}{\sqrt{82}} = 1.988$$

$$\int d(\sin\alpha) = 1 - \sin\left(\frac{\pi}{2} - \varepsilon\right) = 1 - \left(1 + \frac{1}{2}\varepsilon^2\right) = -\frac{1}{2}\varepsilon^2$$

$$\sin\left(\frac{\pi}{2} - \varepsilon\right) = 0.994, \frac{\pi}{2} - \varepsilon = 88^\circ, \varepsilon = 2^\circ = \frac{\pi}{180} \cdot 2 = \frac{2\pi}{180} = \frac{6.283}{180} = 3.483 \times 10^{-2}$$

$$\varepsilon^2 = 1.21 \times 10^{-3} = 0.0012$$

4

$$\frac{s_x}{\beta} = \frac{s_x}{b+\beta} = \frac{s_x}{b} \frac{1}{1+\frac{\beta}{b}} = \frac{s_x}{b} \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right), e^{-\frac{s_x}{\beta}(1-\frac{\beta}{b}+\frac{\beta^2}{b^2})} = 1 - \frac{s_x}{\beta} \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right) + \frac{1}{2} \left(\frac{s_x}{b}\right)^2 \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right)^2$$

$$\frac{s_x}{s-b} = \frac{s_x}{\beta}$$

$$\frac{Y}{Y_0} = \left[1 + \frac{s_x}{b} \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right) \right] \left[1 - \frac{s_x}{b} \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right) + \frac{1}{2} \left(\frac{s_x}{b}\right)^2 \left(1 - \frac{\beta}{b} + \frac{\beta^2}{b^2}\right)^2 \right]$$

$$- \frac{s_x}{\beta} e^{-\frac{s_x}{\beta}}$$

$$= \left[1 + \frac{s_x}{b} \left(1 - \frac{\beta}{b}\right) \left\{ 1 - \frac{s_x}{b} + \frac{s_x}{b} \frac{\beta}{b} + \frac{1}{2} \left(\frac{s_x}{b}\right)^2 \frac{\beta^2}{b^2} - \frac{1}{2} \left(\frac{s_x}{b}\right)^2 \frac{\beta}{b} \right\} \right] - \frac{s_x}{\beta} e^{-\frac{s_x}{\beta}}$$

$$\frac{Y}{Y_0} = \left(1 + \frac{s_x}{b}\right) e^{-\frac{s_x}{\beta}} - \left(1 + \frac{s_x}{s-b}\right) e^{-\frac{s_x}{s-b}}, s-b=\beta, s=b+\beta, \frac{b}{\beta} \ll 1$$

$$\left(1 + \frac{s_x}{b}\right) e^{-\frac{s_x}{\beta}} = \left[1 + \frac{s_x}{b} - \frac{s_x}{b} \frac{\beta}{b}\right] e^{-\frac{s_x}{b}} e^{\frac{s_x}{b} \frac{\beta}{b}} = \left(1 + \frac{s_x}{b} - \frac{s_x}{b} \frac{\beta}{b}\right) \left(1 + \frac{s_x}{b} \frac{\beta}{b}\right) e^{-\frac{s_x}{b}}$$

$$= \left(1 + \frac{s_x}{b} - \frac{s_x}{b} \frac{\beta}{b} + \frac{s_x}{b} \frac{\beta^2}{b^2}\right) e^{-\frac{s_x}{b}} \left(1 + \frac{s_x \beta}{b b} - \frac{s_x \beta^2}{b b^2} + \frac{1}{2} \frac{s_x^2 \beta^2}{b^2 b^2}\right)$$

$$= \left(1 + \frac{s_x}{b} - \cancel{\frac{s_x \beta}{b b}} + \cancel{\frac{s_x \beta^2}{b b^2}} + \cancel{\frac{s_x \beta}{b b}} + \cancel{\frac{s_x^2 \beta^2}{b^2 b^2}} - \cancel{\frac{s_x^2 \beta^2}{b^2 b^2}} - \cancel{\frac{s_x^2 \beta^2}{b^2 b^2}} + \frac{1}{2} \frac{s_x^2 \beta^2}{b^2 b^2}\right) e^{-\frac{s_x}{b}}$$

$$\frac{Y}{Y_0} = \left(1 + \frac{s_x}{b} + \frac{s_x^2 \beta}{b^2 b} + \frac{1}{2} \frac{s_x^2 \beta^2}{b^2 b^2}\right) e^{-\frac{s_x}{b}} - \frac{s_x}{\beta} e^{-\frac{s_x}{\beta}} =$$

$$= \left(1 + \frac{s_x}{b}\right) e^{-\frac{s_x}{b}} + \frac{s_x^2}{b^2} e^{-\frac{s_x}{b}} \left(\frac{\beta}{b} + \frac{1}{2} \frac{\beta^2}{b^2}\right) - \frac{s_x}{\beta} e^{-\frac{s_x}{\beta}}$$

$$\frac{d}{d\beta} \frac{Y}{Y_0} = 0 + \frac{s_x^2}{b^2} e^{-\frac{s_x}{b}} \left(\frac{1}{b} + \frac{\beta}{b^2}\right) + \frac{s_x}{b^2} e^{-\frac{s_x}{b}} - \frac{s_x}{\beta} \frac{s_x}{\beta^2} e^{-\frac{s_x}{b}}$$

$$= \frac{s_x^2}{b^2} e^{-\frac{s_x}{b}} \frac{1}{b} \left(1 + \frac{\beta}{b}\right) - \frac{s_x}{\beta^2} \left(\frac{s_x}{\beta} - 1\right) e^{-\frac{s_x}{b}}$$

$$\frac{s_x}{b} e^{-\frac{s_x}{b}} \left(1 + \frac{\beta}{b}\right) = \frac{b^2}{\beta_m^2} \left(\frac{s_x}{\beta_m} - 1\right) e^{-\frac{s_x}{\beta}}, e^{\frac{s_x}{b}} = \frac{b^2}{\beta_m^2} \frac{\frac{s_x}{\beta_m} - 1}{1 + \frac{\beta_m}{b}} \frac{b}{s_x} e^{\frac{s_x}{b}}$$

$$\frac{s_x}{\beta_m} = 2 \ln \frac{b}{\beta_m} + \ln \left(\frac{s_x}{\beta_m} - 1\right) - \ln \left(1 + \frac{\beta_m}{b}\right) + \ln \frac{b}{s_x} + \frac{s_x}{b}$$

5)

$$b = b_a$$

$$\frac{S_a}{\beta} = 2 \ln \frac{S_a}{\beta} + \ln \left(\frac{S_a}{\beta} - 1 \right) - \ln \left(1 + \frac{\beta}{S_a} \right) + 1$$

$$x = 2 \ln x + \ln \frac{x-1}{1+x} + 1 = 4.6 \log x + 2.3 \log \frac{x-1}{1+x} + 1$$

$$x=10 = 4.6 + 2.3 \log \frac{9}{1.1} + 1 \quad \begin{array}{r} 0.9542 \\ -0.0414 \\ \hline 0.9128 \end{array} \quad \begin{array}{r} 5.6 \\ 2.1 \\ \hline 7.7 \end{array}$$

$$x=7 = 4.6 \times 0.8451 + 2.3 \left(\log 6 - \log 1.1428 \right) + 1 \quad \begin{array}{r} 0.7782 \\ -0.0580 \\ \hline 0.7202 \end{array} \quad \begin{array}{r} 3.885 \\ 1.660 \\ \hline 5.545 \end{array}$$

$$x=6 = 4.6 \times 0.7782 + 2.3 \left(\log 5 - \log 1.167 \right) + 1 \quad \begin{array}{r} 0.6998 \\ 0.0670 \\ \hline 0.6324 \\ 2.1884 \end{array} \quad 6.048 \quad \beta \sim \frac{1}{6} S_a \text{ for } b = S_a$$

$$b = 2 S_a$$

$$\frac{S_a}{\beta_m} = 2 \ln 2 \frac{S_a}{\beta_m} + \ln \left(\frac{S_a}{\beta_m} - 1 \right) - \ln \left(1 + \frac{\beta_m}{2 S_a} \right) + \ln 2 + \frac{1}{2}$$

$$x = 2.3 \left[2 \log 2 x + \log(x-1) - \log(1 + \frac{1}{2x}) \right] + 2.3 \log 2 + 0.5 \quad 0.664$$

$$x=10 = 2.3 \left[2 \times 1.3010 + 0.9542 - 0.0212 \right] + 1.164 \quad \begin{array}{r} 2.6020 \\ 0.9930 \\ \hline 3.5950 \end{array} \quad 7.80$$

$$\begin{array}{r} 2.4082 \\ 0.8451 \\ \hline 3.2533 \\ -0.0263 \\ \hline 3.2270 \end{array} \quad \begin{array}{r} 8.96 \\ 8.96 \\ -8.28 \\ 0.68 \end{array}$$

$$x=8 = 2.3 \left[2 \times 1.2041 + 0.8451 - 0.0263 \right] + 1.164 \quad \begin{array}{r} 7.12 \\ 8.28 \end{array}$$

$$8 + x = 8.28 + x \times 0.34, x \times 0.66 = 0.28 \quad x = \frac{0.28}{0.66} = \frac{14}{33} \approx 0.42$$

$$\beta \sim \frac{1}{8.42} S_a \text{ for } b = 2 S_a$$

$$\frac{F}{F_0} = 1 - \frac{s_a}{b} \times \frac{1}{X} e^{-\frac{bx}{b+s_0}} \Big|_{\frac{s_a}{b+s_0}} = 1 - \frac{s_a}{b} \frac{b+s_0}{s_a} e^{-\frac{s_a}{b+s_0}} + \frac{s_a}{b} \frac{s_0}{s_a} e^{-\frac{s_a}{s_0}}$$

$$\frac{F}{F_0} = 1 - \left(1 + \frac{s_0}{b}\right) e^{-\frac{bx}{b+s_0}} + \frac{s_0}{b} e^{-\frac{bx}{b}} = 1 - 2e^{-\frac{bx}{2b}} + e^{-\frac{bx}{b}} \text{ for } s_0 = b \\ e^{-x} = 1 - x + \frac{x^2}{2}$$

$$s_a < b \quad \frac{F}{F_0} = 1 - 2\left(1 - \frac{s_a}{2b}\right) + 1 - \frac{s_a}{b} = 1 - 2 + \frac{s_a}{b} + 1 - \frac{s_a}{b}$$

$$\frac{F}{F_0} = 1 - 2\left(1 - \frac{s_a}{2b} + \frac{1}{2} \frac{s_a^2}{4b^2}\right) + 1 - \frac{s_a}{b} + \frac{1}{2} \frac{s_a^2}{b^2} = -\frac{1}{4} \frac{s_a^2}{b^2} + \frac{1}{2} \frac{s_a^2}{b^2} = \frac{1}{4} \left(\frac{s_a}{b}\right)^2$$

$$s_a \ll s_0$$

$$\frac{F}{F_0} = 1 - \left(1 + \frac{s_0}{b}\right) \left(1 - \frac{s_a}{b+s_0}\right) + \frac{s_0}{b} \left(1 - \frac{s_a}{s_0}\right) = 1 - \left(1 + \frac{s_0}{b} - \frac{s_a}{b+s_0} - \frac{s_0 s_a}{b(b+s_0)}\right) + \frac{s_0}{b} - \frac{s_a s_0}{b s_0} \\ = 1 - \left(1 - \frac{s_0}{b} + \frac{s_a}{b+s_0} \left(1 + \frac{s_0}{b}\right) + \frac{s_0}{b} - \frac{s_a}{b}\right) = s_a \left(\frac{1}{b+s_0} - \frac{1}{b} + \frac{1}{b+s_0} \frac{s_0}{b}\right) = s_a \frac{b - b - s_0 + s_0}{b(b+s_0)} = 0$$

$$= 1 - \left(1 + \frac{s_0}{b}\right) \left(1 - \frac{s_a}{b+s_0} + \frac{1}{2} \frac{s_a^2}{(b+s_0)^2}\right) + \frac{s_0}{b} \left(1 - \frac{s_a}{s_0} + \frac{1}{2} \frac{s_a^2}{b^2}\right)$$

$$= 1 - \left(1 + \frac{s_0}{b} - \frac{s_a}{b+s_0} - \frac{s_a s_0}{b(b+s_0)} + \frac{1}{2} \frac{s_a^2}{(b+s_0)^2} + \frac{1}{2} \frac{s_a^2}{(b+s_0)^2} \frac{s_0}{b}\right) + \frac{s_0}{b} - \frac{s_a s_0}{b s_0} + \frac{1}{2} \frac{s_a^2 s_0}{b^2 s_0}$$

$$= 1 - \left(1 - \frac{s_0}{b} + \frac{s_a}{b+s_0} + \frac{s_a s_0}{b(b+s_0)} - \frac{1}{2} \frac{s_a^2}{(b+s_0)^2} - \frac{1}{2} \frac{s_a^2}{(b+s_0)^2} \frac{s_0}{b} + \frac{s_0}{b} - \frac{s_a}{b} + \frac{1}{2} \frac{s_a^2 s_0}{b^2 s_0}\right)$$

$$= s_a \frac{b + s_0 - (b + s_0)}{b(b+s_0)} + \frac{1}{2} s_a^2 \frac{(b+s_0)^2 - s_0^2 - b s_0}{b s_0 (b+s_0)^2} = \frac{1}{2} s_a^2 \frac{b^2 + b s_0}{b s_0 (b+s_0)^2} = \frac{1}{2} \frac{s_a^2}{(b+s_0) s_0}$$

$$\frac{F}{F_0} = 1 - \left(1 + \frac{s_0}{b}\right) e^{-\frac{s_a}{b+s_0}} + \frac{s_0}{b} e^{-\frac{s_a}{s_0}}, s_a \ll s_0, \frac{F}{F_0} = \frac{1}{2} \frac{s_a^2}{(b+s_0) s_0}$$

$$s_a = s_a \left[1 + \frac{1}{I_0}\right] = s_a \frac{1 - I_0}{I_0}$$

$$\frac{1}{1-\varepsilon} = 1 + \varepsilon + \varepsilon^2 + \varepsilon^3$$

$$\frac{1}{1+\varepsilon} = 1 - \varepsilon + \varepsilon^2 - \varepsilon^3$$

$$1 = \frac{1 - \varepsilon + \varepsilon^2 - \varepsilon^3}{1 + \varepsilon - \varepsilon^2 + \varepsilon^3}$$

$$b < s \quad \frac{s\alpha}{s+b} = \frac{s\alpha}{s(1+\frac{b}{s})} = \frac{s\alpha}{s} \left(1 - \frac{b}{s}\right) = \frac{s\alpha}{s} - \frac{b s\alpha}{s^2}, e^{-\frac{s\alpha}{s+b}} = e^{-\frac{s\alpha}{s}} \left(1 + \frac{b s\alpha}{s^2}\right)$$

$$\frac{Y}{Y_0} = \left[\left(1 + \frac{s}{b}\right) \left(1 + \frac{s\alpha}{s} - \frac{b s\alpha}{s^2}\right) - \frac{s\alpha}{b} \right] \left(1 + \frac{b s\alpha}{s^2}\right) \left[\left(1 + \frac{s}{b}\right) \left(1 + \frac{s\alpha}{s} - \frac{s\alpha}{b}\right) + \left(1 + \frac{s\alpha}{s}\right) e^{-\frac{s\alpha}{s}} \right]$$

$$\left(1 + \frac{s\alpha}{s} - \frac{b s\alpha}{s^2} + \frac{s}{b} + \frac{s\alpha}{b} - \frac{s\alpha}{s} - \frac{s\alpha}{b}\right) \left(1 + \frac{b s\alpha}{s^2}\right) - \left(1 + \frac{s}{b} + \frac{s\alpha}{s} + \frac{s\alpha}{b} - \frac{s\alpha}{b}\right)$$

$$\frac{1 - b s\alpha}{s^2} + \frac{s}{b} + \frac{b s\alpha}{s^2} - \frac{b^2 s\alpha^2}{s^4} + \frac{s\alpha}{s} - \frac{1}{b} - \frac{s\alpha}{s}$$

$$\frac{Y}{Y_0} = \left(1 + \frac{s - s\alpha}{b} + \frac{s\alpha}{s+b} + \frac{s\alpha s}{b(s+b)}\right) e^{-\frac{s\alpha}{s+b}} - \frac{s}{b} e^{-\frac{s\alpha}{s}}$$

$$\frac{s\alpha}{s+b} = \frac{s\alpha}{s} \frac{1}{1 + \frac{b}{s}} = \frac{s\alpha}{s} \left(1 - \frac{b}{s} + \frac{b^2}{s^2} - \frac{b^3}{s^3}\right) = \frac{s\alpha}{s} - \frac{s\alpha}{s} \frac{b}{s} \left(1 - \frac{b}{s} + \frac{b^2}{s^2}\right)$$

$$e^{-\frac{s\alpha}{s+b}} = e^{\frac{s\alpha}{s}} e^{-\frac{s\alpha b}{s^2}} = e^{-\frac{s\alpha}{s}} \left[1 + \frac{s\alpha b}{s^2} \left(1 + \frac{1}{2} \frac{s\alpha^2 b^2}{s^4} \left(1\right)^2 + \frac{1}{6} \frac{s\alpha^3 b^3}{s^6} \left(1\right)^3\right)\right]$$

$$e^{-\frac{s\alpha}{s+b}} = e^{-\frac{s\alpha}{s+b} + \frac{s\alpha}{s} - \frac{s\alpha}{s}} = e^{-\frac{s\alpha}{s}} e^{\frac{s\alpha}{s} - \frac{s\alpha}{s+b}} = e^{-\frac{s\alpha}{s}} \left[1 + \left(\frac{s\alpha}{s} - \frac{s\alpha}{s+b}\right) + \frac{1}{2} \left(\frac{s\alpha}{s} - \frac{s\alpha}{s+b}\right)^2\right]$$

$$\frac{Y}{Y_0} = e^{-\frac{s\alpha}{s}} \left\{ \left[1 + \frac{s}{b} - \frac{s\alpha}{b} + \frac{s\alpha}{s+b} + \frac{s\alpha s}{b(s+b)}\right] \left[1 + \frac{s\alpha}{s} - \frac{s\alpha}{s+b} + \frac{1}{2} \frac{s\alpha^2}{s^2} - \frac{s\alpha^2}{s(s+b)} + \frac{1}{2} \frac{s\alpha^2}{(s+b)^2} - \frac{s}{b}\right] \right\}$$

$$= e^{-\frac{s\alpha}{s}} \left\{ \left[1 + \frac{s}{b} + \frac{s\alpha}{(s+b)} + \frac{s\alpha}{b} \left(1 - \frac{s}{s+b}\right)\right] \left[1 + \frac{s\alpha}{s} \left(1 - \frac{s}{s+b}\right) + \frac{1}{2} \left(\frac{s\alpha}{s}\right)^2 \left(1 - \frac{s}{s+b}\right)^2\right] - \frac{s}{b}\right\}$$

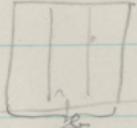
$$= e^{-\frac{s\alpha}{s}} \left\{ \left[1 + \frac{s}{b} + \frac{s\alpha}{s} - \frac{s\alpha b}{s^2} - \frac{s\alpha}{s} - \frac{s}{b} + \frac{s\alpha b}{s^2} + \frac{s\alpha}{s} + \frac{s\alpha^2 b}{s^3} - \frac{s\alpha^2 b}{s^2 s}\right] - \frac{s}{b}\right\}$$

$$= e^{-\frac{s\alpha}{s}} \left\{ 1 + \frac{s}{b} + \frac{s\alpha}{s} - \frac{s\alpha b}{s^2} - \frac{s\alpha}{s} - \frac{s}{b} + \frac{s\alpha b}{s^2} + \frac{s\alpha}{s} + \frac{s\alpha^2 b}{s^3} - \frac{s\alpha^2 b}{s^2 s} \right\}$$

$$\frac{Y}{Y_0} = \left[(1 + \frac{1}{b}) \left(1 + \frac{s\omega}{s+b} \right) - \frac{s\omega}{b} \right] e^{-\frac{s\omega}{s+b}} - \frac{1}{b} e^{-\frac{s\omega}{b}}$$

$$\frac{\bar{Y}}{Y_0} = \frac{1}{2a} \int_{-\frac{b}{2}-a}^{\frac{b}{2}+a} \left[(1 + \frac{1}{b}) \left(1 + \frac{s\omega}{s+b} \right) - \frac{s\omega}{b} \right] e^{-\frac{s\omega}{s+b}} - \frac{1}{b} e^{-\frac{s\omega}{b}} ds$$

$$\frac{v^2}{r} = \frac{(2\omega r + \omega)^2}{r} = 4\omega^2 r + \omega^2 = 40\pi^2 r \omega^2$$



$$\frac{Y}{Y_0} = \left(1 + \frac{s\omega}{s} \right) e^{-\frac{s\omega}{s}} \quad \frac{\bar{Y}}{Y_0} = \frac{1}{2a} \int_{-\frac{b}{2}-a}^{\frac{b}{2}+a} \left(1 + \frac{s\omega}{s} \right) e^{-\frac{s\omega}{s}} ds = \frac{s\omega}{2a} \left. \frac{1}{s} e^{-\frac{s\omega}{s}} \right|_{-\frac{b}{2}-a}^{\frac{b}{2}+a}$$

$$\frac{\bar{Y}}{Y_0} = \frac{s\omega}{2a} \frac{\frac{b}{2}+a}{s\omega} e^{-\frac{s\omega}{\frac{b}{2}+a}} - \frac{s\omega}{2a} \frac{\frac{b}{2}-a}{s\omega} e^{-\frac{s\omega}{\frac{b}{2}-a}} = \left(\frac{b}{4a} + \frac{1}{2} \right) e^{-\frac{s\omega}{\frac{b}{2}+a}} - \left(\frac{b}{4a} - \frac{1}{2} \right) e^{-\frac{s\omega}{\frac{b}{2}-a}}$$

$$2a = \frac{b}{2}, \frac{b}{4a} = 1, \frac{b}{2} + a = 3a, \frac{b}{2} - a = a, \frac{\bar{Y}}{Y_0} = \frac{3}{2} e^{-\frac{s\omega}{3a}} - \frac{1}{2} e^{-\frac{s\omega}{a}}$$

$$\frac{s\omega}{a} < 1, \frac{\bar{Y}}{Y_0} = \frac{3}{2} \left(1 - \frac{1}{3} a \right) - \frac{1}{2} \left(1 - \frac{1}{a} \right) = \frac{3}{2} - \frac{s\omega}{2a} - \frac{1}{2} + \frac{s\omega}{2a} = 1$$

$$\frac{s\omega}{a} = 3, \frac{\bar{Y}}{Y_0} = \frac{3}{2} e^{-1} - \frac{1}{2} e^{-3} = \frac{3}{2} 0,368 - \frac{1}{2} 0,0498 = 0,552 - 0,025$$

$$e^{-\frac{s\omega}{b}} = e^{-\frac{s\omega}{4a}} = e^{-\frac{3}{4}} \approx 0,5$$

$$\bar{\mu} = \frac{\mu_0}{2} \frac{M_0 F}{kT} = 10^{-18} \frac{10 \cdot 40}{4 \cdot 10^{-14}} \cdot 10^{-18} \cdot 10^{-3} = 10^{-24} \text{ fm}$$

$$D = \frac{1}{2} \frac{\bar{\mu}}{m} \frac{\partial F}{\partial z} \frac{l^2}{v^2}, \quad S_{\alpha} = \frac{C \bar{\mu}}{4 k T} \frac{\partial F}{\partial z} = \frac{10^4 \cdot 10^{-21}}{16 \cdot 10^{-14}} \frac{\partial F}{\partial z} = \frac{1}{16} 10^{-3} \frac{\partial F}{\partial z} \text{ cm}$$

$$\frac{\partial F}{\partial z} = 10^3 = 3 \cdot 10^5 \text{ V/cm} \quad S_0 = \frac{\mu^2 l^2}{(4 k T)^2} F \frac{\partial F}{\partial z} = 1,34 \cdot 10^{-6} \left(F \frac{\partial F}{\partial z} \right) \text{ cm}$$

$$\frac{\mu l}{4 k T} = \frac{1,87 \cdot 10^{18} \cdot 10^2}{16 \cdot 10^{-14}} = 1,17 \cdot 10^3$$

$$F \frac{\partial F}{\partial z} = 10^4 \text{ ergs}$$

$$q = C \ln r + \text{konst} \quad q_1 - q_2 = \ln \frac{r_1}{r_2} \quad C = \frac{q_1 - q_2}{\ln \frac{r_1}{r_2}} \sim q_1 - q_2$$

$$F = -\frac{\partial q}{\partial r} = -\frac{C}{r}, \quad \frac{\partial F}{\partial r} = \frac{C}{r^2}, \quad F \frac{\partial F}{\partial r} = \frac{C}{r^3} = \frac{C}{r^3} = \frac{(q_1 - q_2)^2}{r^3} = 10^4, \quad q_1 - q_2 = 10 \text{ ergs} = 10000 \text{ eV}$$

$$\frac{10^2}{r^3} = 10^4, \quad r^3 = 10^{-2} = \frac{10}{10^3}, \quad r = \sqrt[3]{10} \text{ mm} = 2,15 \text{ mm}$$

$$r^2 = a^2 + y^2 \quad \frac{\partial K}{\partial y} = -2I \frac{2a y^2}{r^4} = -2I \frac{4 \cdot 13}{2,69} = -2I \cdot 1,935 = 0,387 i = 23,6 \quad i = 82,37$$

$$r = 1 \text{ cm} \quad y = 1,3a \quad r^2 = a^2 + 1,69a^2 = 2,69a^2 \quad a = \frac{1}{\sqrt{2,69}} = \frac{1}{1,64} = 0,61 \quad y = 1,3a = 0,793$$

$$S_{\alpha}^0 = S_{\alpha_0} \left(1 - \frac{i}{l_0} \right), \quad S_{\alpha}^1 = S_{\alpha_0} \left(1 - \frac{3}{4} \frac{i}{l_0} \right), \quad S_{\alpha}^2 = S_{\alpha_0} \left(1 - \frac{9}{4} \frac{i}{l_0} \right), \quad S_{\alpha}^3 = S_{\alpha_0} \left(1 - \frac{3}{4} \frac{i}{l_0} \right), \quad S_{\alpha}^4 = S_{\alpha_0}$$

$$\frac{d \frac{S_{\alpha}}{l_0}}{d \frac{i}{l_0}} = \frac{d e^{-\frac{S_{\alpha}}{l_0}}}{d \frac{i}{l_0}} = -e^{-\frac{S_{\alpha}}{l_0}} \frac{d \frac{S_{\alpha}}{l_0}}{d \frac{i}{l_0}} = +e^{-\frac{S_{\alpha}}{l_0}} \frac{1}{l_0} \frac{S_{\alpha}}{l_0^2}$$

$\frac{S_{\alpha}}{l_0} = 4$	1,000	1,0000
$+ \frac{3}{4} 0,3679$	$+ 0,2859$	$0,0644$
$+ \frac{1}{2} 0,1353$	$0,0124$	
$+ \frac{1}{4} 0,0498$	1,3560	

$$\frac{d \frac{S_{\alpha}}{l_0}}{d \frac{i}{l_0}} = \frac{S_{\alpha}}{l_0} + e^{-\frac{1}{4} \frac{S_{\alpha}}{l_0}} \frac{3}{4} \frac{S_{\alpha}}{l_0} + e^{-\frac{3}{4} \frac{S_{\alpha}}{l_0}} \frac{2}{4} \frac{S_{\alpha}}{l_0} + e^{-\frac{3}{4} \frac{S_{\alpha}}{l_0}} \frac{1}{4} \frac{S_{\alpha}}{l_0}$$

$$= \frac{S_{\alpha}}{l_0} \left(1 + \frac{3}{4} e^{-\frac{1}{4} \frac{S_{\alpha}}{l_0}} + \frac{1}{2} e^{-\frac{1}{2} \frac{S_{\alpha}}{l_0}} + \frac{1}{4} e^{-\frac{3}{4} \frac{S_{\alpha}}{l_0}} \right)$$

$0,3679$	$-0,0920$
$0,2859$	

$$S_\alpha = -\frac{1}{2} g \frac{l^2}{\alpha^2} + \frac{1}{2} M \frac{2I}{r^2} \frac{l^2}{\alpha^2} - \frac{1}{2} g \frac{l^2}{\alpha^2} = \frac{1}{2} M \frac{2I_0}{r^2} = S_{\alpha_0}$$

$$S_\alpha = -S_{\alpha_0} + S_{\alpha_0} \frac{I}{I_0} = S_{\alpha_0} \left(1 + \frac{I}{I_0} \right) = S_{\alpha_0} \frac{\Delta I}{I_0}$$

$$\frac{M}{M_0} = \left(1 + \frac{S_\alpha}{S_0} \right) e^{-\frac{S_\alpha}{S_0}} \quad \frac{S_\alpha}{S_0} =$$

$$n = n_0 e^{-\frac{c^2}{\alpha^2}} \frac{c^3}{\alpha^3} d\frac{c}{\alpha} = n_0 e^{-x} x dx \quad n_0 \int_{-\infty}^{\infty} e^{-x} x dx = n_0 \int_{-\infty}^{\infty} x de^{-x} = n_0 (x e^{-x} + e^{-x}) \Big|_{-\infty}^{\infty} = n_0$$

$$\begin{aligned} dn &= \gamma ds, \quad d n_0 = \gamma_0 ds_0, \quad s' = s - s_0, \quad ds' = ds = -ds_0 \\ s &= \frac{c}{\alpha}, \quad \frac{s'}{s_0} = \frac{\alpha^2}{c^2} = \frac{1}{x}, \quad ds' = -\frac{s_0 dx}{x^2}, \quad \frac{1}{ds} = \frac{x^2}{s_0} \frac{1}{dx} \end{aligned}$$

$$\gamma = \frac{dn}{ds} = dn_0 e^{-x} x dx \Big|_{ds} = \frac{dn_0}{s_0} e^{-x} x^3$$

$$dn_0 = \gamma_0 ds_0 = -\gamma_0 ds' = -\gamma_0 ds = \gamma_0 \frac{s_0}{x^2} dx$$

$$d\gamma = \gamma_0 \frac{1}{x^2} dx e^{-x} x^3 = \gamma_0 x e^{-x} dx, \quad x = \frac{s_0}{s} = \frac{s_0}{s-s_0}$$

$$\gamma_0 = \text{konst} \quad \gamma = \gamma_0 \int x e^{-x} dx = \gamma_0 (1+x) e^{-x} \Big|_{\frac{s_0}{s-s_0}}^{\frac{s_0}{s-s_0}}$$

$$\gamma = \gamma_0 (1+x) e^{-x} \Big|_{\infty}^{\frac{s_0}{s-s_0}} = \gamma_0 \left(1 + \frac{s_0}{s}\right) e^{-\frac{s_0}{s}}$$

$$d\gamma = \gamma_0 x e^{-x} dx, \quad x = \frac{c^2}{\alpha^2} = \frac{s_0}{s}, \quad s' = b+s, \quad \gamma_0 = \gamma_0^0 \left(1 - \frac{s_0}{b}\right)$$

$$x = \frac{s_0}{s+b}, \quad s+b = \frac{s_0}{x}, \quad b = \frac{s_0}{x} - s, \quad \gamma_0 = \gamma_0^0 \left(1 + \frac{s}{b} - \frac{s_0}{x b}\right) = \gamma_0^0 \left(1 + \frac{s}{b}\right) - \gamma_0^0 \frac{s_0}{b} \frac{1}{x}$$

$$\gamma_0 = \gamma_0^0 \left(1 + \frac{s}{b}\right) \int x e^{-x} dx - \gamma_0^0 \frac{s_0}{b} \int e^{-x} dx = \gamma_0^0 \left[\left(1 + \frac{s}{b}\right) \left(1 + \frac{s_0}{b}\right) e^{-x} \Big|_{\frac{s_0}{b}}^{\frac{s_0}{s+b}} - \frac{s_0}{b} e^{-x} \Big|_{\frac{s_0}{b}}^{\frac{s_0}{s+b}} \right]$$

$$\frac{\gamma_0}{\gamma_0^0} = \left[\left(1 + \frac{s}{b}\right) \left(1 + \frac{s_0}{b}\right) - \frac{s_0}{b} \right] e^{-\frac{s_0}{s+b}} - \left[\left(1 + \frac{s}{b}\right) \left(1 + \frac{s_0}{b}\right) - \frac{s_0}{b} \right] e^{-\frac{s_0}{s}} + \left(1 + \frac{s_0}{b}\right) e^{-\frac{s_0}{s}}$$

$$1 + \frac{s}{b} + \frac{s_0}{s+b} + \frac{s_0 s}{b(s+b)} - \frac{s_0}{b}$$

$$1 + \frac{s_0}{s} + \frac{s}{s+b} - \frac{s_0}{b}$$

$$t = \frac{l}{c} = \frac{s}{v}, s = \frac{v}{c} l, ds = -\frac{v}{c} l dc, \left| \frac{dc}{c} \right| = \frac{ds}{l v} \frac{c}{v} = \frac{b}{l} \frac{c}{v}$$

$$\Delta t = \frac{b}{v} \quad t = \frac{l}{c} \quad \frac{\Delta t}{t} = \frac{b c}{l v} \quad r = 1 \text{ cm} \quad 2\pi r = 6.3 \text{ cm} \quad v = 6.3 \times 10^4 \frac{\text{cm}}{\text{sec}} \quad r = 10^4$$

$$\frac{dy}{dx} = \frac{c e^{-\frac{c^2}{\alpha^2}} c^3}{\alpha^3} \frac{dc}{\alpha} + \frac{c}{\alpha} = x, \frac{dy}{dx} = 3x^2 e^{-x^2} - 2x^4 e^{-x^2}, x_m^2 = \frac{3}{2}$$

$$\sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$

$$\frac{y_m}{1864} = e^{-\frac{3}{2}} \cdot \frac{3}{2} \sqrt{\frac{3}{2}} \sim 0.2231 \times 1.5 \cdot 1.2245 \sim 0.410$$

$$0.0973 : 0.410 = 0.237\% \quad \sqrt{\frac{3}{2}} = 1.224, \quad 100\%$$

$$\frac{c}{\alpha} = 1, \quad y = e^{-1} \cdot 1^3 = 0.3679, \quad \frac{y}{y_m} = 0.90$$

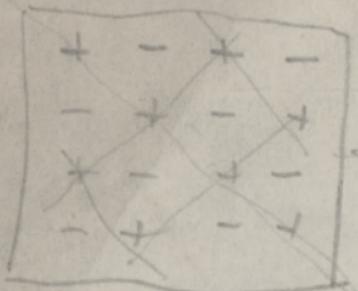
$$\frac{6.20}{15.39}$$

$$1 \quad 90\%$$

$$\frac{c}{\alpha} = \frac{1}{2}, \quad y = e^{-\frac{1}{4}} \cdot \frac{1}{8} = 0.4488 : 8 = 0.0973, \quad \frac{y}{y_m} \sim 0.1$$

$$\frac{1}{2} = 0.5 \quad 24\%$$

$$\frac{c}{\alpha} \ll 1, \quad y = \left(1 - \frac{c^2}{\alpha^2}\right) \frac{c}{\alpha}^3, \quad \frac{c}{\alpha} = \frac{1}{4}, \quad y \sim \frac{1}{64} (1 - \frac{1}{16}) \sim 0.0156 \cdot 0.9375 \sim 0.0146, \quad \frac{y}{y_m} = \frac{1}{4} = 0.25 \quad 3.5\%$$

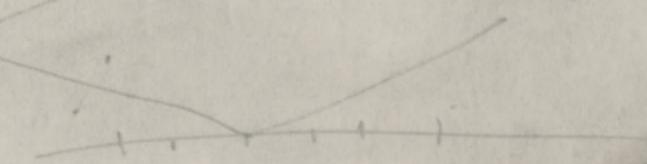
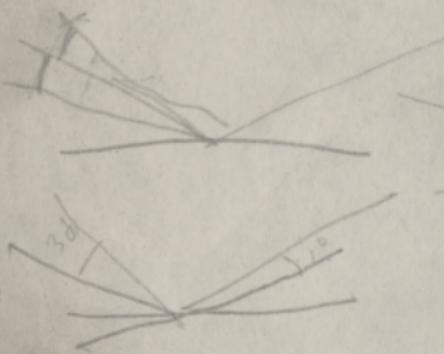
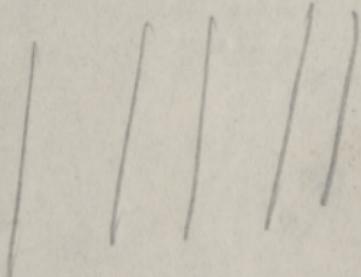


$$\begin{array}{r} -1.8 \\ +0.6 \\ 2.4 \cdot \frac{1}{2} \\ \hline 140 \end{array}$$

$$1.92 \times 10^{-2}$$

$$1.745 \times 10^{-2}$$

1.6



$$\textcircled{2} \quad mvr = \frac{h}{2\pi}, \quad 2\pi r^2 v c = \mu, \quad 2\pi r v = \nu, \quad \nu = \frac{\theta}{2000}, \quad \mu = \frac{1}{2} r v e = \frac{e}{2} r \nu$$

$$v_r = \frac{1}{m} \frac{h}{2\pi}, \quad \mu = \frac{1}{2} \frac{e}{m} \frac{h}{2\pi} = \frac{1}{2} \frac{e}{m} \nu \quad \nu = mvr$$

$$e = \frac{F}{N} = \frac{9.6 \times 10^3}{6.0 \times 10^{23}} = 1.6 \times 10^{-20} \text{ el.m.cgs.} = 4.8 \times 10^{-10} \text{ el.st.cgs.} \quad \frac{h}{2\pi} = \frac{6.5 : 6.3}{20} = 1.032 \times 10^{-27}$$

$$m = 9 \times 10^{-28} \text{ g}, \quad \frac{e}{m} = \frac{1.6 \times 10^{-20}}{9 \times 10^{-28}} = 1.76 \times 10^7 \text{ el.m.cgs.} \quad \mu = 0.88 \times 10^{-7} \times 1.032 \times 10^{-27} = 0.91 \times 10^{-28}$$

$$\mu_n = \frac{0.9 \times 10^{-20}}{1800} = 0.5 \times 10^{-23} \mu_N = 3 \text{ cg's} \quad M_0 = \mu N = 5.46 \times 10^3 = 5,460 \text{ c.g.s.}$$

$$\mu H = h \nu, \quad \nu = \frac{1}{2} \frac{e}{m} \frac{h}{2\pi} \frac{H}{h} = \frac{H}{4\pi} \frac{e}{m} = \frac{1.76 \times 10^7}{12.566} H = 1.4 \times 10^6 H$$

$$\nu_1 = \frac{\mu}{h} H = \frac{h}{2\pi}, \quad h = 2\pi \nu, \quad \nu_1 = \frac{1}{2\pi} \frac{\mu}{H} H$$

$$\frac{14.6}{12.566} : \frac{12.566}{5.026} = 1.4$$

$$F = e(\mathcal{H} + d\mathcal{H}) - e\mathcal{H} = e d\mathcal{H}, \quad d\mathcal{H} = l \frac{d\mathcal{L}}{ds}, \quad F = e l \frac{d\mathcal{L}}{ds} = \mu \frac{d\mathcal{L}}{ds}$$

$$\left(\frac{3}{2} - \frac{s}{a}\right)(1+x) + \frac{s_x}{a} \frac{1}{1+x}$$

$$\left(\frac{3}{2} - \frac{s}{a}\right) + \frac{1}{1+x} \frac{s_x}{a} = \frac{3}{2} - \frac{s}{a} + \frac{s_x}{a} \frac{s+b}{s_x+s+b}$$

$$\left(\frac{3}{2} - \frac{s}{a} + \frac{s_x}{a} \frac{s-\frac{3}{2}a}{s_x+s-\frac{1}{2}a}\right) F\left(\frac{s_x}{s-\frac{1}{2}a}\right)$$

$$-\left(\frac{3}{2} - \frac{s}{a} + \frac{s_x}{a} \frac{s-\frac{3}{2}a}{s_x+s-\frac{3}{2}a}\right) F\left(\frac{s_x}{s-\frac{3}{2}a}\right)$$

$$14.5 : 13 = 1.1154 \quad \frac{14.5}{13} = 1.1154$$

$$\begin{array}{r} 13 \\ \overline{)14.5} \\ 13 \\ \hline 15 \\ \overline{)20} \\ 13 \\ \hline 7 \\ \overline{)70} \\ 65 \\ \hline 50 \end{array}$$

$$\frac{14.5}{9} = 1.61 \quad \frac{14.5}{9} = 1.61$$

$$\begin{array}{r} 9 \\ \overline{)14.5} \\ 9 \\ \hline 55 \\ \overline{)55} \\ 0 \end{array}$$

$$\frac{14.5}{7} = 2.07143 \quad \frac{14.5}{7} = 2.07143$$

$$\begin{array}{r} 7 \\ \overline{)2350} \\ 21 \\ \hline 35 \\ \overline{)935} \\ 915 \\ \hline 15 \end{array}$$

$$9 : 23.5 = 0.38298$$

$$\frac{M05}{1950} \quad M : 21.5 = 0.32558$$

$$\frac{1880}{1900} \quad \frac{645}{550} \quad 130 : 275 = 0.47272$$

$$\begin{array}{r} 410 \\ \overline{)2300} \\ 215 \\ \hline 150 \\ \overline{)1850} \\ 175 \\ \hline 100 \\ \overline{)1250} \\ 105 \\ \hline 200 \\ \overline{)1450} \\ 1420 \\ \hline 30 \\ \overline{)2000} \\ 1925 \\ \hline 75 \\ \overline{)550} \\ 525 \\ \hline 25 \\ \overline{)1925} \\ 1925 \\ \hline 0 \end{array}$$

$$110 : 255 = 0.43137$$

$$\begin{array}{r} 110 \\ \overline{)1045} \\ 1045 \\ \hline 50 \\ \overline{)450} \\ 450 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 800 \\ \overline{)765} \\ 765 \\ \hline 35 \\ \overline{)25} \\ 25 \\ \hline 0 \end{array}$$

$$\gamma_0 = \frac{q^2}{2} \pi_0^2 = \frac{m(2\pi c N)^2}{2} \gamma_e^2, d_0 = \frac{M}{2} 4\pi c^2 v^2 \gamma_e^2 \text{ erg}$$

$$J_0 = 250 \times 40 \times 10^{24} \times 4 \times 10^{-16} \times 2.4 \times 10^{-8} \text{ cal} = 4 \times 10^{28} \times 2.4 \times 10^{-24} \sim 10^5 \text{ cal}$$

$$1046 \quad 0.041 \quad 866 \quad 1151 \quad 1046358 \quad 13915 \quad 0.089025$$

$$- \quad 84 \quad \underline{1482} \quad 348 \quad \underline{0.076010} \quad 258 \quad \underline{0.089278}$$

$$0.08926 \quad 411 \quad 41 \quad 411 \quad 0.11333 \quad 461$$

$$-0.06464 \quad 423 \quad 42 \quad 423 \quad 0.08928 \quad 471$$

$$\underline{0.02161:5} \quad 433 \quad 43 \quad 434 \quad 2405:5 \quad 481$$

$$0.00432 \quad 442 \quad 44 \quad 444 \quad 481 \quad 491$$

$$450 \quad 45 \quad 453 \quad 501 \quad 2405$$

$$2159 \quad 215 \quad 2162$$

$$0.13956 \quad 510 \quad 509 \quad 1819 \quad 112722 \quad 1.53846 \quad 2318 \quad 208$$

$$0.11333 \quad 519 \quad 517 \quad 607 \quad 154 \quad 354$$

$$\underline{2623:5} \quad 528 \quad 525 \quad 113329 \quad 0.139565$$

$$534 \quad 533 \quad 546 \quad 541$$

$$525 \quad 546 \quad 2640 \quad 2625$$

$$1.42854 \quad 2842 \quad 0.164458 \quad 0.16446 \quad 548$$

$$1.43 \quad 2820:5 \quad 0.13956 \quad 554$$

$$565 \quad 574 \quad 564 \quad 2820$$

$$1.9333 \quad 3448:3 \quad 0.198853 \quad 585 \quad 19440 \quad 564$$

$$1149 \quad -1140 \quad 2194406 \quad 592 \quad 16446 \quad 4$$

$$-0.164458 \quad 29948:5 \quad 599 \quad 606 \quad 2994$$

$$1.25 \quad 22920 \quad 618 \quad 1.17644 \quad 265271 \quad 260406 \quad 26212 \quad 648 \quad 648$$

$$19440 \quad 624 \quad 353 \quad 260406 \quad 1420 \quad 22920 \quad 654 \quad 653$$

$$3150:5 \quad 630 \quad 4865 \quad 262126 \quad 3292:5 \quad 660 \quad 658$$

$$630 \quad 636 \quad 659 \quad 659 \quad 666 \quad 664$$

$$642 \quad 3150 \quad 3900 \quad 3900$$

$$296900 \quad 1.05263 \quad 6429 \quad 333244 \quad 33154 \quad 695 \quad 695$$

$$291321 \quad 0.296280 \quad 674 \quad -1690 \quad 29628 \quad 1699 \quad 1699$$

$$5579:9 \quad 0.26212 \quad 679 \quad 33158 \quad 3529:5 \quad 403 \quad 403$$

$$6200 \quad 3416:5 \quad 684 \quad 406 \quad 404 \quad 410$$

$$620 \quad 683 \quad 688 \quad 411 \quad 411 \quad 411$$

$$5380 \quad 3410 \quad 691 \quad 3515 \quad 3529$$

$$0.36468 \quad 721 \quad 40511 \quad 435 \quad 830 \quad 0.403429$$

$$-0.33154 \quad 724 \quad 36488 \quad 740 \quad 316$$

$$3631:5 \quad 727 \quad 3723:5 \quad 745 \quad \underline{405113}$$

$$726 \quad 729 \quad 745 \quad 745$$

$$730 \quad 3723$$

$$1926:3 \quad 469484 \quad 2071 \quad 430 \quad 0.85648 \quad 0.81310 \quad 857 \quad 863$$

$$642 \quad 642 \quad 332 \quad -0.81310 \quad 0.44013 \quad 958 \quad 865$$

$$770126 \quad 4332:4 \quad 098 \quad 4332:4 \quad 4297:5 \quad 860 \quad 864$$

$$866 \quad 859 \quad 861 \quad 861 \quad 864$$

$$869 \quad 4297 \quad 4329$$

2382	0.90004	841	1.21306	906
212	-0.85642	842	-1.16764	904
143	4365.5	843	4539.5	908
069	843	844	908	909
		845		909
		4365		4539

2546	620	0.94402	874
599	-0.90004	878	
821	4395.5	879	
	879	880	
		881	
		4395	

2717	989422	883	0.719424	1618
	1162	884		388
1	988260	885	644662	
-94402		886	686	
4424.5	886		646986	
885	4424		6440	

2892	031472	888
1282	889	
1.03246	890	
0.98826	891	
4450.5	892	
890		
4450		
4450	4450	

$$\int e^{-x} x dx = \int x de^{-x} = xe^{-x} \Big|_{x_1}^{x_2} - \int e^x dx = xe^{-x} + e^x \Big|_{x_2}^{x_1}$$

3041	141
04609	893
04450	894
03246	895
4444.5	896
895	4444

3252	123505	898
	1028	
122488	899	
04450	900	
4498.5	900	
980	901	
	4498	

3449	618
164050	
116467	902
122488	903
4519.5	904
904	905
	905

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With $l=100$ cm we have

$$s_\alpha = \frac{3}{5} \times (M/T) \text{ mm.} \quad (1a)$$

For Cs ($M=132.9$; $T=450^\circ\text{K}$): $s_\alpha=0.177$ mm. Fig. 3 gives the distribution of the intensity in the vertical direction for a beam of 0.04 mm width (beam without half-shadow, detecting wire very thin). s is the distance from the center of the beam, i/i_0 the ratio of the ion current i at the position s to i_0 for the undeflected beam, that is, the straight beam of atoms not influenced by any force.

The available intensity J_0 is in a very rough approximation given by

$$J_0 = \frac{2 \times 10^{-5}}{(MT)^{\frac{1}{2}}} \frac{h}{r^2 \text{ cm}^2 \text{ sec.}}^3$$

where $r=2l$ is the length of the beam and h the height of the ovenslit (in this case h is horizontal).⁴ With $M=132.9$; $T=450^\circ\text{K}$; $2l=r=2 \times 10^2$ cm, $h=0.2$ cm:

$$J_0 = 4 \times 10^{-13} (\text{mol/cm}^2 \text{ sec.})$$

If the diameter of the detecting wire is 4×10^{-3} cm and the effective length 2×10^{-1} cm, J_0 corresponds to an ion current $i_0=3 \times 10^{-11}$ amp.

³ O. Stern, Zeits. f. Physik 39, 755 (1926); U.z.M. 1.

⁴ J_0 depends also on the product of the width b of the ovenslit and the pressure p in the oven. But because of the condition that the mean free path λ in the oven should not be smaller than b , this product is constant. In the above equation it is assumed that for all substances in the first approximation $\lambda=1/10$ mm for $p=1/10$ mm.

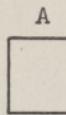


FIG. 1.

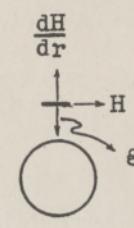


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The magnetic field may be produced by a current I flowing through a wire underneath and parallel to the beam. Let d be the distance between the center of the beam and the center of the wire. Then at the place of the beam the field strength H is $2I/d$ and the inhomogeneity $dH/dr = -2I/d^2$. H is horizontally, dH/dr vertically directed (Fig. 2). The magnetic force $F_m = \mu(dH/dr)$ exerted on a magnetic dipole has also the vertical direction. Thereby μ is the component of the magnetic moment of the dipole in the direction of H (horizontal in our case).⁵ For alkali atoms in a strong field μ has only the two values $+\mu_0$ and $-\mu_0$ (μ_0 Bohr magneton). In our case we have to deal with a very weak field where we have many more components. But this does not make any difference in the essential point as we shall see later. So let us assume for the moment that we have only the two components $+\mu_0$ and $-\mu_0$.⁶ Then for one-half of the atoms the magnetic force has the same direction as the force of gravity, for the other half of the atoms the opposite direction. For these atoms it will be possible to choose $|dH/dr| = 2I_0/d^2$ so that the magnetic force just cancels the force of gravity. These atoms will get no acceleration at all and move strictly in straight lines. I_0 is determined by the equation

$$mg = \mu_0 |dH/dr| = \mu_0 (2I_0/d^2). \quad (2)$$

To find I_0 we can employ different methods. The most straightforward procedure seems to be the

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At this point we can see at once why the splitting of the beam into many components, 16 for Cs, by a weak field does not matter. The component with the largest value of μ has always

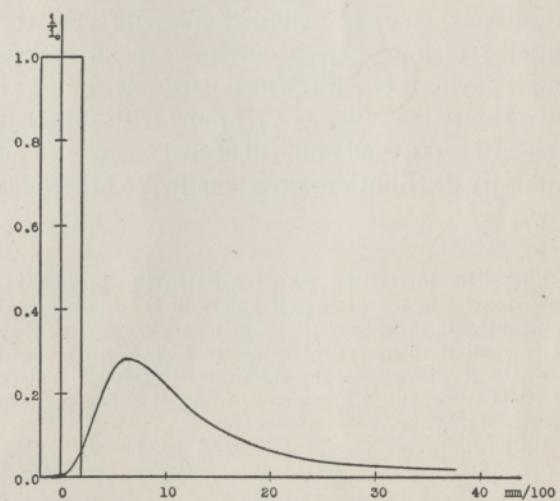


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the moment μ_0 .⁷ But this component is the only one we are concerned with because only for this one the deflection has an upward direction as long as $I - I_0$ does not become too large (till about $I - I_0 < \frac{1}{3}I_0$).

Another method to determine I_0 would be to place the detecting wire directly in the path of the straight beam (Fig. 1, C'') and measure i as a function of I . Then i should have a maximum for $I = I_0$ because if I is larger or smaller than I_0 we deflect atoms upward or downward and diminish the intensity.⁸ The other components do not disturb us in this case either because they give no maximum of i for $I = I_0$ but only a monotonic increase of i with I . Of course, also here i can be easily calculated as a function of I .

It seems that I_0 could be determined very accurately by either one of these methods. This should make possible a very exact measurement of $N\mu_0$. Eq. (2) gives:

$$\mu_0 = mgd^2/2I_0 \quad \text{or} \quad N\mu_0 = M_0 = Mg^2/2I_0$$

(N Avogadro's number, M molecular weight).

Since M and g are well known the accuracy of the result will probably depend mainly on the accuracy of d , that is of the alignment of the arrangement.

To calculate numerical values we write (2) in the form

$$|dH/dr| = (M/M_0)g = 2I_0/d^2. \quad (2a)$$

For Cs we have

$$|dH/dr| = (132.9/5550) \times 980 = 23.5 \text{ gauss/cm}$$

and for $d = 1 \text{ cm}$

$$I_0 = \frac{1}{2} \times 23.5 \text{ e.m.u.} = 117.5 \text{ amp.}$$

Corrections for the finite height h of the beam and the magnetic field of the earth are small (quadratic terms) and can easily be taken into account. Furthermore, the beam must be placed

⁷ Exactly, $\mu_0 \pm$ magnetic moment of the nucleus. Since this moment is of the order of magnitude $10^{-3}\mu_0$ it has to be known only very roughly. On the other hand it may be possible in the future to determine nuclear moments in this way.

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in the north-south direction. In this case the Coriolis force produced by the rotation of the earth has no vertical component. Otherwise this force amounts to some tenths of one percent of the force of gravity even for the atoms with the velocity α .

NUCLEAR MOMENTS

It is quite interesting to consider the numerical values for a similar experiment with H_2 molecules. For the deflection by gravity Eq. (1a) gives

$$s_\alpha = \frac{3}{5} \frac{M}{T} \frac{3}{5} \frac{2}{60} \frac{1}{50} \text{ mm}$$

if we take $T = 60^\circ\text{K}$. For the compensating inhomogeneity we get from Eq. (2a) taking $N\mu$ equal to 5 nuclear magnetons per mole

$$|dH/dr| = \frac{M}{N\mu} \frac{2}{15} g = \frac{980}{15} = 131 \frac{\text{gauss}}{\text{cm}}$$

still quite a convenient value for a wire field.

But in this case it will be necessary to take into account the diffraction of the de Broglie waves for the interpretation of the measurements. The wave-length λ_α of a molecule with the velocity α is

$$\lambda_\alpha = \frac{h}{m\alpha} = \frac{Nh}{(2RTM)^{\frac{1}{2}}} = \frac{30.7}{(TM)^{\frac{1}{2}}} 10^{-8} \text{ cm.}$$

For this wave-length the distance s_d of the first diffraction maximum from the beam at the place of the detector is

$$s_d = l = \frac{b}{b} \times \frac{30.7}{(MT)^{\frac{1}{2}}} \times 10^{-8} \text{ cm,}$$

where b is the width of the collimating slit and l the distance between the collimating slit and the detector. For H_2 at 60°K we get:

$$\lambda_\alpha = 2.8 \times 10^{-8} \text{ cm and with } b = 1/100 \text{ mm,}$$

$$s_d = 2.8 \times 10^{-3} \text{ cm}$$

compared with $s_\alpha = 2 \times 10^{-3} \text{ cm}$. For Cs, however, we have

$$\lambda_\alpha = 0.125 \times 10^{-8} \text{ cm and with } b = 2 \times 10^{-3} \text{ cm,}$$

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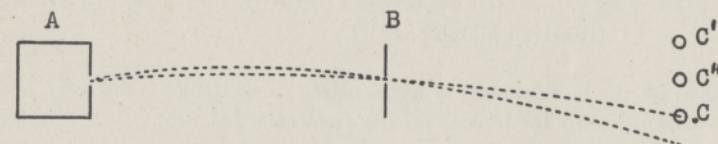


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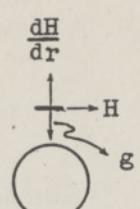


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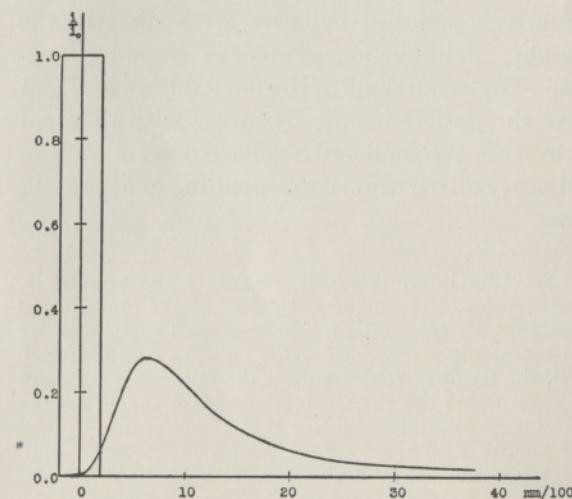


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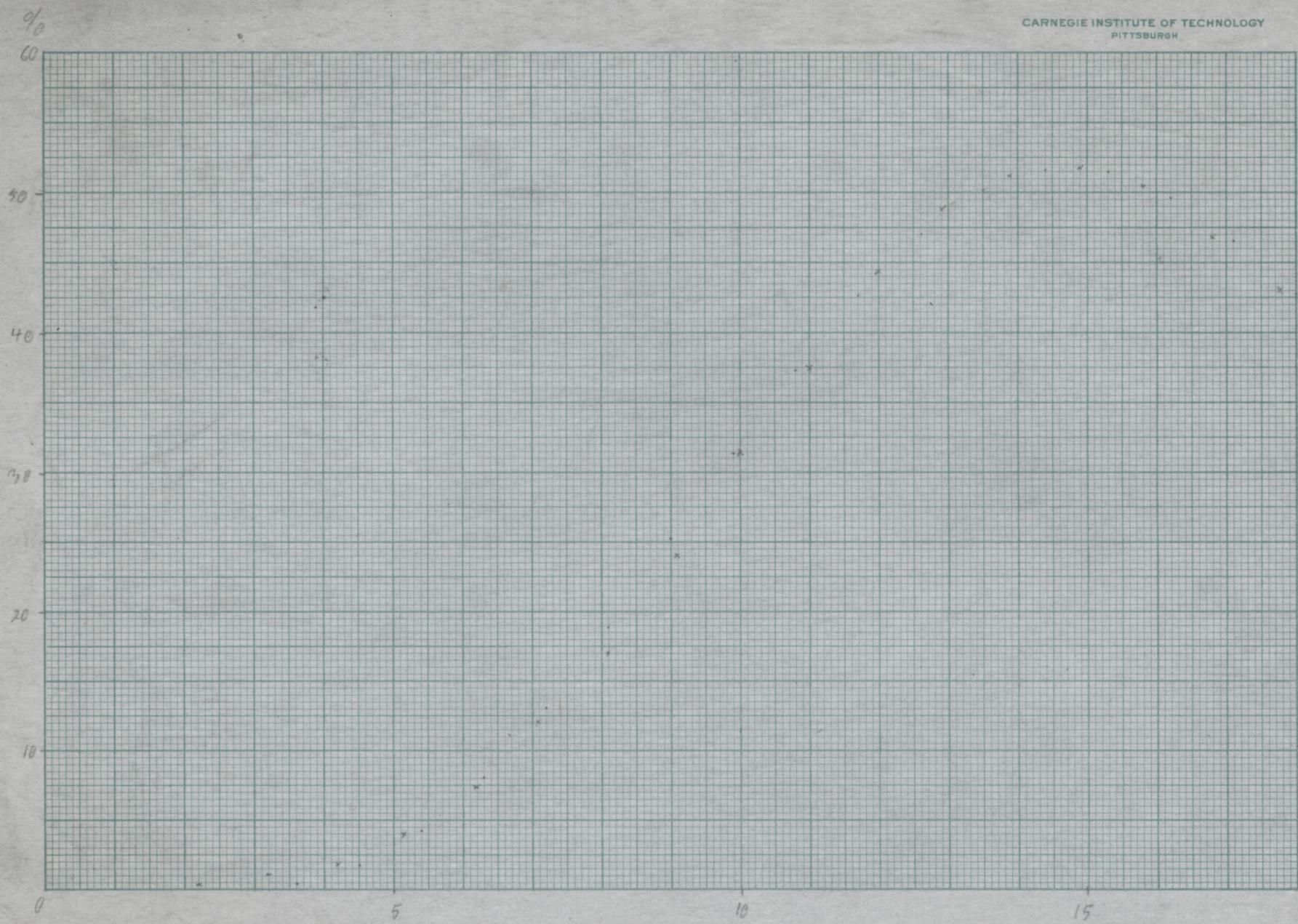
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$$\lambda_\alpha = 0.125 \times 10^{-8} \text{ cm and with } b = 2 \times 10^{-3} \text{ cm,}$$

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CARNEGIE INSTITUTE OF TECHNOLOGY
PITTSBURGH



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$$\begin{array}{r}
 0.45864 \\
 0.45384 \\
 \hline
 0.008480
 \end{array}
 \begin{array}{r}
 +7.5 \\
 +12.4 \\
 \hline
 4.9
 \end{array}$$

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 \hline
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 +20.1 \\
 \hline
 4.1
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$$\begin{array}{r}
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 \hline
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 \begin{array}{r}
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 25.9 \\
 \hline
 5.4
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258

$$\begin{array}{r}
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$$\begin{array}{r}
 0.46380 \\
 0.46146 \\
 \hline
 0.00284
 \end{array}
 \begin{array}{r}
 2.9 \\
 28.9 \\
 \hline
 26.0
 \end{array}$$

$$\begin{array}{r}
 0.46380 \\
 0.46146 \\
 \hline
 0.00284
 \end{array}
 \begin{array}{r}
 2.9 \\
 28.9 \\
 \hline
 26.0
 \end{array}$$

$$\begin{array}{r}
 0.45230 \\
 0.45408 \\
 \hline
 0.00498
 \end{array}
 \begin{array}{r}
 14.9 \\
 8.5 \\
 \hline
 6.4
 \end{array}$$

$$\begin{array}{r}
 464 \\
 22 \\
 \hline
 489
 \end{array}$$

$$\begin{array}{r}
 0.45408 \\
 0.45341 \\
 \hline
 0.00365
 \end{array}
 \begin{array}{r}
 8.4 \\
 12.2 \\
 \hline
 3.8
 \end{array}$$

$$\begin{array}{r}
 45 \\
 44 \\
 \hline
 44
 \end{array}$$

$$\begin{array}{r}
 0.75341 \\
 0.75221 \\
 \hline
 0.00120
 \end{array}
 \begin{array}{r}
 12.2 \\
 14.7 \\
 \hline
 2.5
 \end{array}$$

$$\begin{array}{r}
 186 \\
 194 \\
 \hline
 383
 \end{array}
 \begin{array}{r}
 186 \\
 195 \\
 \hline
 381
 \end{array}
 \begin{array}{r}
 172 \\
 195 \\
 \hline
 367
 \end{array}
 \begin{array}{r}
 164 \\
 197 \\
 \hline
 361
 \end{array}$$

$$\begin{array}{r}
 0.45221 \\
 0.44434 \\
 \hline
 0.00484
 \end{array}
 \begin{array}{r}
 14.8 \\
 20.0 \\
 \hline
 5.2
 \end{array}$$

$$\begin{array}{r}
 485 \\
 49 \\
 \hline
 534
 \end{array}$$

$$\begin{array}{r}
 224 \\
 194 \\
 \hline
 421
 \end{array}$$

$$\begin{array}{r}
 0.44434 \\
 0.43581 \\
 \hline
 1156
 \end{array}
 \begin{array}{r}
 14 \times 10^{-3} \\
 17 \times 10^{-3} \\
 \end{array}
 \begin{array}{r}
 0.6 \\
 -0.6 \\
 \end{array}
 \begin{array}{r}
 10.2 \times 10^{-3} \\
 \end{array}$$

$$\begin{array}{r}
 16.3 \\
 24.4 \\
 \hline
 40.7
 \end{array}
 \begin{array}{r}
 14.8 \\
 24.5 \\
 \hline
 39.3
 \end{array}
 \begin{array}{r}
 16.2 \\
 24.5 \\
 \hline
 48.7
 \end{array}$$

$$\begin{array}{r}
 8.3 \\
 24.5 \\
 \hline
 32.8
 \end{array}
 \begin{array}{r}
 13.6 \\
 24.5 \\
 \hline
 38.1
 \end{array}
 \begin{array}{r}
 16.6 \\
 24.5 \\
 \hline
 41.1
 \end{array}
 \begin{array}{r}
 17.2 \\
 25.5 \\
 \hline
 41.7
 \end{array}
 \begin{array}{r}
 16.3 \\
 24.5 \\
 \hline
 40.8
 \end{array}
 \begin{array}{r}
 13.3 \\
 24.5 \\
 \hline
 34.8
 \end{array}
 \begin{array}{r}
 9.9 \\
 24.5 \\
 \hline
 33.9
 \end{array}$$

$$\begin{array}{r}
 3.32\% \\
 4.01\%
 \end{array}
 \begin{array}{r}
 24.5 \\
 \hline
 41.4
 \end{array}
 \begin{array}{r}
 16.9 \\
 24.5 \\
 \hline
 42.8
 \end{array}
 \begin{array}{r}
 17.9 \\
 24.9 \\
 \hline
 42.8
 \end{array}$$

$$2 \frac{\lambda}{d} = \frac{x}{200}$$

green light

$$d = \frac{400 \lambda}{x} = \frac{400 \times 5.2 \times 10^{-4}}{x} = \frac{20.8}{x} 10^{-2} \text{ nm} \quad \frac{400 \times 5.89 \times 10^{-4}}{x} = \frac{23.56}{x} 10^{-2} \text{ nm}$$

green lower coll. slit: $x = \frac{19\frac{1}{2}}{4} = 4.88 \quad d = 4.26 \times 10^{-2} \text{ nm}$

upper " : $x = \frac{18\frac{1}{2}}{4} = 4.625 \quad d = 4.50 \times 10^{-2} \text{ nm} \quad 471 \ 465 \ 465 \quad (20\%)$

Na l. c. s. : $22\frac{3}{8} \quad x = 5.60 \quad d = 4.20 \times 10^{-2}$

u. " " : $20\frac{1}{8} \quad x = 5.03 \quad d = 4.69 \times 10^{-2}$

$$\frac{22\frac{3}{8}}{25} : 4 = 5.624 \\ \frac{20\frac{1}{8}}{25} : 4 = 5.56$$

u. f. s. : $22\frac{1}{2} \quad x = 5.62 \quad d = 4.19 \times 10^{-2}$

l. fs : $22\frac{1}{4} \quad x = 5.56 \quad d = 4.23 \times 10^{-2}$

$$0.23 \times 18 \quad 0.14 \times 18 \\ \frac{2.3}{2.3} \quad \frac{1.84}{1.84} \\ \frac{4.32}{4.32}$$