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Ø.M.K.

Professor Otto Stern,

c/o. Professor W. Pauli,

Physikalisches Institut der
Eidgenössischen Techn.Hochschule

Gloriastrasse 35

Z ü r i c h

SVEJTS,

Aage Bohr.

UNIVERSITETETS INSTITUT
FOR
TEORETISK FYSIK
BLEGDAMSVEJ 15, KØBENHAVN Ø.

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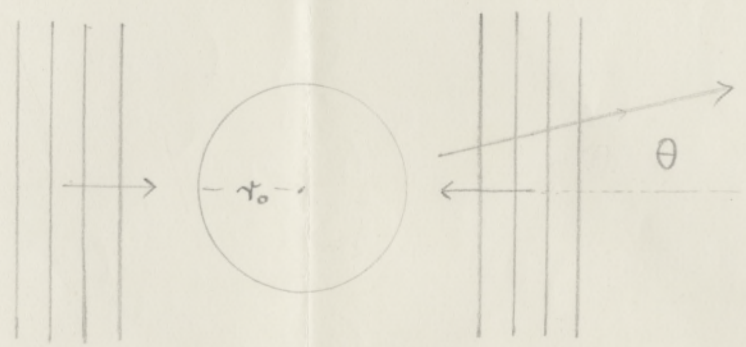
DEN January 8, 1947.

Dear Professor Stern,

As you may have heard from Professor Pauli, with whom the problem was talked over before he left, the question raised by my father during the discussion about your planned experiments to test the anomalies in the scattering of identical particles have been looked into somewhat further. It appears that the effects which the theory predicts for molecular beams of such comparatively large velocities will actually be exceedingly small and probably very difficult to observe. The calculations of Massey and Mohr (Proc.Roy.Soc. 141, 434, 1933), the results of which are quoted in the book of Mott and Massey (Fig.45) and which might seem to give, for identical atoms, a scattering cross-section for small angles only about half that obtained for dissimilar atoms, do not directly allow of such a comparison.

That the effect must be very small is clear from simple arguments. In fact, the quantum mechanical symmetry requirements for identical particles (we here consider such as obey Bose-Einstein statistics) amount to the collision in relative coordinates being represented not by a single wave, but by the superposition of two waves travelling in opposite directions.



The wave scattered into the angle θ (see Figure) will, therefore, result partly from a "forward" scattering of the waves coming from the left and a "backward" scattering of the opposite wave train, and the anomaly in the scattering law arising from the identity of the particles is simply that due to the interference of these two scattered waves.

Now, for the problems in question, where $\bar{\lambda}$ is very small compared with the atomic diameter r_0, the forward scattering into very small angles ($\theta \sim \frac{\bar{\lambda}}{r_0}$) will by far exceed the backward scattering, and it is therefore immediately evident that the interference effect must be very small. Actually, it may be shown, using the model of the rigid spheres, that the ratio between the amplitudes of the forward and backward scattered waves asymptotically approaches the value $\frac{r_0}{\bar{\lambda}}$ for $\theta = 0$ and $r_0 \gg \bar{\lambda}$. The deviations in the scattering cross-section can, therefore, at most amount to $2\frac{\bar{\lambda}}{r_0}$. This quantity is about 1% for Hg atoms at $T \sim 300^0$. Moreover, however, even this small effect would not be observable in an arrangement like that suggested, since there will be a phase difference between the forward and backward scattered waves which depends strongly on the ratio $r_0 : \bar{\lambda}$. Already very small variations in the relative velocity ($\sim \frac{\bar{\lambda}}{r_0}$), in fact, revert the effect which will, therefore, be completely obscured for not strictly monochromatic beams.

With the usual notation of $f(\theta)$ for the factor to $\frac{1}{r} e^{ikr}$ in the asymptotic expression for the scattered wave, the curves of Massey and Mohr give the values of $|f(\theta)|^2$ (for dissimilar particles) and of $\frac{1}{2}|f(\theta) + f(\pi-\theta)|^2$ (for identical particles). Actually, however, in order to find the deviations in the scattering law in the two cases, one should compare $|f(\theta)|^2 + |f(\pi-\theta)|^2$ with $|f(\theta) + f(\pi-\theta)|^2$. Using the computed data of Massey and Mohr, one finds for $\theta = 0$, in the particular case of $r_0 = 20\bar{\lambda}$, a difference of only about 5-10%, which fits well in with the general expectations.

It would presumably, as is also mentioned by Massey and Mohr, be more favourable to investigate the scattering at $\theta = 90^0$, in which case, as is immediately seen, the symmetrization leads always, irrespective of velocity, to a doubling of the cross-section. In this connection, it must, however, be remembered that not only will it be necessary to measure over very small angular intervals (of the order of $\frac{\bar{\lambda}}{r_0}$), but also, the ~~above~~ results refer to the scattering in relative coordinates and, for instance, for a collision between two non-monochromatic molecular beams, the direction corresponding to $\theta = 90^0$ for one type of collision may correspond to widely different values of θ for collisions between atoms of other initial velocities.

My father asks me to send you also his best regards to you and to the Paulis.

Yours sincerely,

Aage Bohr.

$$\mathcal{C} = \ln\frac{(2\pi m)^{\frac{3}{2}} k^{\frac{5}{2}}}{h^3} = \frac{R\ln R - C_p + S_0}{R} = \ln R - \frac{5}{2} + \frac{S_0}{R}$$

$$S_0 = R\ln\frac{(2\pi m)^{\frac{3}{2}} k^{\frac{5}{2}}}{h^3} - R\ln R + \frac{5}{2}R = Nk\ln\frac{(2\pi m k)^{\frac{3}{2}}}{N h^3} + \frac{5}{2}kN - Nk\ln\frac{(2\pi m k)^{\frac{3}{2}}}{h^3} + \frac{3}{2}kN - k\ln N!$$

$$S = R\ln V + \frac{3}{2}R\ln T + S_0 = kN\ln\frac{V}{N} + \frac{3}{2}kN\ln T + \cancel{kN\ln N} + kN\ln\frac{(2\pi m k)^{\frac{3}{2}}}{h^3} + \frac{3}{2}kN - \cancel{kN\ln N} + kN$$

$$= kN\ln\frac{V}{N} + \frac{3}{2}kN\ln T + \frac{5}{2}kN + kN\ln\frac{(2\pi m k)^{\frac{3}{2}}}{h^3}$$

$N = n$, [illegible]: $S = kn\ln\frac{V}{n} + \frac{3}{2}kn\ln T + \frac{3}{2}kn + kn\ln\frac{(2\pi m k)^{\frac{1}{2}}}{h^3}$

$$S = \int_0^T \frac{C}{T}\,dT = \int_0^T \frac{d\mathcal{E}}{T} = \frac{\mathcal{E}}{T}\Big|_0^T + \int_0^T \frac{\mathcal{E}}{T^2}\,dT = \frac{\mathcal{E}}{T} + k\ln\sum e^{-\frac{\varepsilon_i}{kT}} = \frac{\mathcal{E}}{T} + k\ln n!$$

$$\mathcal{E} = \frac{\sum \varepsilon_i e^{-\frac{\varepsilon_i}{kT}}}{\sum e^{-\frac{\varepsilon_i}{kT}}} = kT^2\frac{d\ln\sum}{dT}$$

$$S = \frac{\mathcal{E}}{T} + k\ln\int e^{-\frac{\mathcal{E}(q_1\cdots p_n)}{kT}}\,dq_1\cdots dp_n$$

Gas

$$\mathcal{E} = \frac{p_1^2}{2m} + \cdots \frac{p_n^2}{2m} : \quad \int = \int dq_1\cdots\int dq_n \int e^{-\frac{p_1^2}{2mkT}}dp_1 \cdots \int e^{-\frac{p_n^2}{2mkT}} = V^n (2\pi m kT)^{\frac{1}{2}n} \Big| : n!\,h^n$$

$$S = kn\ln V + k\tfrac{1}{2}n\ln(2\pi m kT) + \tfrac{1}{2}kn\,\big|\, - kn(\ln n - 1) - kn\ln h$$

$$S_{kl} = R\ln V + \frac{1}{2}R\ln T + R\ln(2\pi m k)^{\frac{1}{2}} + \frac{1}{2}R, \quad S_q = R\ln V + \frac{1}{2}R\ln T + R\ln\frac{(2\pi m k)^{\frac{1}{2}}}{Nh} + \frac{3}{2}R$$

Solid

$$\mathcal{E} = \frac{1}{2}(2\pi\nu)^2 m(q_1^2 + \cdots q_n^2) + \frac{p_1^2}{2m} + \cdots \frac{p_n^2}{2m} ; \quad \int = \int_{-\infty}^{+\infty} e^{-\frac{(2\pi\nu)^2 m q_1^2}{2kT}}dq_1 \cdots \int_{-\infty}^{+\infty} e^{-\frac{p_1^2}{2mkT}}dp \cdots = \left(\frac{2kT}{(2\pi\nu)^2 m}\right)^{\frac{1}{2}}\pi^{\frac{1}{2}}(2mkT)^{\frac{1}{2}}\pi^{\frac{1}{2}} = \left(\frac{kT}{\nu}\right)^n \Big| n!$$

$$S_{kl} = kn + kn\ln\frac{kT}{\nu} + k\ln n!\,\big|\, - k\ln n! - kn\ln h \quad S_q = R + R\ln\frac{kT}{h\nu}$$

	Gas	Solid
kl.	$R\ln V + \frac{1}{2}R\ln T + \frac{1}{2}R + R\ln(2\pi m k)^{\frac{1}{2}}$	$R + R\ln\frac{kT}{\nu} + k\ln N!$
qu.	$R\ln V + \frac{1}{2}R\ln T + \frac{3}{2}R + R\ln\frac{(2\pi m k)^{\frac{1}{2}}}{Nh}$	$R + R\ln\frac{kT}{h\nu}$

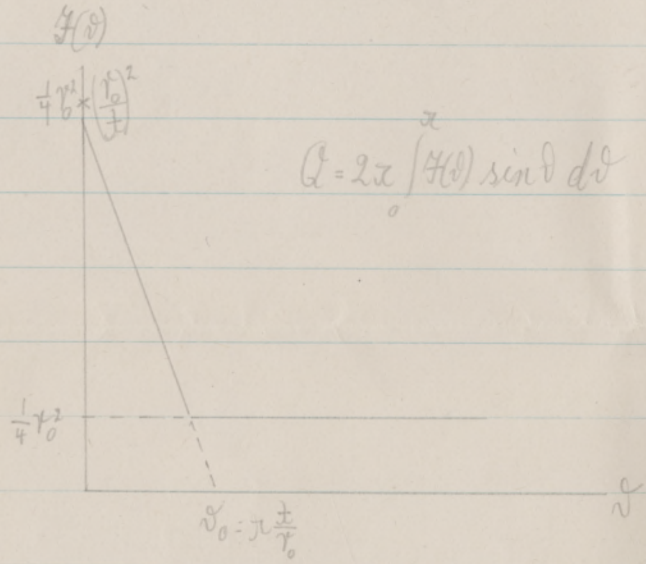
$$S_g = S_s + \frac{\lambda}{T}$$

$$R \ln V_g + \cancel{\tfrac{3}{2} R \ln T} + \cancel{\tfrac{5}{2} R} + R \ln \frac{(2\pi m k)^{\frac{3}{2}}}{N h^3} = \cancel{3R} + \cancel{3R \ln \frac{kT}{h\nu}} \pm \frac{\lambda}{T}$$

$$\ln V_g = \frac{3}{2} \ln T + \frac{1}{2} + \frac{\lambda}{RT} + \ln \frac{\cancel{k^3}^{\,k^{\frac{3}{2}}} N \cancel{h^3}}{\cancel{h^3} \nu^3 (2\pi m)^{\frac{3}{2}} \cancel{k^{\frac{3}{2}}}}$$

$$V_g = \left(\frac{kT}{2\pi m}\right)^{\frac{3}{2}} \frac{N}{\nu^3} e^{\frac{\lambda + \frac{1}{2} RT}{RT}} \qquad p_g = \frac{RT}{V_g} = RT \frac{(2\pi m)^{\frac{3}{2}} \nu^3}{RT (kT)^{\frac{1}{2}}} e^{-\frac{\lambda + \frac{1}{2} RT}{RT}} = \frac{(2\pi m)^{\frac{3}{2}} \nu^3}{(kT)^{\frac{1}{2}}} e^{-\frac{\lambda + \frac{1}{2} RT}{RT}}$$

$$= \frac{(2\pi M)^{\frac{3}{2}} \nu^3}{N (RT)^{\frac{1}{2}}} e^{-\frac{\lambda_0}{RT}}$$



$$Q = 2\pi \int_0^\pi \mathcal{F}(\vartheta) \sin\vartheta \, d\vartheta \qquad k = \frac{2\pi}{\lambda} = \frac{1}{\lambda}$$

$$\psi \sim e^{ikz} + \frac{1}{r} e^{ikr} f(\vartheta), \quad f(\vartheta) = \frac{1}{2ik} \sum_{n=0}^{n=\infty} (2n+1) \left[e^{2i\eta_n} - 1 \right] P_n(\cos\vartheta)$$

$$dw = \mathcal{C} e^{-\frac{\mathcal{E}}{kT}} dq_1 \cdots dp_n, \quad 1 = \mathcal{C}\int e^{-\frac{\mathcal{E}}{kT}} d\tau, \quad \mathcal{C} = \frac{1}{\int_0^\infty dw}$$

$$S_1 - S_2 = \int_{T_2, v_2}^{T_1, v_1} \frac{dQ}{T}, \quad S_{T_1} - S_{T_2} = \int_{T_2}^{T_1} \frac{d\mathcal{E}}{dT}\frac{dT}{T} = \int_{T_2}^{T_1} \frac{d\mathcal{E}}{T} = \frac{\mathcal{E}}{T}\Big|_{T_2}^{T_1} - \int_{T_2}^{T_1} \mathcal{E}\, d\frac{1}{T} = \frac{\mathcal{E}}{T}\Big|_{T_2}^{T_1} + \int_{T_2}^{T_1} \frac{1}{T^2} d\mathcal{E} = \frac{\mathcal{E}}{T_1} - \frac{\mathcal{E}}{T_2} + k\int_{T_2}^{T_1} \frac{d\ln}{dT} dT$$

$$\mathcal{E} = \frac{\int_0^\infty \mathcal{E}\, dw}{\int_0^\infty dw} = \frac{\int_0^\infty \mathcal{E} e^{-\frac{\mathcal{E}}{kT}} d\tau}{\int_0^\infty e^{-\frac{\mathcal{E}}{kT}} d\tau} = kT^2 \frac{d \ln \int_0^\infty e^{-\frac{\mathcal{E}}{kT}} d\tau}{dT} \qquad S_1 - S_2 = \frac{\mathcal{E}_1}{T_1} - \frac{\mathcal{E}_2}{T_2} + k \ln \int_0^\infty dw_{T_1} - k \ln \int_0^\infty dw_{T_2}$$

$$T \to \infty \quad \mathcal{E} \sim T \quad S_1 - S_2 = k \ln \frac{\int_0^\infty dw_{T_1}}{\int_0^\infty dw_{T_2}}$$

Ideal Gas: $$V^n \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\frac{p_1^2 + \cdots + p_n^2}{2mkT}} dp_1 \cdots dp_n = V^n \sqrt{2mkT}\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^n = V^n (2\pi m kT)^{\frac{n}{2}}$$

$$S_1 - S_2 = nk \ln V_1 + \frac{n}{2} k \ln(2\pi m k T_1) - nk \ln V_2 - \frac{n}{2} k \ln(2\pi m k T_2)$$

Faktor n! in S klass. auffüllen!

$$S_{kl} = nk \ln V + \frac{n}{2} k \ln T + nk \ln(2\pi m k)^{\frac{1}{2}} + n\frac{k}{2} \left(\text{wegen } \frac{\mathcal{E}}{T}\right)$$

$$+ k \ln n! = k n \ln n - kn, \quad \cancel{S' = nk \ln \frac{V}{n} + \frac{n}{2} k \ln T - nk \ln n + nk \ln(2\pi m k)^{\frac{1}{2}} + kn \ln n - kn + \frac{k}{2}n}$$

$$S_{kl} = nk \ln \frac{V}{n} + \frac{n}{2} k \ln T + nk \ln(2\pi m k)^{\frac{1}{2}} + \underbrace{nk \ln n - nk}_{k \ln n!} + \frac{3}{2} nk = nk \ln \frac{V}{n} + \frac{n}{2} k \ln T + nk \ln(2\pi m k)^{\frac{1}{2}} - \frac{1}{2} kn$$

quantentheor.: $$S_{kl} - k \ln n!\, h^n = S_q = nk \ln \frac{V}{n} + \frac{n}{2} k \ln T + \frac{3}{2} kn + nk \ln \frac{(2\pi m k)^{\frac{1}{2}}}{h}$$

Reson.: $$\mathcal{E} = \frac{p^2}{2m} + \frac{(2\pi\nu)^2 m}{2} q^2, \quad \int_{-\infty}^{\infty} e^{-\frac{p^2}{2mkT}} dp = (2\pi m kT)^{\frac{1}{2}}, \quad \int_{-\infty}^{\infty} e^{-\frac{(2\pi\nu)^2 m}{2kT} q^2} dq = \left(\frac{2kT\pi}{(2\pi\nu)^2 m}\right)^{\frac{1}{2}}, \quad \int \cdot dp \times \int dq = \frac{2\pi kT}{2\pi\nu} = \frac{kT}{\nu}$$

$$S_{kl} = nk \ln \frac{kT}{\nu} + nk + k \ln n! \qquad S_q = S_{kl} - k \ln n!\, h^n = nk \ln \frac{kT}{h\nu} + nk$$

wegen $\frac{\mathcal{E}}{T}$

$$\mathcal{E} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad s = \int_0^T \frac{d\mathcal{E}}{dT}\frac{dT}{T} = \int_0^T \frac{d\mathcal{E}}{T} = \frac{\mathcal{E}}{T}\Big|_0^T - \int_0^T \mathcal{E}\, d\frac{1}{T} = \frac{\frac{h\nu}{T}}{e^{\frac{h\nu}{kT}} - 1} - k\int_0 \frac{1}{e^{\frac{h\nu}{kT}} - 1} d\frac{h\nu}{kT} = k\left(\frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1} - \int_\infty^{\frac{h\nu}{kT}} \frac{dx}{e^x - 1}\right)$$

$$\frac{d \ln(1 - e^{-x})}{dx} = \frac{e^{-x}}{1 - e^{-x}} = \frac{1}{e^x - 1} \qquad s = k\left[\frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1} - \ln\left(1 - e^{-\frac{h\nu}{kT}}\right)\right] \quad \frac{h\nu}{kT} \ll 1: \; s = k\left[1 - \ln \frac{h\nu}{kT}\right]$$

$$-\frac{1}{2}\frac{h\nu}{kT} \qquad + \frac{1}{2}\frac{h\nu}{kT}$$

$$\frac{h}{p} = \lambda = \frac{2l}{n}, \quad \mathcal{E}_n = \frac{p^2}{2m} = n^2 \frac{h^2}{8l^2 m}, \quad -\frac{u}{T} + s = k \ln \sum_{n=0}^{n=\infty} e^{-n^2 \frac{h^2}{8l^2 m kT}} = k \ln \sum e^{-\alpha^2 n^2} = k \ln \frac{1}{\alpha}\int_0^\infty e^{-x^2} dx = k \ln \frac{\sqrt{\pi}}{2\alpha} = k \ln \frac{l\sqrt{2\pi m kT}}{h}$$

$$-n\frac{k}{2} + S = nk \ln \frac{V}{n} + \frac{n}{2} k \ln T + nk \ln \frac{(2\pi m k)^{\frac{1}{2}}}{h}$$ (kn fehlt, weil jedes Atom in $\frac{l}{n}$)