A CHAOTIC ATTRACTOR FROM CHUA'S CIRCUIT

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Abstract A chaotic attractor has been observed with an extremely simple autonomous circuit. It is third order and has only one nonlinear element ; a 3-segment piecewise-linear resistor. The attractor appears to have interesting structures that are different from Lorenz's and Rössler's.

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Our purpose here is to report that a chaotic attractor has been observed with an extremely simple autonomous circuit. It is third order and has only one nonlinear element ; a 3-segment piecewise-linear resistor. It is a simplified version of a circuit suggested by Leon Chua of Berkeley, who was visiting Waseda, October 1983 — January 1984.

Consider the circuit of Figure 1(a) where the constitutive relation of the nonlinear resistor is given by Figure 1 (b). The dynamics is described by

$$\begin{cases} C_1 \dfrac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - g(v_{C_1}) \\ C_2 \dfrac{dv_{C_2}}{dt} = G(v_{C_1} - v_{C_2}) + i_L \\ L \dfrac{di_L}{dt} = -v_{C_2} \end{cases} \tag{1}$$

where v_{C_1}, v_{C_2} and i_L denote voltage across C_1, voltage across C_2 and current through L, respectively. Figure 2 shows the chaotic attractor observed by solving (1) with

$$1/C_1 = 10, \quad 1/C_2 = 0.5, \quad 1/L = 7, \quad G = 0.7. \tag{2}$$

Figures 2 (a), 2 (b) and 2 (c) are the projections of the attractor onto the (i_L, v_{C_1})-plane, (i_L, v_{C_2})-plane and (v_{C_2}, v_{C_1})-plane, respectively. (The fourth order Runge- Kutta was used with step size 0.02). It is interesting to observe that a hyperbolic periodic orbit (not a stable limit cycle) is present outside the attractor. (Newton iteration was used)

If the reader feels uncomfortable with the function g of Figure 1(b) in that it is not eventually passive and there are initial conditions with which (1) diverges, he can simply replace Figure 1(b) with Figure 3. If B_p=14, it has no effect on the attractor and on the hyperbolic periodic orbit, because $|v_{C_1}(t)| < 14$ for all $t \geq 0$ on the attractor and on the hyperbolic periodic orbit. The only difference is the appearance of a large stable limit cycle as shown in Figure 4, where (1) does not diverge with any initial condition ($B_p = 14$, $m_o = 5$). Note that the existence of the hyperbolic periodic orbit is quite natural, since the chaotic attractor's domain of attraction is bounded, there must be an object which separates the domain of attraction from other initial conditions.

The attractor persists in a strong manner ; the shape does not seem to change qualitatively with fairly large variations of parameters. It appears to have interesting structures that are different from Lorenz's [1] and Rössler's [2]. We note that (1) is, in a sense, simpler than the Lorenz and Rössler equations, in that the latter have product of two variables while g of Figure 1(b) is of a single variable and 3-segment piecewise-linear. Because of this simplicity, one can do several interesting analyses that are impossible for Lorenz and Rössler equations.

Many more interesting structures have been observed with parameter values different from (2). Details including Lyapunov exponents, 1-dimensional maps, bifurcations and circuit realizations will be reported in later papers.

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REFERENCES

[1] E. Lorenz, "Deterministic non-periodic flows", J. Atmosphere Science, vol. 20, pp 130-141, 1963.

[2] O. Rössler, "An equation for continuous chaos", Phys. Lett., vol. 57A, pp 397-398, 1976.

FIGURE CAPTIONS

Figure 1. A third order autonomous circuit with chaotic attractor ; (a) circuitry, (b) constitutive relation of the nonlinear resistor.

Figure 2. The chaotic attractor and hyperbolic periodic orbit; (a) projection onto the (i_L, v_{C_1})-plane, (b) projection onto the (i_L, v_{C_2})-plane, (c) projection onto the (v_{C_2}, v_{C_1})-plane.

Figure 3. A modified constitutive relation of the nonlinear resistor.

Figure 4. The trajectories with modified resistor constitutive relation; (a) projection onto the (i_L, v_{C_1})-plane, (b) projection onto the (i_L, v_{C_2})-plane, (c) projection onto the (v_{C_2}, v_{C_1})-plane.

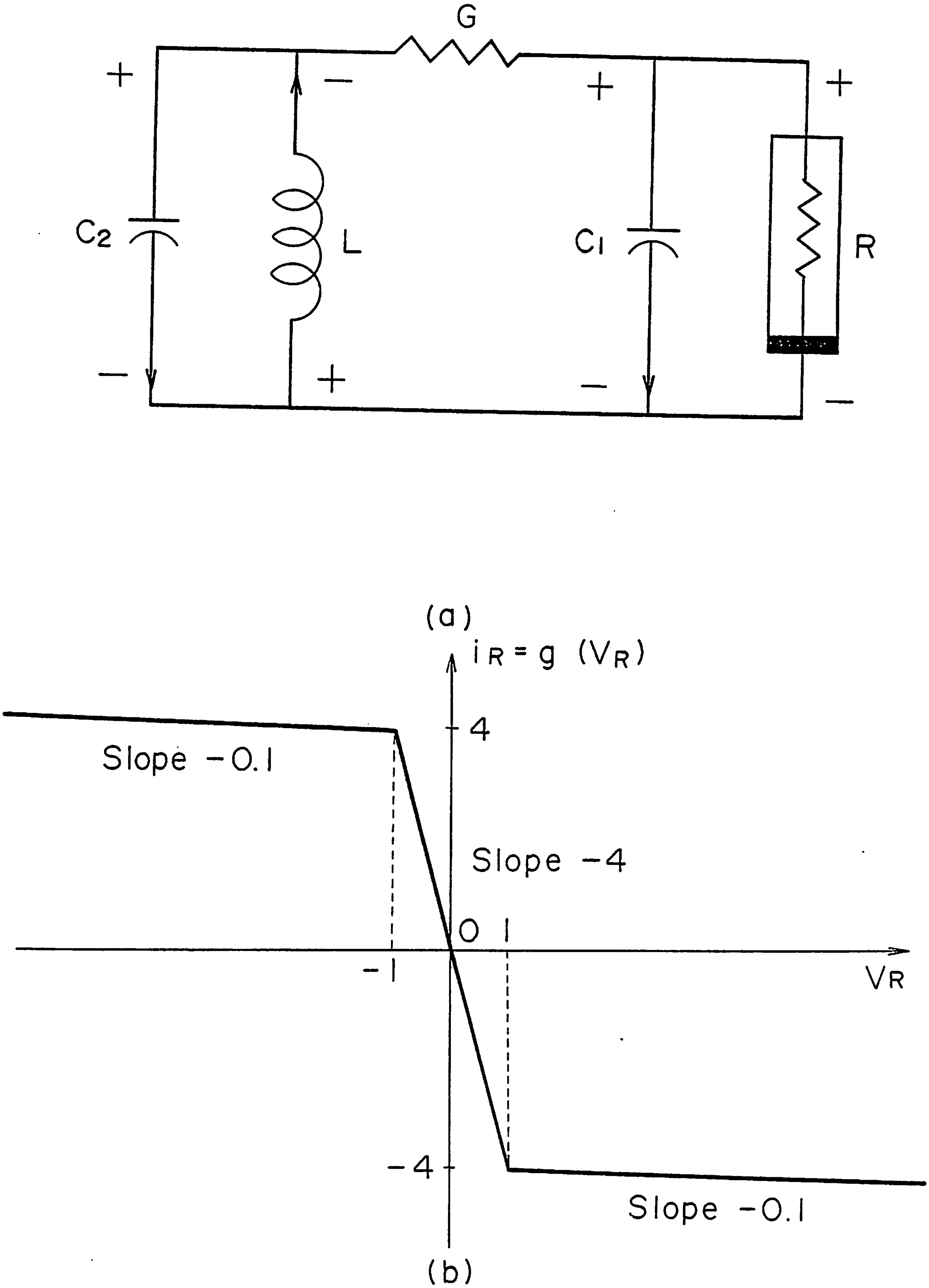


Fig. 1

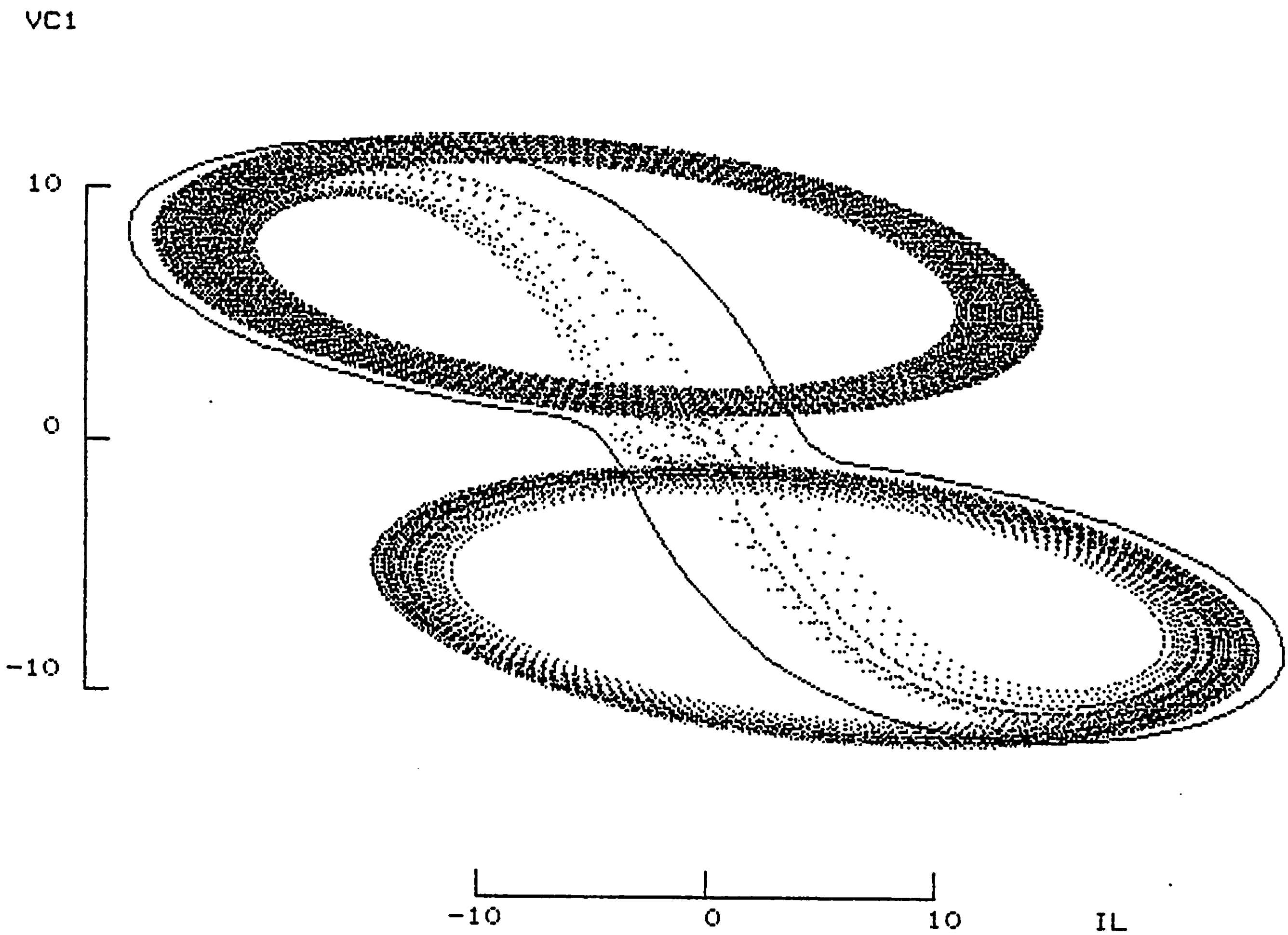


Fig. 2(a)

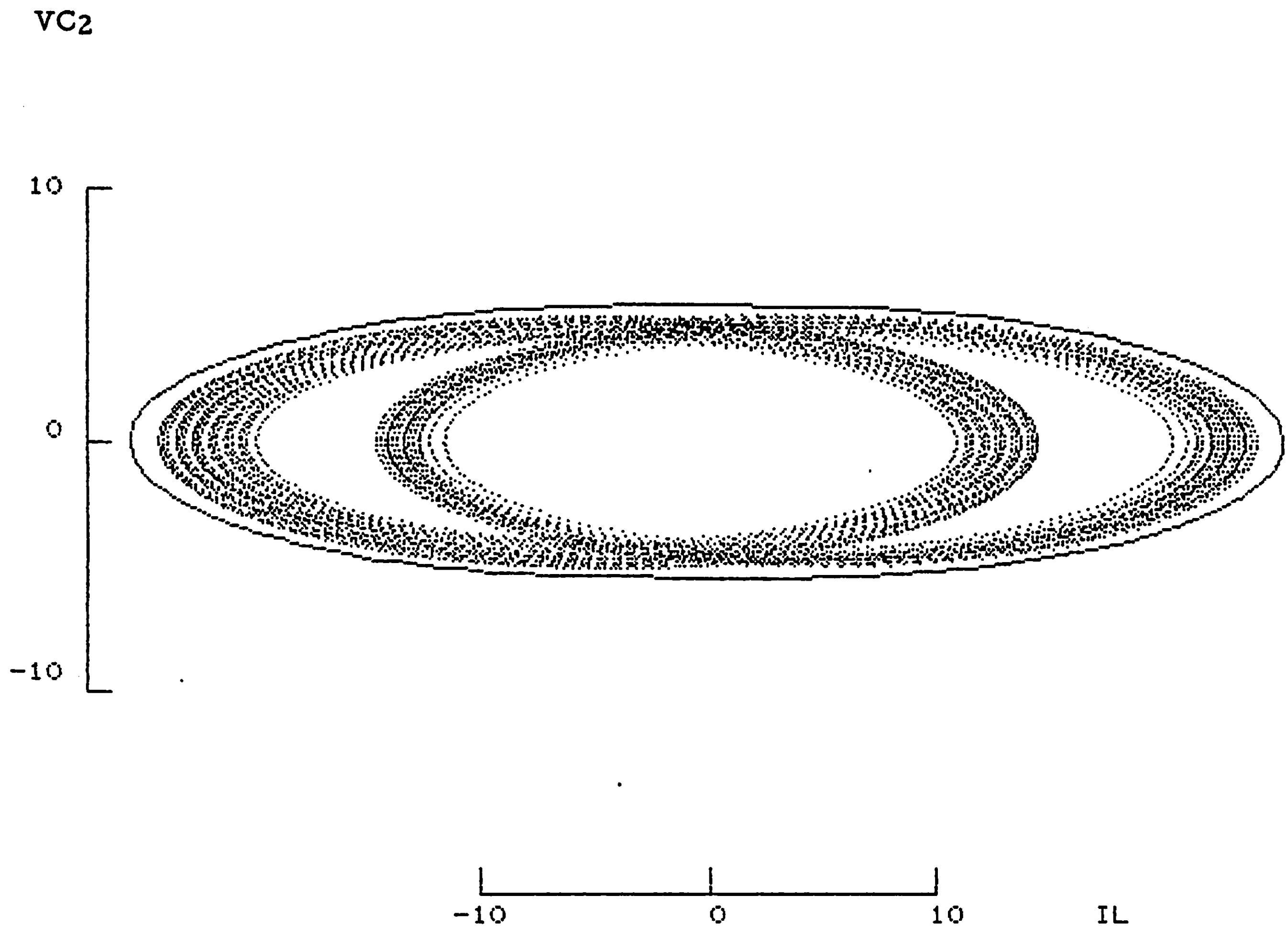


Fig. 2(b)

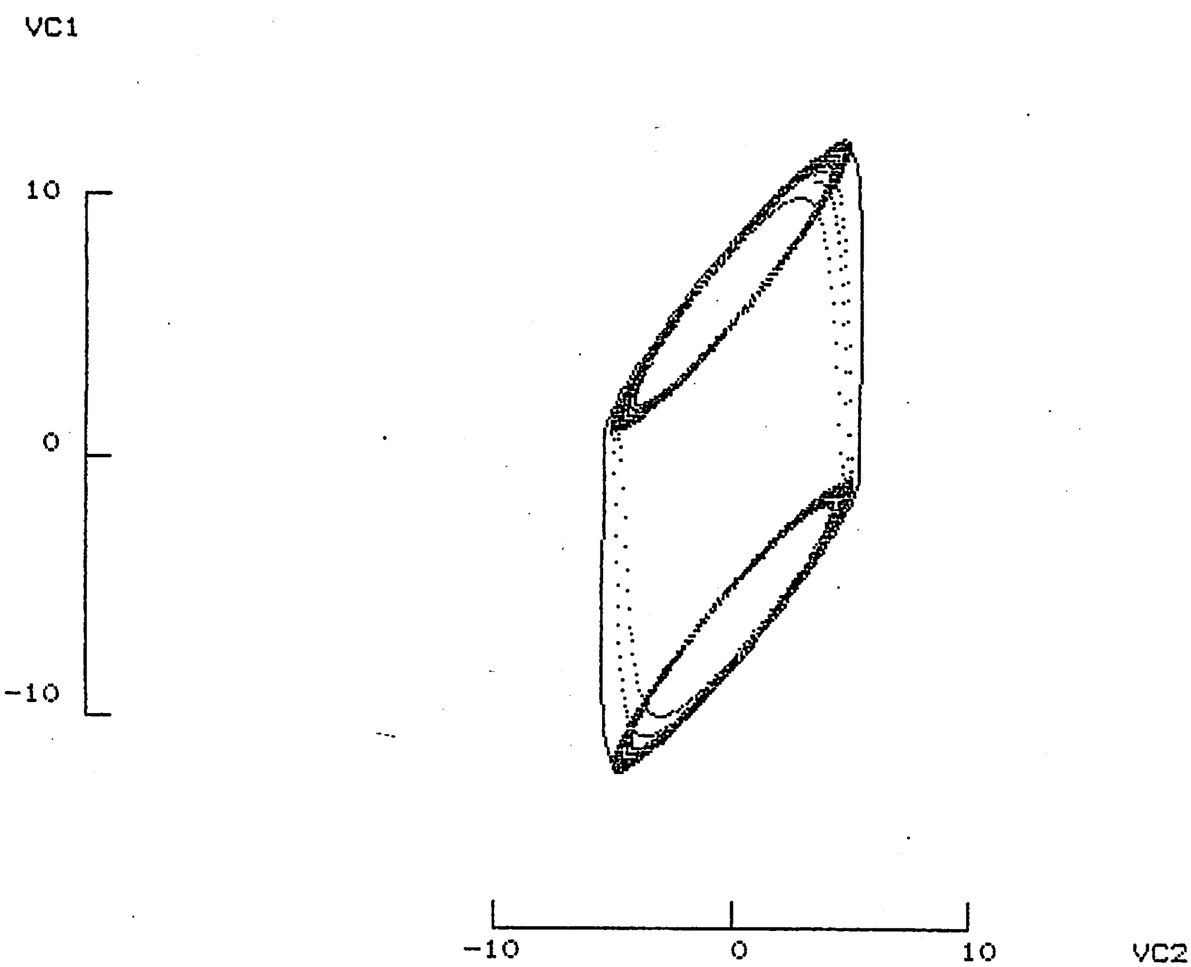
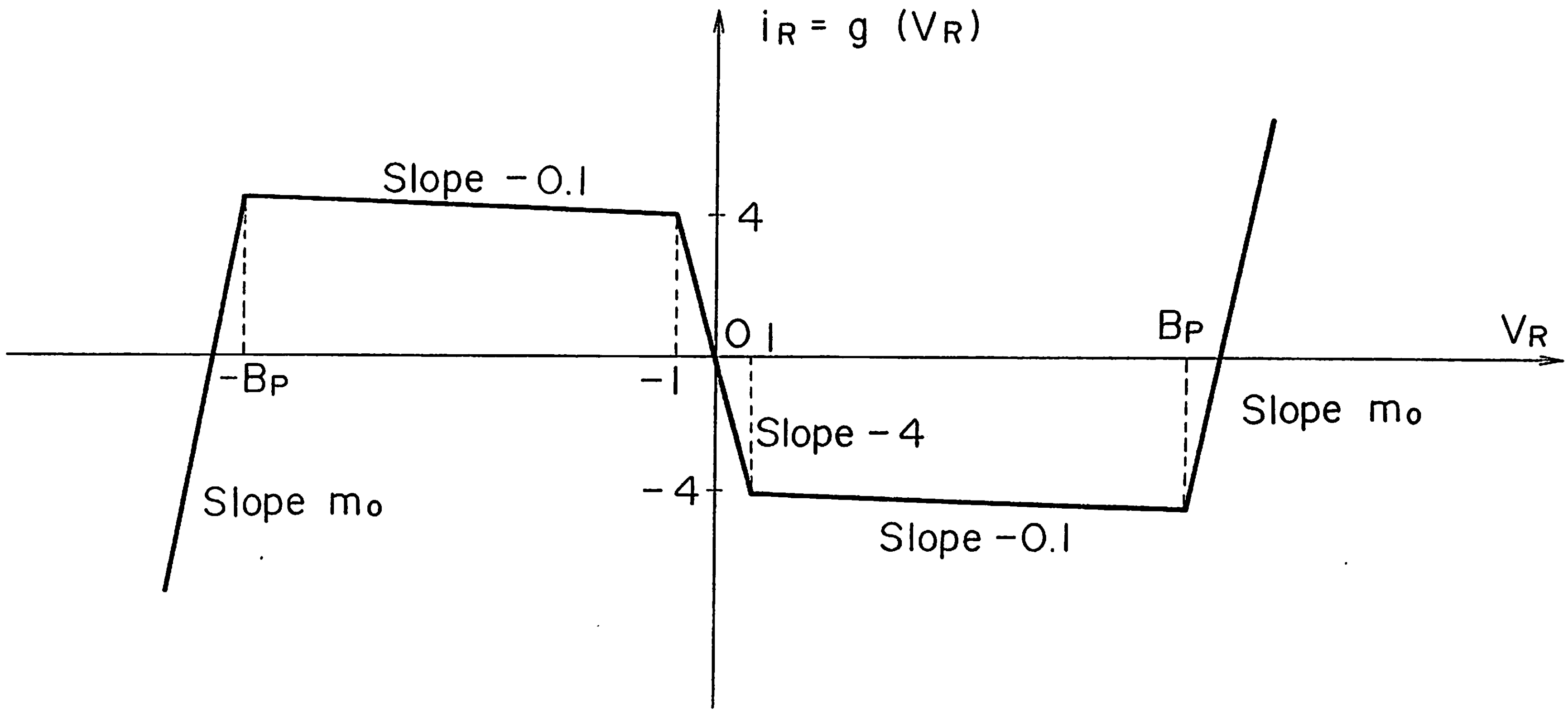


Fig. 2(c)



$i_R = g(V_R)$
Slope −0.1
4
0
1
B_P
V_R
$-B_P$
−1
Slope m_o
Slope −4
−4
Slope m_o
Slope −0.1

Fig. 3

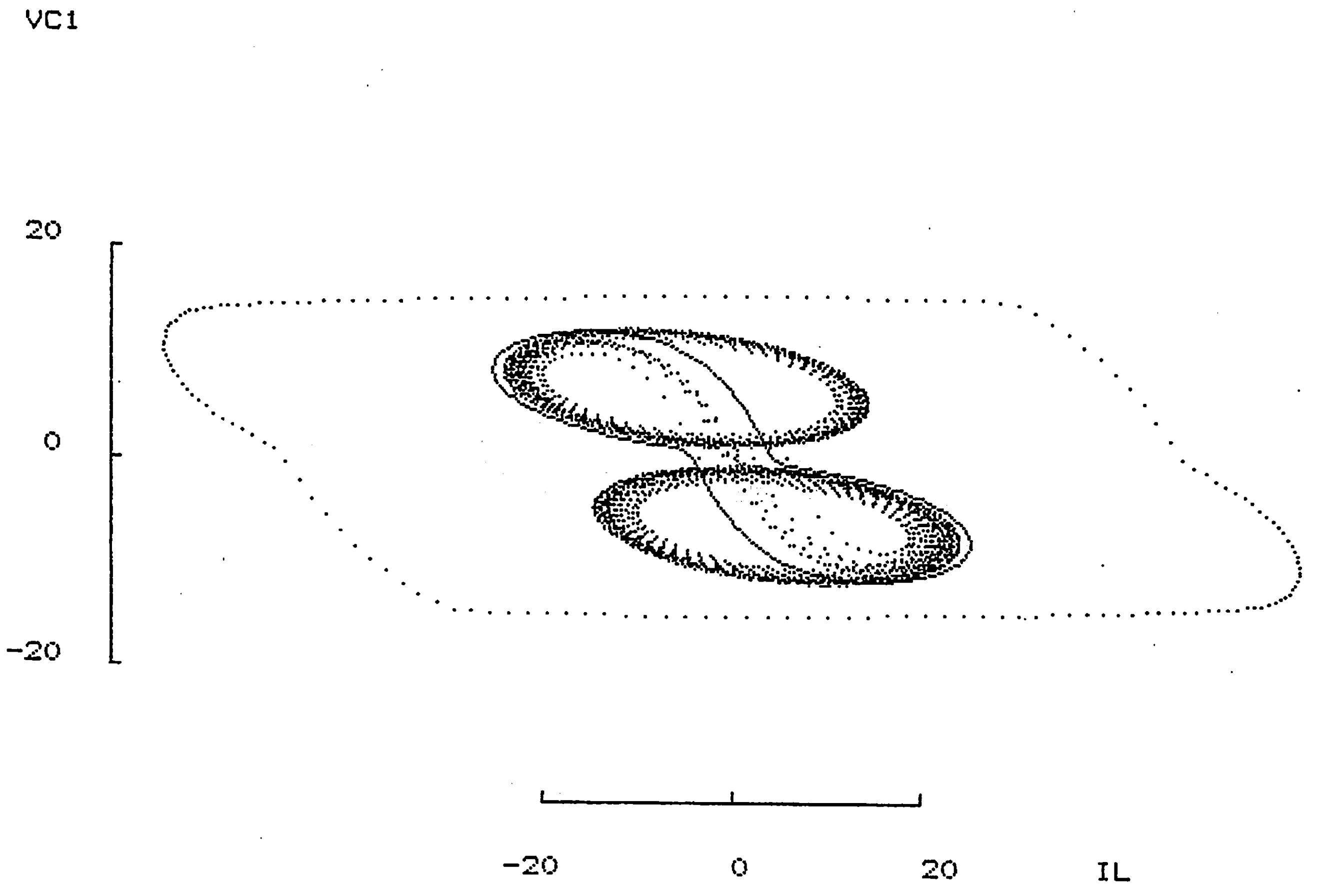


Fig. 4(a)

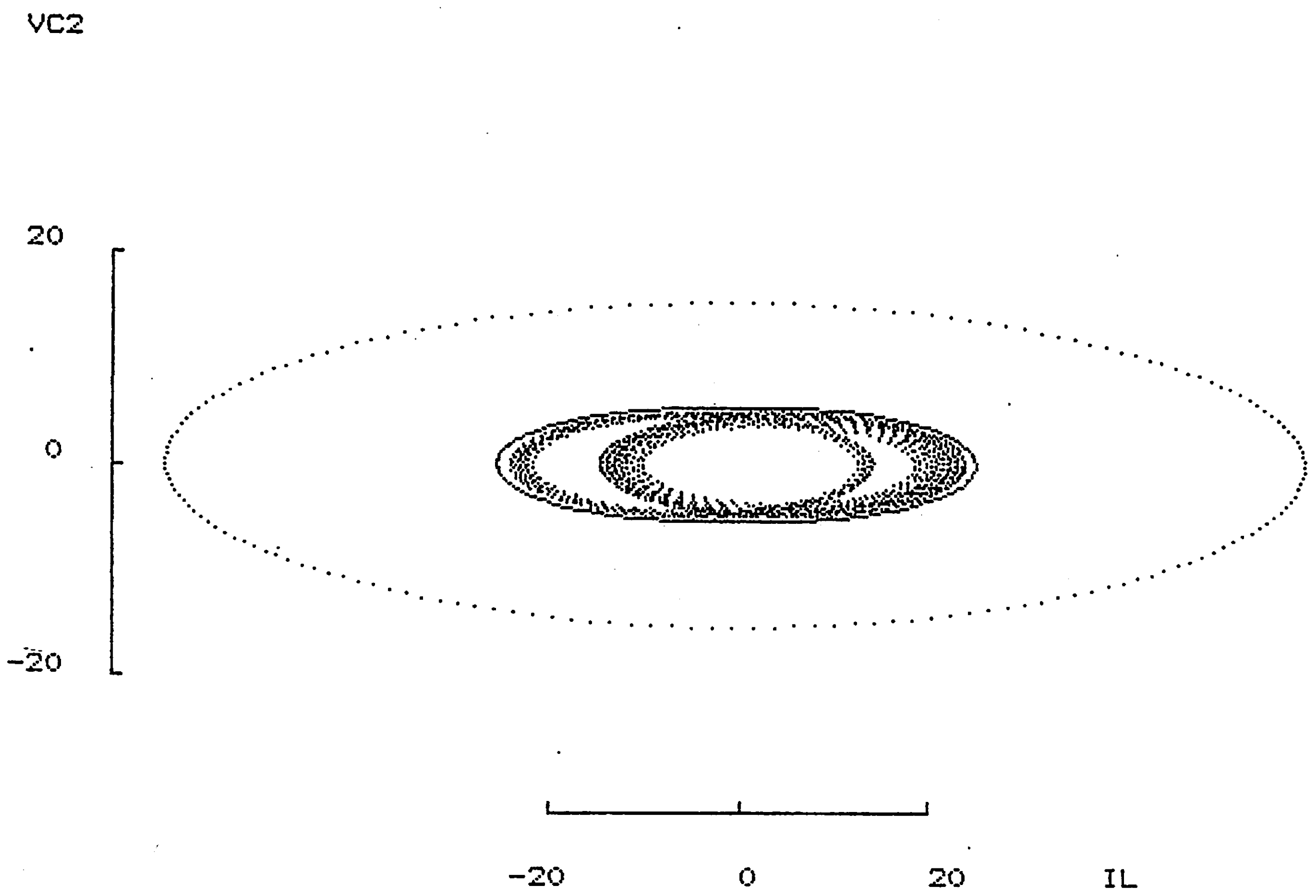


Fig. 4(b)

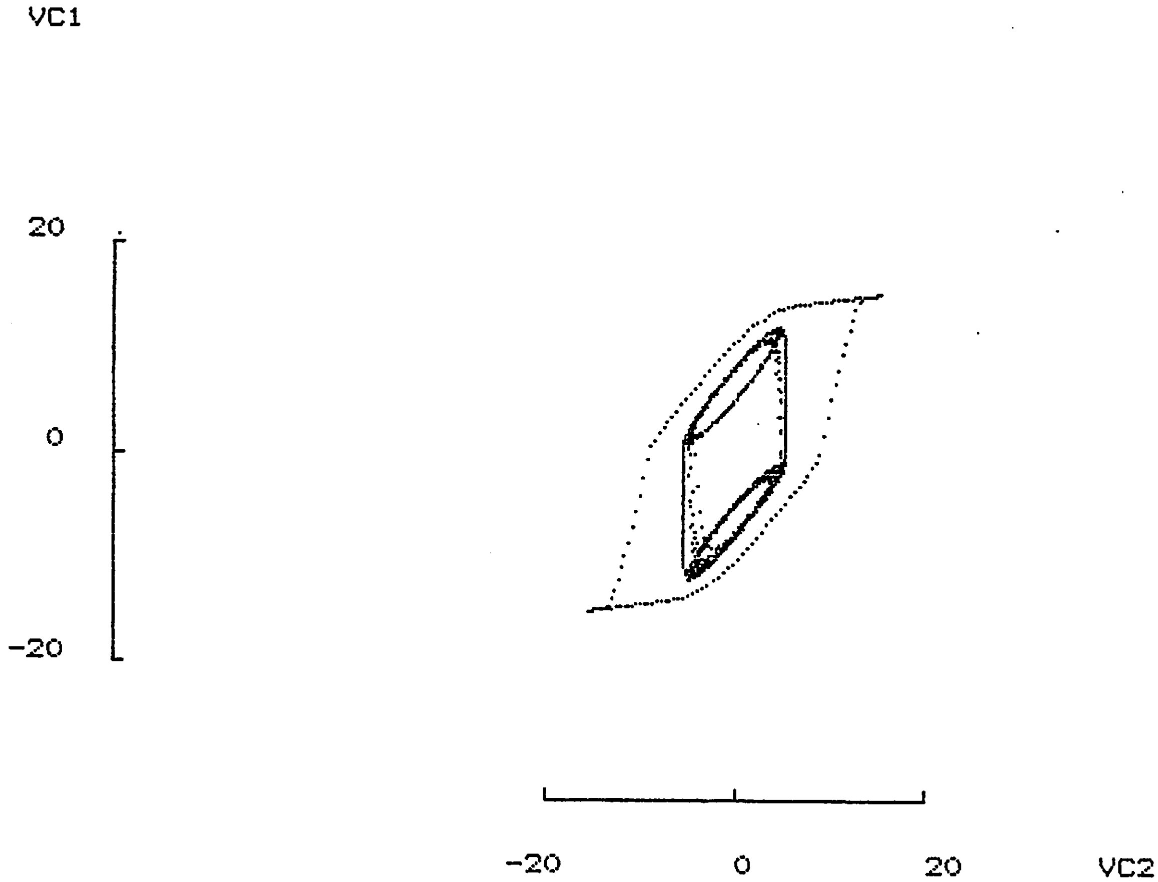


Fig. 4(c)