DISCRETE TIME ADAPTIVE CONTROL UTILIZING PRIOR INFORMATION

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Discrete Time Adaptive Control Utilizing Prior Information

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ABSTRACT

We present schemes for discrete time adaptive control of a linear time invariant systems, which is partially known, along with an analysis of their convergence properties.

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1 Problem Statement

A great deal of effort has been devoted to establishing conditions for the robust stability of adaptive control algorithms. There are two sets of approaches to this issue: In the first approach, an internal signal in the adaptive loop is made persistently exciting to guarantee exponential stability of the scheme. Robustness of the scheme follows as a consequence of the robustness of exponential stability. In the second approach, the adaptive algorithm is modified, using for instance, a deadzone or forgetting factor in the adaptation law to prevent the algorithm from responding to spurious signals such as those arising from noise and unmodeled dynamics. Both approaches model the plant to be controlled as being completely unknown. In this paper, we discuss the control of system which are partially known (in a sense that is made explicit shortly). It seems intuitively that the control algorithm could be robust if this prior information could be incorporated into the adaptive controller [2,3,4]. With this as motivation, we present two adaptive control algorithms for 'partially known ' systems.

The system to be controlled is a single input-single output time-invariant system of the form

$$\frac{y}{u} = \frac{n_0 + \sum_{i=1}^{l} \alpha_i n_i}{d_0 - \sum_{j=1}^{k} \beta_j d_j} \tag{1.1}$$

where α_i , β_j 's are unknown constants, n_i , d_j 's are known polynomials in z^{-1} (discrete

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time) or s (continuous time). The model (1.1) is general enough for several kinds of 'partially known' systems -we give three examples:

(1) Network functions of RLC circuits with some elements unknown. Consider the circuit of Fig.1 with the resistor R unknown (drawn as a two port to exhibit the unknown resistance).

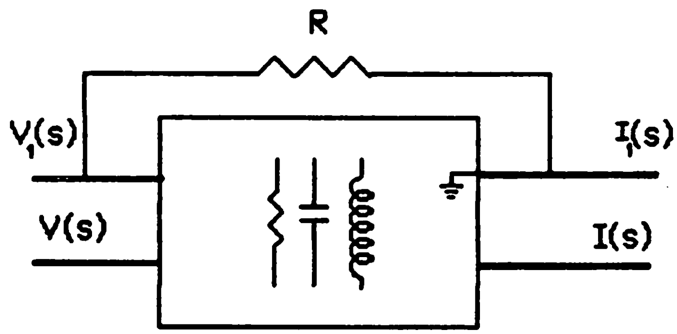

Fig. 1

If the short circuit admittance matrix of the two port in Fig.1 is

$$\begin{bmatrix} I \\ I_1 \end{bmatrix} = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix} \begin{bmatrix} V \\ V_1 \end{bmatrix} \tag{1.2}$$

A simple calculation yields the admittance function

$$\frac{I(s)}{V(s)} = \frac{y_{11}+R(y_{11}y_{22}-y_{12}y_{21})}{1+Ry_{22}}$$

which is of the form (1.1). Circuits with more than one unknown element can be drawn as multiports to show that the admittance function is of the form (1.1)

(2) Interconnection of several systems with unknown interconnection gains. Consider the simple discrete time configuration of Fig.2 with the polynomials n and d known.

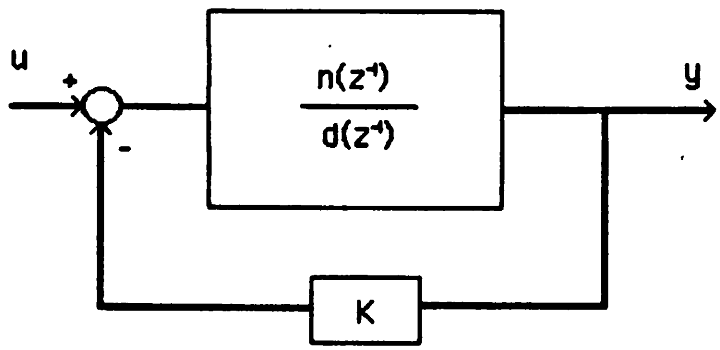

Fig. 2

The closed loop transfer function is of the form (1.1) $\frac{n}{d+kn}$.

(3) Systems with some known poles and zeros. Consider the system of Fig.3, with unknown plant but known actuator and sensor dynamics

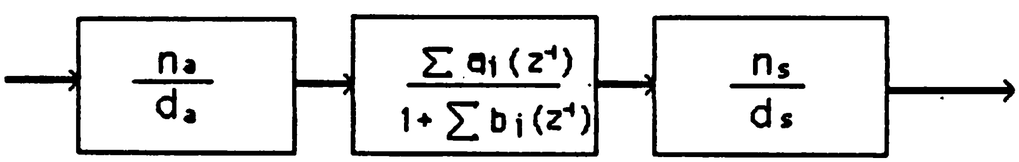

Fig. 3

The overall transfer function is written as

$$\frac{\sum a_i n_a n_s z^{-i}}{d_a d_s + \sum b_j d_a d_s z^{-j}}$$

which is of the form (1.1) since $n_a n_s z^{-i}$, $d_a d_s z^{-j}$ known.

In this paper we will focus attention on discrete time systems. Our methods are an extension of those proposed by Goodwin et al in [6] and the method of proof is identical to that in [6]. The novelty of our paper is the set up in which the methods are applied. The layout of our paper is as follows: In section 2, we discuss parameter identification for systems of the form (1.1). Section 3 contains the new adaptive control laws with the convergence analysis and section 4 a simple simulation example comparing our law with that of [6] which does not use prior information.

2 Parameter Identification

Parameter identification is the first step in controlling an unknown plant. Consider a discrete time system of the form (1.1), that is

$$\frac{y(t)}{u(t)}=\frac{n_0(z^{-1})+\sum_{i=1}^{l}\alpha_i n_i(z^{-1})}{(1+d_0(z^{-1}))-\sum_{j=1}^{k}\beta_j d_j(z^{-1})}=\frac{n(z^{-1})}{d(z^{-1})} \tag{2.1}$$

where α_i, β_j's are unknown parameters, d_j, n_i are known *nth* and *mth* order polynomials in the unit delay operator z^{-1}.

$$d_j(z^{-1})=d_{j1}z^{-1}+...+d_{jn}z^{-n} \qquad j=0,1,...,k \tag{2.2}$$

$$n_i(z^{-1})=n_{i1}z^{-1}+...+n_{im}z^{-m} \qquad i=0,1,...,l \tag{2.3}$$

The identification problem is to identify β_j, α_i from input-output measurements of the system.

Definition 2.1 A system of the form (2.1) with unknown parameters α_i, β_j is said to be **identifiable** if and only if there exists at least one input u(t) such that the unknown parameters can be uniquely determined based on input-output measurements.

The following result is from Bai and Sastry [2]:

Theorem 2.1 Assume that $n(z^{-1})$ and $d(z^{-1})$ in equation (2.1) are coprime. Then, the necessary and sufficient condition for the system (2.1) to be identifiable is that the following matrix D is full column rank.

$$D=\begin{bmatrix} d_{11} & d_{k1} & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ d_{1n} & d_{kn} & 0 & 0 \\ 0 & 0 & n_{11} & n_{l1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & n_{1m} & n_{lm} \end{bmatrix} \tag{2.4}$$

We summarize the methods of [2]. Rearranging equation (2.1), we get

$$(1+d_0(z^{-1}))y(t)-n_0(z^{-1})u(t)$$

$$=\sum_1^k \beta_j d_j(z^{-1})y(t)+\sum_1^l \alpha_i n_i(z^{-1})u(t) \tag{2.5}$$

Define the following signal vectors

$$z_0(t)=(1+d_0(z^{-1}))y(t)-n_0(z^{-1})u(t) \tag{2.6}$$

$$z^T(t-1)=(d_1(z^{-1})y(t),\ldots,d_k(z^{-1})y(t),n_1(z^{-1})u(t),\ldots,n_l(z^{-1})u(t)) \tag{2.7}$$

$$\theta_1^T=(-d_{01},\ldots,-d_{0n},n_{01},\ldots,n_{0m}) \tag{2.8}$$

$$\theta_0^T=(\beta_1,\ldots,\beta_k,\alpha_1,\ldots,\alpha_l) \tag{2.9}$$

$$\phi^T(t-1)=(y(t-1),\ldots,y(t-n),u(t-1),\ldots,u(t-m)) \tag{2.10}$$

Then it follows that

$$z_0(t)=\theta_0^T z(t-1)=\theta_0^T D^T \phi(t) \tag{2.11}$$

Let $\hat{\theta}(t)$ denote the parameter estimate at time t. Then since $z_0(t)$ and $z(t-1)$ are obtainable from the input and output , we may construct the equation error

$$e(t)=\hat{\theta}(t-1)z(t-1)-z_0(t) \tag{2.12}$$

with $\tilde{\theta}(t)=\hat{\theta}(t)-\theta_0$ denoting the parameter error, we see that

$$e(t)=\tilde{\theta}^T(t-1)z(t-1) \tag{2.13}$$

Equation (2.13) is linear in the parameter error, so that any one of a number of standard techniques for parameter update (see [7]) may be used. Two of them which we are going to use in the next section are:

The Projection Type Algorithm

The update law

$$\hat{\theta}(t)=\hat{\theta}(t-1)+\frac{z(t-1)}{1+z^T(t-1)z(t-1)}(z_0(t)-\hat{\theta}(t-1)z(t-1)) \tag{2.14}$$

is referred to as projection type law. It is well known [1,7] that this algorithm has following properties.

$$||\hat{\theta}(t)-\theta_0|| \leqslant ||\hat{\theta}(t-1)-\theta_0|| \leqslant ||\hat{\theta}(0)-\theta_0|| \quad \text{for } any\ t \geqslant 1 \tag{2.15}$$

$$\lim_{t\to\infty}\frac{\tilde{\theta}^T(t-1)z(t-1)^2}{1+z^T(t-1)z(t-1)}=0 \tag{2.16}$$

The Least Squares Type Algorithm (with Covariance Resetting)

The least squares type algorithm with covariance resetting is given by

$$\hat{\theta}(t)=\hat{\theta}(t-1)+\frac{P(t-2)z(t-1)}{1+z^T(t-1)P(t-2)z(t-1)}(z_0(t)-z^T(t-1)\hat{\theta}(t-1)) \tag{2.17a}$$

$$\mathrm{P(t-1)} = \begin{cases} P(t-2)-\dfrac{P(t-2)z(t-1)z^T(t-1)P(t-2)}{1+z^T(t-1)P(t-2)z(t-1)} & \text{if } t \neq 0,t_1,t_2,\ldots \\ k_i I & \text{if } t=0,t_1,t_2,\ldots \end{cases} \tag{2.17b}$$

where $0<k_{\min}\leqslant k_i \leqslant K_{\max}<\infty$. In (2.17b), covariance resetting occurs at $\{0,t_1,t_2,\ldots\}$.

It is not difficult to show that both algorithms have exponential convergence rates when z(t) is persistently exciting, i.e. there is some $N, \alpha_1, \alpha_2 > 0$ such that

$$\alpha_1 I \geqslant \sum_{k+1}^{k+N} z(t)z^T(t) \geqslant \alpha_2 I$$

3 Adaptive Control of 'Partially Known' Systems

Consider the system of the form (2.1). An adaptive control law is to be designed to stabilize this system and to cause the output y(t) to track a given reference sequence $y^*(t)$ i.e. we require y(t) and u(t) to be bounded and

$$\lim_{t\to\infty}(y(t)-y^*(t))=0$$

The following assumptions will be made about the system (2.1)

(1) $n_{01}+\alpha_1 n_{11}+\ldots+\alpha_{l1}n_{l1}\neq 0$

This implies that the pure delay in the transfer function (2.1) is known and equal to 1. This is for simplicity alone in our analysis, the extension to the case where pure delay is greater than 1 (but known) follows readily.

(2) $n(z^{-1})$has all zeros strictly inside the closed unit disk i.e. the system is inverse stable.

(3) $y^*(t)$ is known a priori and bounded.

Control Algorithm Using Projection Type Identification Law

From equation (2.5),(2.8) and (2.11), we have

$$y(t+1)=\theta_1^T\phi(t)+\theta_0^T z(t)$$

$$=\theta_1^T\phi(t)+\theta_0^T D^T\phi(t) \tag{3.1}$$

We choose the projection type estimation law (2.14) and a control law specified implicitly by

$$y^*(t+1)=\theta_1^T\phi(t)+\hat{\theta}(t)^T D^T\phi(t) \tag{3.2}$$

(A minor modification is necessary to ensure that the coefficient of u(t) in (3.2) is nonzero. This can be achieved in the same way as in [6] and does not affect the current analysis.) Then, we have

Theorem 3.1 (Convergence Theorem)

Subject to assumptions 1), 2) and 3), consider the control law (3.2), together with the projection type estimation law (2.14), applied to the system (2.1). Then, y(t) and u(t) are bounded and

$$\lim_{t\to\infty}(y(t)-y^*(t))=0 \tag{3.3}$$

Proof: Define the output error by

$$e_y(t)=y(t)-y^*(t) \tag{3.4}$$

It follows from (3.1) and (3.2) that

$$e_y(t)=-\tilde{\theta}^T(t-1)z(t-1) \tag{3.5}$$

Now using equation (2.16), we have

$$\lim_{t\to\infty}\frac{e^2_y(t)}{1+z^T(t-1)z(t-1)}=0 \tag{3.6}$$

Note that

$$\frac{e^2_y(t)}{1+z^T(t-1)z(t-1)}\geq\frac{e^2_y(t)}{1+\sigma_{\max}(DD^T)\phi^T(t-1)\phi(t-1)} \tag{3.7}$$

By assumptions 2) and 3), we have as in [6] that

$$||\phi(t-1)|| \leq c_1 + c_2 \max_{1 \leq \tau \leq t} |e_y(\tau)| \qquad (3.8)$$

for some $0 \leq c_1 < \infty$, $0 < c_2 < \infty$. The conclusion now follows from equation (3.7) and (3.8) using the key technical lemma in [6] and by noting that boundedness of $||\phi(t)||$ ensures boundedness of y(t) and u(t).

Control Algorithm Using Least Squares Type Identification Law (with Covariance Resetting)

If the least squares type estimation law (2.17) is used , then we get the same result.

Theorem 3.2 Subject to assumptions 1), 2) and 3), consider the control law (3.2), together with the least squares type estimation law (2.17), applied to the system (2.1). Then y(t) and u(t) are bounded and

$$\lim_{t \to \infty} (y(t) - y^*(t)) = 0$$

Proof: The proof proceeds by an argument similar to that in [5]. Define

$$e_y(t) = y(t) - y^*(t)$$

Then from [5], we have

$$\lim_{t \to \infty} \frac{e^2{}_y(t)}{1 + k_{\max} z^T(t-1) z(t-1)} = 0 \qquad (3.10)$$

The remainder of the proof is same as that of theorem 3.1.

We have shown the global stability of two adaptive control algorithms. Note that nothing has been said about the convergence rate of the output and the parameter convergence. However, if $n(z^{-1})$ and $d(z^{-1})$ in (2.1) are coprime and the D matrix in (2.4) has full column rank, then the persistency of excitation of z(t) follows from the sufficient richness of input u(t) (i.e. u(t) has sufficient spectral content, see [1]). This implies that the control algorithm, with either projection type or least squares type with covariance resetting parameter update, has exponential convergence rate both for the output error and parameter error.

4 Simulation

To illustrate the methods of last section, consider the following example

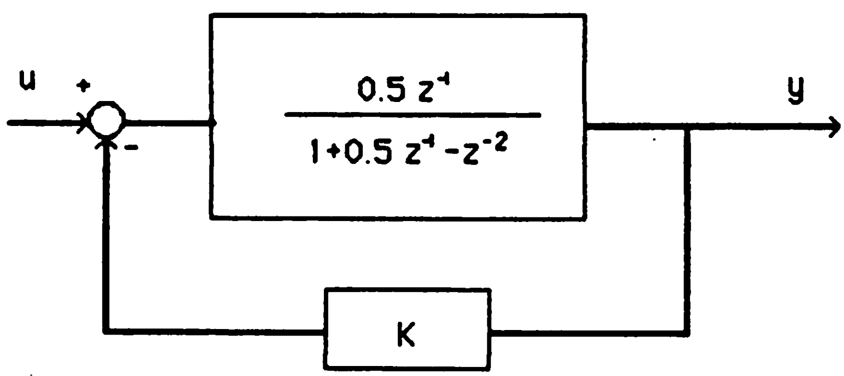

Fig. 4

where k is unknown. The closed loop transfer function is

$$\frac{0.5z^{-1}}{(1+0.5z^{-1}-z^{-2})+k\,0.5z^{-1}}.$$

Fig.5 shows the plant output under the adaptive control algorithm of (3.2) with projection type update law and the plant output under the adaptive control algorithm without using prior information respectively. (for the simulation, $k=1$, and $y^*(t)\equiv 1$). The algorithm using prior information has faster convergence rate and better transient performance.

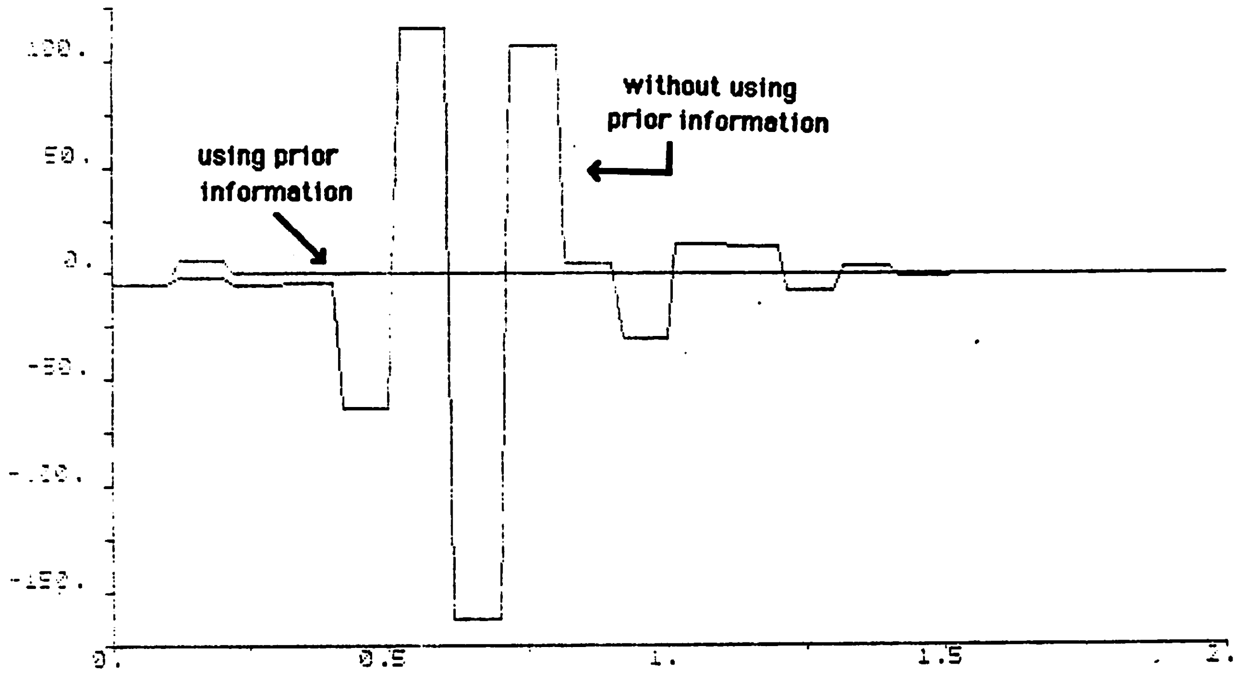

Fig.5

5 Concluding Remarks

In this paper we have presented two algorithms for discrete time adaptive control which utilize prior information about the plant, including some known poles and zeros. If the plant is completely unknown, the algorithms are identical to those proposed by Goodwin et al in [6]. However the algorithm presented here have better transient performance and faster convergence rate when the system is partially known. In future work we will show the added robustness margins obtainable from our scheme.

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