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Kinematics and Control of Multifingered Hands with Rolling Contact†

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ABSTRACT

In this paper, we derive the kinematics of rolling contact for two surfaces of arbitrary shape rolling on each other. Applying these kinematic equations to two planar multifingered hands manipulating some object of arbitrary shape, a scheme is presented for the control of these hands, which is in fact a generalization of the computed torque method of control of robot manipulators. In implementing the control, we require that all applied forces lie within the friction cone of the object so that sliding does not occur. We illustrate the theory with graphics simulations of the control law applied to the system dynamics for two examples.

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1. Introduction

An important feature of multifingered hands is their ability to perform fine motion manipulation, especially when the manipulator operates in a crowded environment. Most current control schemes for multi-fingered hands , for example [4], assume the contact type between the fingertip and the object to be of the *point contact with friction* type.

In this work, we consider the manipulation of objects of arbitrary shape by multi-fingered hands, where the contact between the object and the fingers are rolling contacts, i.e. the fingertip rolls without slipping on the surface of the object. The kinematics, prehensility, dynamics, and control of these systems are developed in this paper.

Previous work on rolling contacts, to our knowledge, can be found in Kerr [1], Montana [2], and Cai and Roth [3]. Kerr discusses how to compute the movement of the fingers in order to produce a given displacement of the object. Kinematic equations are derived from the constraint that the fingertip and object velocities are equal at the point of contact. Control of such a hand is not considered. Montana [2] studies the kinematics of contact from a geometric point of view. He does not, however, study the effects of the kinematics of a finger attached to the fingertip. Cai and Roth [3] study the roll-slide motions between two curves under planar motion. The kinematic equations for the contact point evolution are derived. To our knowledge, there has been no previous work on the control of multifingered hands with rolling contacts.

A brief outline of the paper is as follows: In Section 2 we derive the kinematics of rolling in $\mathbb{R}^3$ using velocity constraints and normal constraints between the surfaces; the developement closely parallels that of [1]. In Section 3 we derive relationships between the joint torques and velocities of the fingers and the net force and velocity of the body being manipulated. Section 4 gives the control scheme along with a proof of its convergence. Section 5 presents simulation results for the application of our control law to two kinematically different planar hands.

2. The Kinematics of Rolling

When one surface rolls on top of another, the trajectory of the contact point on each surface depends in an essential way on the geometry of the surfaces. In this section, we derive the differential equations that specify the evolution of the point of contact. Since our eventual goal is to apply the theory to the manipulation of objects by multi-jointed fingers with rolling contacts, we will refer to one of the bodies in contact as the *object* and the other as the *finger*.

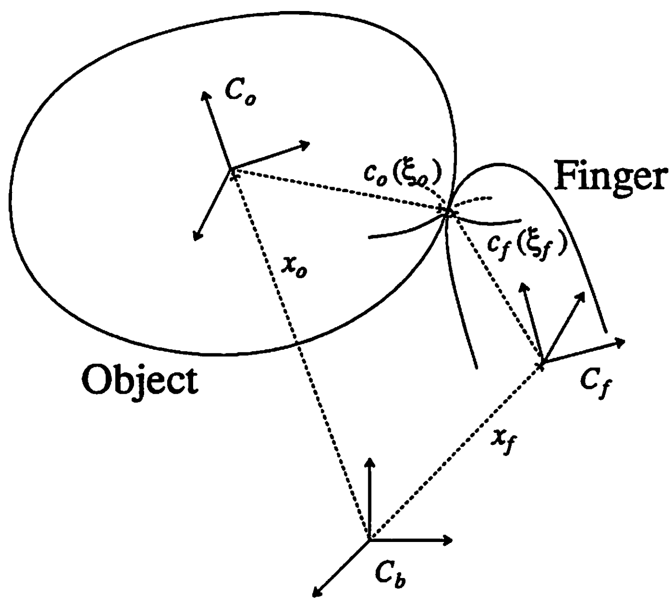

Figure 2.1

Consider a finger in contact with an object as shown in Figure 2.1. Denote by C_o a coordinate frame attached to the center of mass of the object, and by C_f one attached to the finger. Parameterize the object and the fingertip surface (locally) in C_o and C_f, as $c_o(\xi_o)$ and $c_f(\xi_f)$ respectively. Note that $\xi_o, \xi_f \in \mathbb{R}^2$ if the surfaces in question are in $\mathbb{R}^3$. We will also be interested in the manipulation of objects in the plane ($\mathbb{R}^2$), in which case $\xi_o, \xi_f \in \mathbb{R}$. Further, let C_b be an inertial base frame. Define $x_o, x_f \in \mathbb{R}^3$ to be the positions of the origins of C_o, C_f in the base frame, and $R_o, R_f \in SO(3)$ (the group of 3×3 orthogonal matrices with positive determinant) to be the rotation matrices giving the orientations of C_o, C_f in the base frame C_b, respectively. It follows from elementary considerations that a point on the object with coordinate $c_o(\xi_o)$ in the object frame has (base) coordinates given by

$$x_o + R_o c_o(\xi_o). \tag{2.1}$$

Frames C_o and C_f both move relative to the base frame C_b so that x_o, x_f, R_o, and R_f are all functions of time. The velocity of the origin of frame C_o has a translational component $v_o \in \mathbb{R}^3$ given by

$$v_o(t) = \dot{x}_o(t) \tag{2.2}$$

and a rotational component $\omega_o = (\omega_{o1}, \omega_{o2}, \omega_{o3})^T \in \mathbb{R}^3$ such that

$$\dot{R}_o = \omega_o \times R_o = \begin{bmatrix} 0 & -\omega_{o3} & \omega_{o2} \\ \omega_{o3} & 0 & -\omega_{o1} \\ -\omega_{o2} & \omega_{o1} & 0 \end{bmatrix} R_o . \tag{2.3}$$

Note that the matrix $\dot{R}_o R_o^{\mathrm{T}}$ is skew-symmetric because $R_o(t)$ is orthogonal. The matrix in (2.3) is referred to as $(\omega_o\times)$ since its action on a vector $y \in \mathbb{R}^3$ is precisely $\omega_o \times y$. Similarly, v_f and ω_f are the translational and rotational components of velocity for the fingertip.

The act of one surface rolling without slipping on top of another, yields three constraints on certain parameters relating the two bodies: the position of the contact, the velocity of the point of contact, and the surface normals at the point of contact. We use these constraints to determine the evolution of the contact point. First, since we assume that there is no slipping at the point of contact, the velocity of the point of contact *on* the object must equal the velocity of the point of contact *on* the fingertip (with reference to the base frame), i.e.,

$$v_o + \omega_o \times R_o c_o = v_f + \omega_f \times R_f c_f \,. \tag{2.4}$$

Equation (2.4) may be rewritten as

$$U_o \begin{bmatrix} v_o \\ \omega_o \end{bmatrix} = U_f \begin{bmatrix} v_f \\ \omega_f \end{bmatrix} \tag{2.5}$$

where

$$U_o \triangleq \begin{bmatrix} I & -(R_o c_o \times) \end{bmatrix} \,,\; U_f \triangleq \begin{bmatrix} I & -(R_f c_f \times) \end{bmatrix} .$$

Equation (2.5) represents the two different expressions for the velocity of the point of contact between the body and the fingertip.

Next, since the fingertip and object stay in contact, we may express the coordinates of the point of contact in two ways, i.e.,

$$x_o(t) + R_o(t) c_o(t) = x_f(t) + R_f(t) c_f(t) \,. \tag{2.6}$$

Differentiating (2.6) yields (using (2.3))

$$v_o(t) + \omega_o(t) \times R_o(t) c_o(t) + R_o(t)\dot{c}_o(t) = v_f(t) + \omega_f(t) \times R_f(t) c_f(t) + R_f(t)\dot{c}_f(t) \,. \tag{2.7}$$

Subtracting equation (2.4) from (2.7) yields

$$R_o(t)\dot{c}_o(t) = R_f(t)\dot{c}_f(t) \,. \tag{2.8}$$

To show the dependence of c_o and c_f on the surface parameters ξ_o and ξ_f respectively, equation (2.8) may be rewritten as

$$R_o(t)\frac{\partial c_o}{\partial \xi_o}\dot{\xi}_o = R_f(t)\frac{\partial c_f}{\partial \xi_f}\dot{\xi}_f \,. \tag{2.9}$$

In $\mathbb{R}^3$, equation (2.9) represents three equations in the four unknowns $\dot{\xi}_o, \dot{\xi}_f$. Equations (2.8) and (2.9) may be interpreted as a constraint on the tangent vectors of the contact curves on the fingertip and object.

Finally, since the two surfaces touch at the point of contact, they must have equal and opposite outward unit normal vectors. Thus, with $\hat{n}_o$ and $\hat{n}_f$ being the outward unit normal vectors to the surface of the object and fingertip, respectively, at the point of contact, we have

$$R_o \hat{n}_o = -R_f \hat{n}_f \,. \tag{2.10}$$

Differentiating (2.10) yields (again using (2.3))

$$\omega_o \times R_o \hat{n}_o + R_o \frac{\partial \hat{n}_o}{\partial \xi_o} \dot{\xi}_o + \omega_f \times R_f \hat{n}_f + R_f \frac{\partial \hat{n}_f}{\partial \xi_f} \dot{\xi}_f = 0 \,. \tag{2.11}$$

Equations (2.6) - (2.11) above correspond to equations (6.8) - (6.13) of [1].

Combining equations (2.9) and (2.11) yields

$$\begin{bmatrix} R_o \dfrac{\partial c_o}{\partial \xi_o} & -R_f \dfrac{\partial c_f}{\partial \xi_f} \\ R_o \dfrac{\partial \hat{n}_o}{\partial \xi_o} & R_f \dfrac{\partial \hat{n}_f}{\partial \xi_f} \end{bmatrix} \begin{bmatrix} \dot{\xi}_o \\ \dot{\xi}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ (R_o \hat{n}_o \times) & (R_f \hat{n}_f \times) \end{bmatrix} \begin{bmatrix} \omega_o \\ \omega_f \end{bmatrix} . \tag{2.12}$$

Equations (2.12) represent six equations which may be solved for $\dot{\xi}_o$ and $\dot{\xi}_f$ given ω_o and ω_f. While it appears that there are more equations than unknowns in (2.12), it may be verified that the equations are internally consistent.

2.1. Simplification to the Planar Case

It is of particular interest to specialize and simplify the preceding development to the case where rolling occurs in a plane (see Figure 2.2). Then C_o, C_f represent *planar* coordinate frames, $v_o, v_f \in \mathbb{R}^2$, $\omega_o, \omega_f \in \mathbb{R}$, and $c_o(\xi_o), c_f(\xi_f)$ are now parameterized by $\xi_o, \xi_f \in \mathbb{R}$. Then $U_o \in \mathbb{R}^{2\times 3}$ in equation (2.5) is given by

$$U_o = \left[I \;\middle|\; \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} R_o c_o \right] \tag{2.13}$$

and similarly for U_f. When c_o and c_f are plane curves parameterized by arclength with the same orientation (eg., ξ increases as the curve is traversed counterclockwise), equation (2.9) reduces to an arc length constraint. Since the tangent vectors (of unit length for an arc length parametrization) are just a rotated copy of the unit outward normal vectors and planar rotation is a

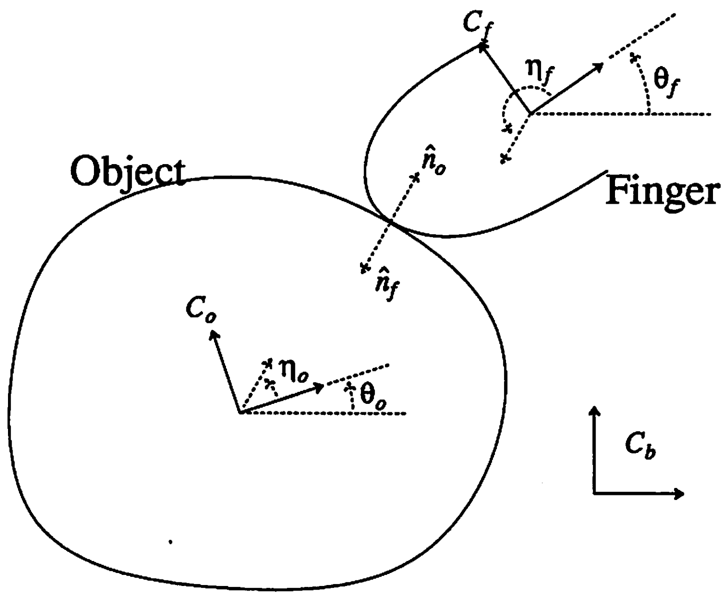

Figure 2.2

commutative operation, the normal constraint (2.10) also implies a constraints on tangents:

$$R_o \frac{dc_o}{d\xi_o} = -R_f \frac{dc_f}{d\xi_f} \tag{2.14}$$

Then, since $\frac{dc_o}{d\xi_o}$ and $\frac{dc_f}{d\xi_f}$ are nonzero vectors, (2.14) can be used with the tangency constraint (2.9) to obtain

$$\dot{\xi}_o = -\dot{\xi}_f \,, \tag{2.15}$$

a derivative form of the arc length constraint which holds for planar manipulation.

To simplify (2.11), let η_o, η_f be the angles (i.e., the orientation) of $\hat{n}_o$, $\hat{n}_f$ with respect to the frames C_o, C_f, respectively. Then we may write the normal constraint as (see Figure 2.2)

$$\eta_o + \theta_o = \eta_f + \theta_f + k\pi \tag{2.16}$$

for some k, an odd integer. Differentiating (2.16), using (2.15) and the fact that $\dot{\theta}_o = \omega_o$ and $\dot{\theta}_f = \omega_f$, yields

$$\left[\frac{d\eta_o}{d\xi_o} + \frac{d\eta_f}{d\xi_f} \right] \dot{\xi}_o = \omega_f - \omega_o \,. \tag{2.17}$$

From differential geometry (see [5]) and straightforward calculations, $\frac{d\eta_o}{d\xi_o}$ and $\frac{d\eta_f}{d\xi_f}$ may be seen to be the curvatures of the object and fingertip, respectively, at the point of contact when ξ_o and ξ_f are arclength parameters.

3. Grasping and Manipulability

We will study the grasping and manipulability of an object being contacted by m fingers, each with a rolling contact. Let the m finger frames be $C_{f1}, C_{f2}, \cdots, C_{fm}$, and let the contact points have coordinates (in each finger frame) $c_{f1}, c_{f2}, \cdots, c_{fm}$, respectively. Let the corresponding points on the body be given by $c_{o1}, c_{o2}, \cdots, c_{om}$ with respect to the body frame, C_o. Using the notation of Section 2, the matrices $U_{oi}, i = 1,\ldots,m$ map the velocity $(v_o^{\mathrm{T}}, \omega_o^{\mathrm{T}})^{\mathrm{T}}$ of the object frame C_o to the velocities of the contact points, $c_{oi}, i = 1,\ldots,m$, that is

$$v_{ci} = U_{oi}\begin{bmatrix} v_o \\ \omega_o \end{bmatrix},$$

where v_{ci} denotes the velocity of the ith contact point. Stacking the U_{oi}s, we get

$$\begin{bmatrix} v_{c1} \\ v_{c2} \\ \vdots \\ v_{cm} \end{bmatrix} = \begin{bmatrix} U_{o1} \\ U_{o2} \\ \vdots \\ U_{om} \end{bmatrix}\begin{bmatrix} v_o \\ \omega_o \end{bmatrix} \tag{3.1}$$

$$\triangleq G^{\mathrm{T}}\begin{bmatrix} v_o \\ \omega_o \end{bmatrix}.$$

Note that $G^{\mathrm{T}} \in \mathbb{R}^{3m\times 6}$. A dual relation to (3.1) is obtained by considering the effect of forces, $f_{c1}, \ldots, f_{cm} \in \mathbb{R}^3$, applied at the points c_{oi}, on the body at the origin of the frame C_o. Using the principle of virtual work, the desired transformation is found to be

$$\begin{bmatrix} f_o \\ \tau_o \end{bmatrix} = G\begin{bmatrix} f_{c1} \\ f_{c2} \\ \vdots \\ f_{cm} \end{bmatrix}. \tag{3.2}$$

The origin of the body frame C_o is frequently chosen to be the center of mass of the body. G depends both on the location of the contact points and the current body orientation. The matrix $G \in \mathbb{R}^{6\times 3m}$ is referred to as the *grasp* matrix: forces in the null space of G correspond to those forces that can be exerted at the contact points without causing a net force-moment on the body. These are referred to as *internal* forces. Equations (3.1) and (3.2) provide valid relations if the fingers remain in contact with the object and there is no slipping between the two surfaces. Such a condition will occur only if the contact forces lie in a friction cone at each contact point -- i.e., the tangential component of the contact force is less than or equal to the coefficient of friction, μ, times the normal component of the contact force. Let $FC_i \subset \mathbb{R}^3$ denote the friction cone at the i th contact point, i.e., if f_{ci}^n and f_{ci}^t are the normal and tangential components of f_{ci}, then

$$FC_i = \left\{ f_{ci} \in \mathbb{R}^3 : \| f_{ci}^t \| \leq \mu \| f_{ci}^n \| \right\}. \tag{3.3}$$

Define $FC = FC_1 \times \cdots \times FC_m$.

For the purpose of grasping, we would like to have the ability to withstand any disturbance force-moment pair on the object. The mathematical characterization of this ability is

$$G(FC) = \mathbb{R}^6, \tag{3.4}$$

i.e., the grasp map should map the friction cone *onto* $\mathbb{R}^6$, so that a given force-moment on the body can be achieved by an appropriate choice of contact forces lying in the friction cone. This property of a grasp has been called *grasp stability* [6] or *force closure* [7]. In this paper, we refer to condition (3.4) as the *force closure* condition.

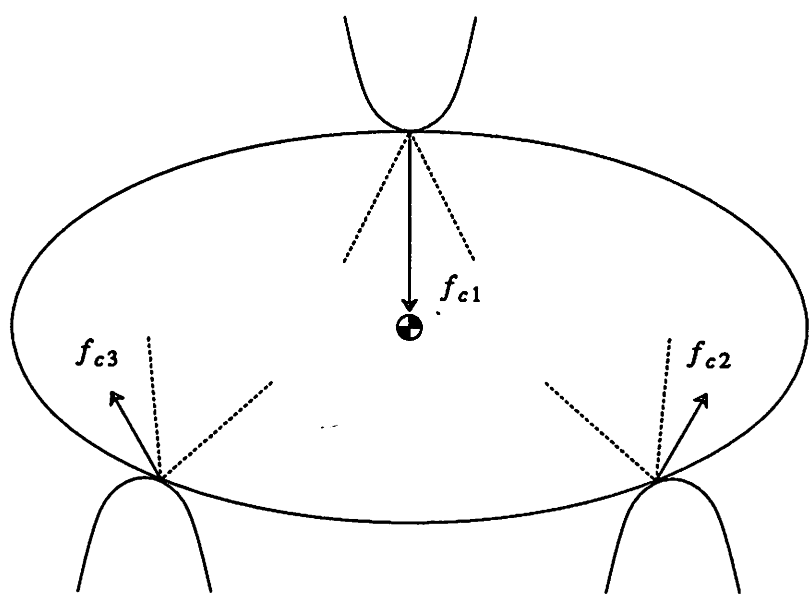

Figure 3.1

Internal forces represent the ability to apply tension and compression to an object. In order to be able to firmly grasp an object, it is desirable that the internal forces lie in the interior of the friction cone. Mathematically, this condition may be stated as

$$\eta(G)\cap\overset{\circ}{FC}\neq\varnothing \tag{3.5}$$

where $\overset{\circ}{FC}$ is the interior of the friction cone and $\eta(\cdot)$ denotes the null space of a matrix. Figure 3.1 shows an example of a grasp in which the internal or grasping force (indicated by the arrows) lies outside the friction cone (the dotted lines), so that the applying this force to the object will result in sliding or slipping at one or more contact points. When condition (3.5) is satisfied, we can bring any given vector of contact forces into the friction cone by adding a sufficiently large force in the null space of G. In this paper, we refer to condition (3.5) as the *prehensility* condition. One application of prehensility is as follows: let $f_{c1}, \ldots, f_{cm}$ be a set of contact forces that results in a certain net force and moment to the body. Consider the case where $f_c \notin FC$, thereby rendering the possibility of the contacts slipping. Then if condition (3.5) is satisfied, one can choose a large enough grasping force, $f_g \in \eta(G)\cap\overset{\circ}{FC}$, so that the sum of f_g and f_c results in the same net body force and moment and provides a feasible, non-slipping contact.

Thus far, we have discussed the kinematics of rolling contact without any mention of the kinematics of the manipulator attached to the rolling fingertip. In a multi-fingered hand, each finger is, in effect, a manipulator. Consider a multi-fingered hand with rolling contact at the fingertips. Let the ith finger have n_i joints and let $q_i \in \mathbb{R}^{n_i}$ denote the generalized (linear and rotational) coordinates of the links of the manipulator. Then, by differentiating the forward kinematic map of the manipulator, one obtains the following relationship between $\dot{q}_i$ and the velocity $(v_{fi}^T, \omega_{fi}^T)^T$ of the ith finger coordinate frame

$$\begin{bmatrix} v_{fi} \\ \omega_{fi} \end{bmatrix} = J_i(q_i)\dot{q}_i \,. \tag{3.7}$$

Here $J_i(q_i) \in \mathbb{R}^{6\times n_i}$ is the Jacobian matrix of the ith manipulator (finger). As in Section 2, we may express the velocity of the point of contact for finger i as

$$\begin{aligned} v_{ci} &= U_{fi} J_i \dot{q}_i \\ &\triangleq \tilde{J}_i \dot{q}_i \,. \end{aligned} \tag{3.8}$$

The matrix $\tilde{J}_i \in \mathbb{R}^{3\times n_i}$ maps the joint velocities $\dot{q}_i$ to the velocity at the point of contact. Dual to the relationship (3.8) is the relationship between the fingertip contact force f_{ci} and the vector of torques $\tau_i \in \mathbb{R}^{n_i}$ at the joints of finger i:

$$\tau_i = \tilde{J}_i^T f_{ci} \,. \tag{3.9}$$

Aggregating the relationships (3.8), (3.9) for $i = 1, \ldots, m$ yields

$$\begin{aligned} v_c &= \tilde{J}\dot{q} \\ \tau &= \tilde{J}^{\mathrm{T}} f_c \end{aligned} \tag{3.10}$$

with $\tilde{J}$ = block $\mathrm{diag}(\tilde{J}_1, \tilde{J}_2, \ldots, \tilde{J}_m) \in \mathbb{R}^{3m \times (n_1 + \cdots + n_m)}$. When $\tilde{J}$ is square (each finger has 3 degrees of freedom) and invertible (the m manipulators are not in a singular configuration), we can invert (3.10) to get

$$f_c = (\tilde{J}^{-1})^{\mathrm{T}} \tau . \tag{3.11}$$

In turn, the force and moment applied to the object are given by

$$\begin{bmatrix} f_o \\ \tau_o \end{bmatrix} = G f_c = G \tilde{J}^{-\mathrm{T}} \tau . \tag{3.12}$$

The singular values of $G\tilde{J}^{-\mathrm{T}} \in \mathbb{R}^{6 \times 3m}$ measure how easy it is to apply forces and moments to the body using the joint torques. Also, since

$$\dot{q} = \tilde{J}^{-1} G^{\mathrm{T}} \begin{bmatrix} v_o \\ \omega_o \end{bmatrix} \tag{3.13}$$

and the singular values of $\tilde{J}^{-1} G^{\mathrm{T}}$ are the same as those of $G\tilde{J}^{-\mathrm{T}}$, the same singular values provide a measure of the joint velocities required to produce a given body velocity. The largest singular value of $G\tilde{J}^{-\mathrm{T}}$ may be referred to as a *force closure index* and the smallest as a *manipulability index.*

4. Dynamics and Control

In this section, we consider the problem of controlling the position and orientation of an object in $\mathbb{R}^2$ by the appropriate application of torques at the finger joints. Thus, we specialize the development of the preceding sections to the planar case. Denote by $X = \begin{bmatrix} x \\ \theta \end{bmatrix}$ the position ($x \in \mathbb{R}^2$) and orientation ($\theta \in \mathbb{R}$) of the object frame C_o relative to the palm or base frame. Then $\dot{X} = \begin{bmatrix} v \\ \omega \end{bmatrix}$ is the velocity (linear and angular) of the body frame. The dynamics of the body are expressed as

$$M_o \ddot{X} = F_o \tag{4.1}$$

where $M_o = \mathrm{diag}(m_o, m_o, j_o)$ is the mass-inertia matrix of the body and $F_o = (f_o^{\mathrm{T}}, \tau_o)^{\mathrm{T}}$ is the force and torque applied at the origin of the object frame. This, in turn, is related to the forces applied by the fingers at the respective contact points by

$$F_o = G f_c \tag{4.2}$$

where $f_c = (f_{c1}^{\mathrm{T}}, \ldots, f_{cm}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{2m}$ is the vector of components of each of the m forces exerted by the fingers at their respective contact points. Combining equations (4.1) and (4.2), the contact forces needed to generate the body acceleration $\ddot{X}$ are given by

$$f_c = G^+ M_o \ddot{X} + f_{\mathrm{I}} \tag{4.3}$$

where $G^+ = G^{\mathrm{T}}(GG^{\mathrm{T}})^{-1}$ is the pseudo-inverse of G (assuming force closure of the grasp G) and f_{I} is an internal force (belonging to the null space of G). The dynamics of each finger have the form

$$M_i(q_i)\ddot{q}_i + N_i(q_i, \dot{q}_i) = \tau_i - \tilde{J}_i^{\mathrm{T}} f_{ci} \tag{4.4}$$

where $M_i(q_i) \in \mathbb{R}^{n_i \times n_i}$ is the positive definite moment of inertia matrix for the ith finger, $N_i(q_i, \dot{q}_i) \in \mathbb{R}^{n_i}$ is a vector of gravity, coriolis, and friction terms, and $\tau_i \in \mathbb{R}^{n_i}$ is the vector of input joint torques. (Note that the term $-\tilde{J}_i^{\mathrm{T}} f_{ci}$ in (4.4) is the torque, given by equation (3.9), due to the force $-f_{ci}$ that the object is *applying* to the fingertip). In this section, we will make the added simplification that there are only two joints per finger (i.e., there are as many joints as there are independent forces that can be applied at the contact points). For m fingers, the equations (4.4) may be aggregated to give

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau - \tilde{J}^{\mathrm{T}} f_c \tag{4.5}$$

with

$$M(q) = \text{block diag}(M_1(q_1), \ldots, M_m(q_m)) \in \mathbb{R}^{(n_1 + \cdots + n_m) \times (n_1 + \cdots + n_m)},$$

$$N(q, \dot{q}) = (N_1(q_1, \dot{q}_1)^{\mathrm{T}}, \ldots, N_m(q_m, \dot{q}_m)^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{n_1 + \cdots + n_m},$$

and

$$\tau = (\tau_1^{\mathrm{T}}, \ldots, \tau_m^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{(n_1 + \cdots + n_m)}.$$

Combining equations (4.3) and (4.5), we get

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau - \tilde{J}^{\mathrm{T}}(G^+ M_o \ddot{X} + f_{\mathrm{I}}). \tag{4.7}$$

Also, using equations (3.1) and (3.8), we get the velocity constraint

$$\tilde{J}\dot{q} = G^T \dot{X}. \tag{4.8}$$

Differentiating (4.8), we obtain the acceleration constraint

$$\dot{\tilde{J}}\dot{q} + \tilde{J}\ddot{q} = \dot{G}^{\mathrm{T}}\dot{X} + G^{\mathrm{T}}\ddot{X} . \tag{4.9}$$

Note that the grasp map G is a time varying map since the contact points change as the fingers move on the object. Now, provided that the system does not go through a singular configuration, $\tilde{J}^{-1}$ will exist so that

$$\ddot{q} = \tilde{J}^{-1}(\dot{G}^{\mathrm{T}}\dot{X} + G^{\mathrm{T}}\ddot{X} - \dot{\tilde{J}}\dot{q}) . \tag{4.10}$$

Using this in (4.7) yields

$$(M\tilde{J}^{-1}G^{\mathrm{T}} + \tilde{J}^{\mathrm{T}}G^{+}M_o)\ddot{X} = \tau - M\tilde{J}^{-1}(G^{\mathrm{T}}\dot{X} - \dot{\tilde{J}}\dot{q}) - \tilde{J}^{\mathrm{T}}f_{\mathrm{I}} - N . \tag{4.11}$$

The control problem is to get $X(t)$ to track a given desired trajectory $X_d(t)$ asymptotically. The constraint on the contact forces at the fingertips is that they lie within a friction cone so that there is no sliding and contact between the fingers and the object is maintained. The proposed control law for this purpose has three terms:

(1) A non-linearity cancellation term of the form

$$N + M\tilde{J}^{-1}(\dot{G}^{\mathrm{T}}\dot{X} - \dot{\tilde{J}}\dot{q}) .$$

(2) Proportional and Derivative Error feedback terms to give the system linear error dynamics

$$[M\tilde{J}^{-1}G^{\mathrm{T}} + \tilde{J}^{\mathrm{T}}G^{+}M_o](\ddot{X}_d + K_v\dot{E} + K_pE)$$

where $K_v = k_vI$, $K_p = k_pI$ with k_v, k_p scalars chosen such that $s^2 + k_vs + k_p$ is Hurwitz. Also $E \triangleq X_d - X$ so that $\dot{E} = \dot{X}_d - \dot{X}$.

(3) A term of the form $\tilde{J}^{\mathrm{T}}f_{\mathrm{N}}$, with f_{N} chosen in the null space of G so as to keep the contact forces within the friction cones at each point of contact.

The following is the main result of this section.

Theorem 4.1. Consider an object being manipulated in the plane by m fingers each having 2 degrees of freedom with dynamics given by (4.7). Consider the control law

$$\begin{aligned}\tau = [M\tilde{J}^{-1}G^{\mathrm{T}} + \tilde{J}^{\mathrm{T}}G^{+}M_o](\ddot{X}_d + K_v\dot{E} + K_pE) + N \\ + M\tilde{J}^{-1}(\dot{G}^{\mathrm{T}}\dot{X} - \dot{\tilde{J}}\dot{q}) + \tilde{J}^{\mathrm{T}}f_{\mathrm{N}}\end{aligned} \tag{4.12}$$

with f_{N} belonging to the null space of G. If the grasp maintains force closure and each of the fingers avoids singularities over the entire trajectory, then the tracking error and the tracking error rate converge to zero. Further, if the prehensility condition (3.5) is satisfied, f_{N} can be chosen so as to keep the contact forces within the friction cone at the points of contact.

Proof. Since the trajectory does not take the system through any configuration singularities, $\tilde{J}^{-1}$ exists and the control law (4.12) is well defined over the entire trajectory. Using (4.12) in (4.11) yields

$$[M\tilde{J}^{-1}G^{\mathrm{T}} + \tilde{J}^{\mathrm{T}}G^{+}M_o](\ddot{E} + K_v\dot{E} + K_pE) = \tilde{J}^{\mathrm{T}}(f_{\mathrm{I}} - f_{\mathrm{N}}). \tag{4.13}$$

Premultiplying (4.13) by $G\tilde{J}^{-\mathrm{T}}$ yields

$$[M_o + G\tilde{J}^{-\mathrm{T}}M\tilde{J}^{-1}G^{\mathrm{T}}](\ddot{E} + K_v\dot{E} + K_pE) = 0 \tag{4.14}$$

since $GG^{+} = I$ and $f_I - f_N$ lies in the null space of G.

Now, M_o is positive definite and $G\tilde{J}^{-\mathrm{T}}M\tilde{J}^{-1}G^{\mathrm{T}}$ is semi-definite (positive definite, in fact, since G^{T} is onto) so that (4.14) implies that

$$\ddot{E} + K_v\dot{E} + K_pE = 0. \tag{4.15}$$

Appropriate selection of K_v, K_p implies that $E, \dot{E} \rightarrow 0$.

The force f_{N} in the null space of G is chosen to guarantee that the contact force for each finger, f_{ci}, remains within the friction cone using the prehensility condition. □

5. Simulations

In this section, we present simulations of two planar hands manipulating an object, using the control law of equation (4.12). In these simulations, all rolling constraints are strictly enforced without checking that the resulting forces lie in the friction cone. The simulation can then be analyzed to see where the friction cone requirement is violated and note what range of grasping forces will be necessary to place the contact forces in the friction cone.

A graphics package has been developed to show the motion of the fingers and object through time in a stick-figure movie. We have included several frames for each example showing the configuration of the hand-object system. Additionally, the contact force at each contact point is drawn as a line segment showing both magnitude and direction. Viewing these pictures, it is easy to see when the forces stray from the friction cone.

To find a grasping force in $\eta(G) \cap \overset{\circ}{FC}$, we have tried the following heuristic: Let f_{normal} be the force such that each contact point force has unit magnitude in the direction of the object inward normal. Set f_{N} to the projection of f_{normal} onto the null space of G. This projection is accomplished using the projection operator

$$(I - G^{+}G).$$

If the resulting force has a nonzero component at each contact point which lies within the friction cone, we can bring arbitrary contact forces into the friction cone by adding a sufficiently large multiple of f_N to the contact force. This procedure is only a heuristic. The general problem of finding a force in $\eta(G) \cap \overset{\circ}{FC}$ as well as determining when the prehensility condition (3.5) is satisfied is the subject of ongoing research.

(1) Elliptical fingertips on an elliptical object

A planar two-fingered hand was considered. Each finger consists of two links of unit length with revolute joints and an elliptically-shaped fingertip rigidly attached to the end of the second link. The object to be manipulated is also elliptical in shape. The matrices used in the velocity constraint equation (4.8) are given as follows:

Each elliptical curve has the form $c(\xi) = \begin{bmatrix} a\cos(\xi) \\ b\sin(\xi) \end{bmatrix}$ (ξ, here, is *not* arc length). Thus, $\tilde{J} = \mathrm{diag}(\tilde{J}_1, \tilde{J}_2)$ with

$$\tilde{J}_i = U_{fi} J_i = \left[\begin{array}{c|c} I & \begin{matrix} -b_f \sin(\xi_{fi}) \\ a_f \cos(\xi_{fi}) \end{matrix} \end{array} \right] \begin{bmatrix} -s_{i12} - s_{i2} & -s_{i2} \\ c_{i12} + c_{i2} & c_{i2} \\ \hline 1 & 1 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$$

where $s_{i12} = \sin(\theta_{i1} + \theta_{i2}), c_{i2} = \cos(\theta_{i2})$, etc. Also, $G^{T} = \begin{bmatrix} U_{o1} \\ U_{o2} \end{bmatrix}$, where

$$U_{oi} = \left[\begin{array}{c|c} I & \begin{matrix} -b_o \sin(\xi_{oi}) \\ a_o \cos(\xi_{oi}) \end{matrix} \end{array} \right] \in \mathbb{R}^{2 \times 3}.$$

The control law (4.12) was used to control the corresponding dynamic system and a number of frames from the resulting simulation are shown. The desired trajectory of the object center of mass is a circle while the orientation varies as a sinusoid. Figure 5.1 shows frames from the simulation with grasping force, f_N, set to zero. Note that the contact forces often stray from the friction cone. This would normally result in sliding or loss of contact. Figure 5.2 contains frames from the simulation run again with the grasping force, f_N, determined using the heuristic described above. Now, the contact forces remain within the friction cone so that the same result might have been achieved by a real system.

(2) Circular fingertips on an elliptical object

We also considered the control of a planar 3-fingered hand, made up of 3 circular revolute fingertips which can translate linearly along a rail rigidly fixed to a platform. The three rails originate from a common point and are separated by $\frac{2\pi}{3}$ in orientation. We fix the base frame, C_b, at the common center point. In this case, $\tilde{J} = \text{diag}(\tilde{J}_1, \tilde{J}_2, \tilde{J}_3)$, where

$$\tilde{J}_i = U_{fi} J_i = \left[\begin{array}{c|c} I & \begin{matrix} -r\sin(\xi_{fi} + \theta_{fi}) \\ r\cos(\xi_{fi} + \theta_{fi}) \end{matrix} \end{array} \right] \begin{bmatrix} \cos(\alpha_i) & 0 \\ \sin(\alpha_i) & 0 \\ 0 & 1 \end{bmatrix}$$

where α_i is the orientation of the i th rail with respect to frame C_b. The grasp matrix is

$$G = [\, U_{o1}{}^T \;\; U_{o2}{}^T \;\; U_{o3}{}^T \,]$$

where U_{oi}, $i = 1,2,3$ are as given in example 1.

The control law (4.12) was again used to control the finger-object dynamic system. The desired trajectory of the object center of mass is a *figure 8* with the orientation varying as a sinusoid. Figure 5.3 shows frames from the resulting simulation with grasping force, f_N, set to zero. Again, the contact forces often stray from the friction cone which might allow the fingers to slip or slide. Figure 5.2 contains frames from the simulation run again with the grasping force, f_N, determined using the heuristic described above. The contact forces now remain within the friction cone so that this trajectory could have been produced by a real system.

6. Conclusion

In this work, we have considered the manipulation of objects of arbitrary shape by multi-fingered hands, where the contact between the object and the fingers are rolling contacts. The kinematics, prehensility, dynamics, and control of these systems have been developed in this paper. Simulations were provided to show the effectiveness of the control scheme.

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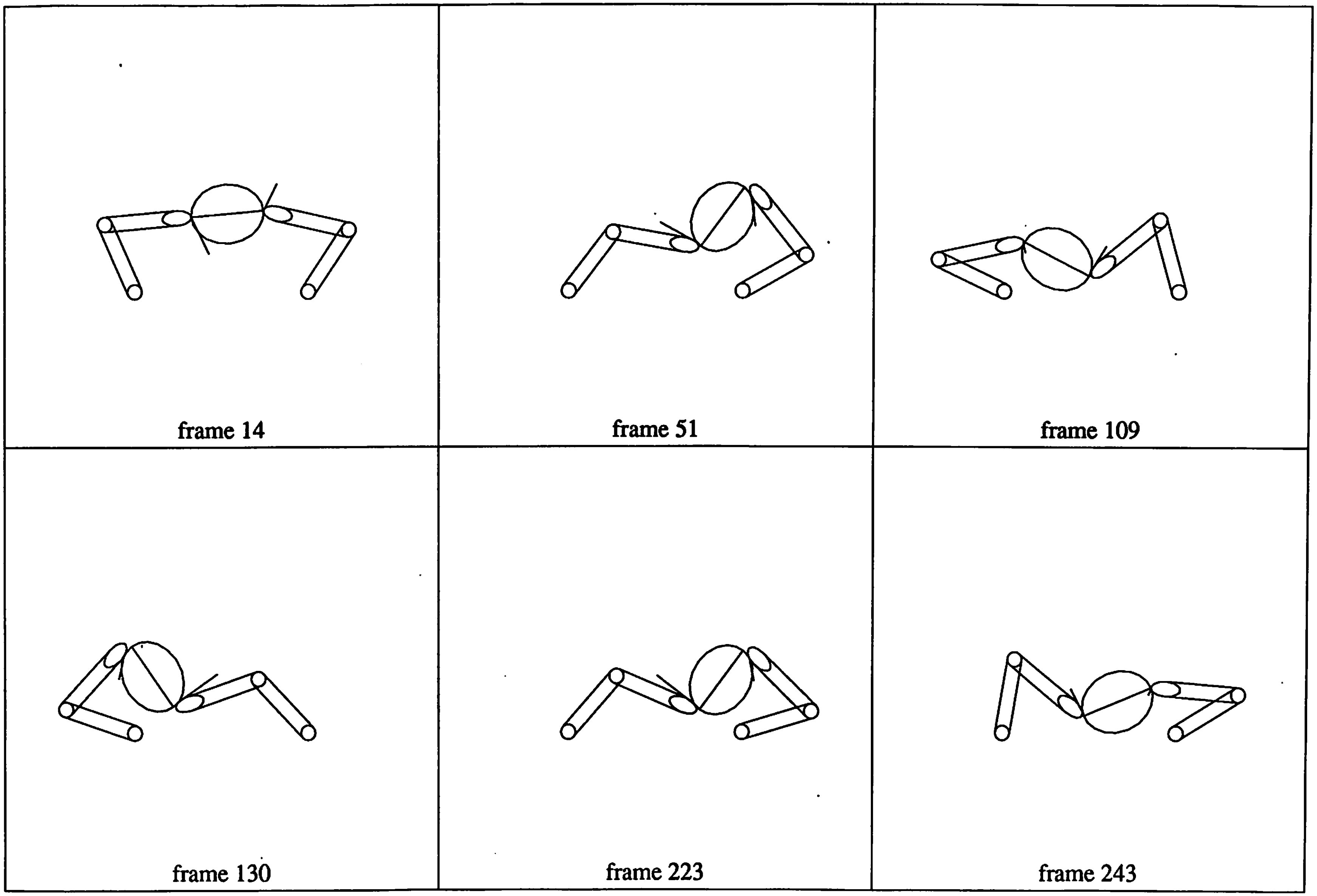

Figure 5.1

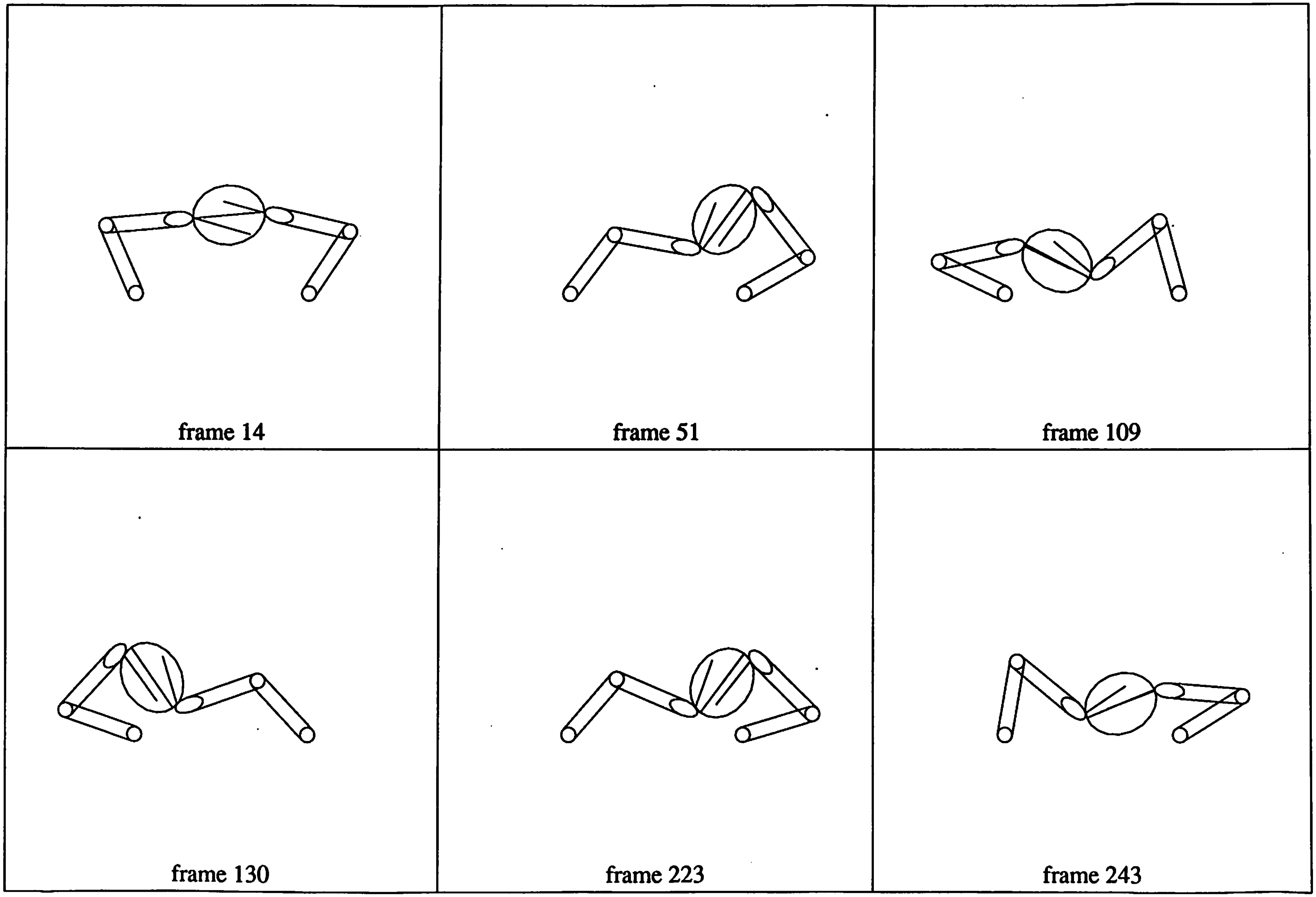

Figure 5.2

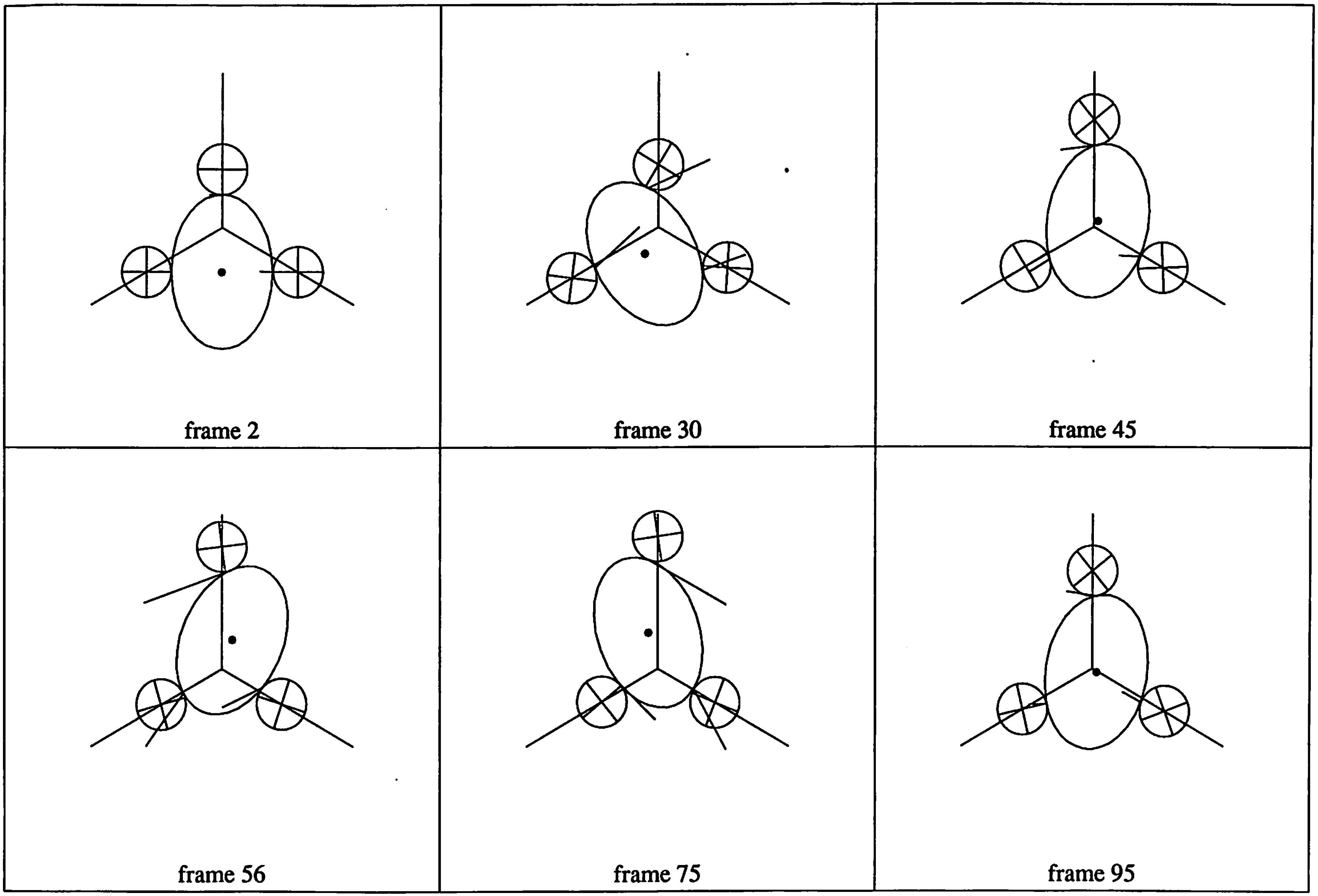

Figure 5.3

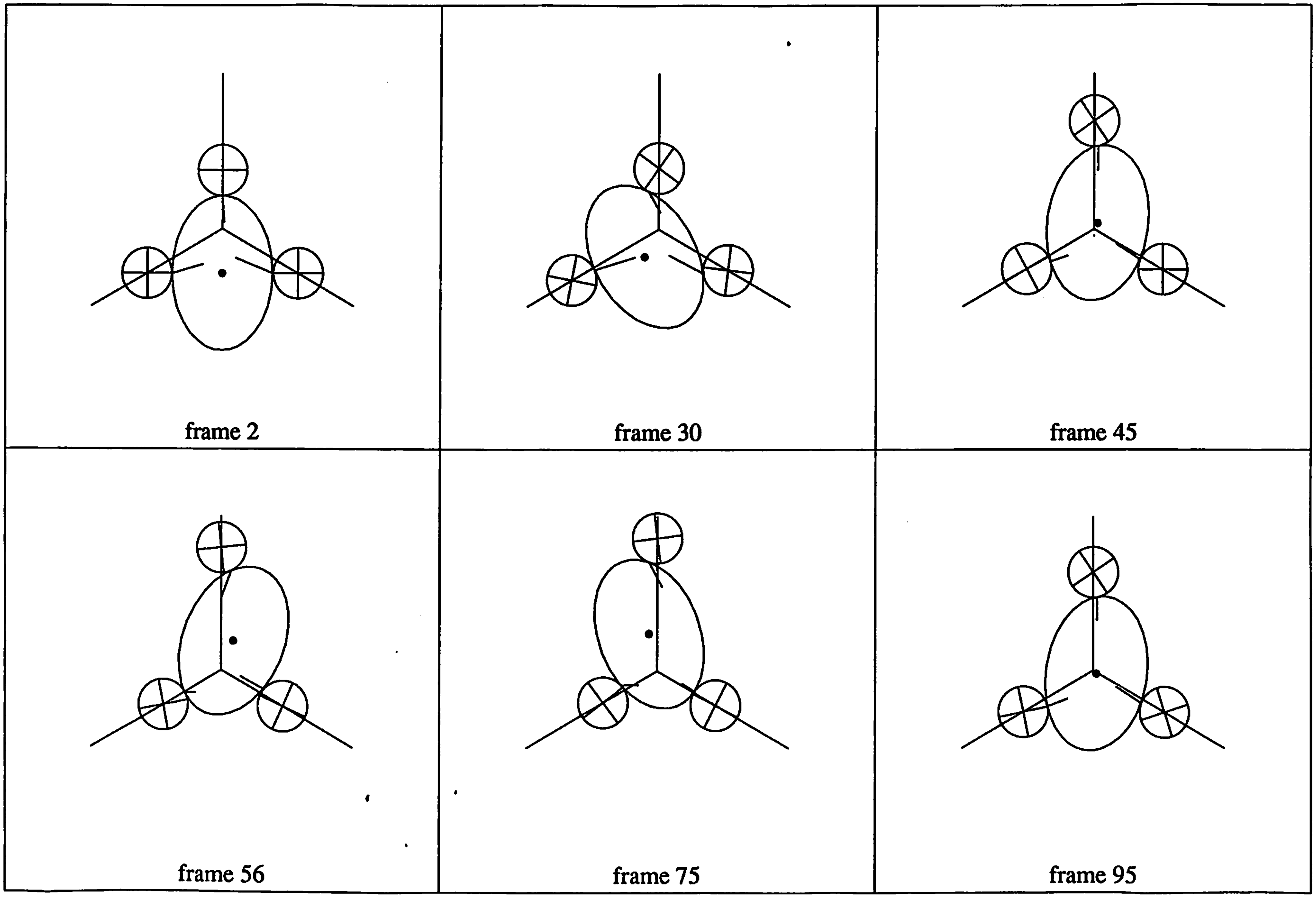

Figure 5.4