CYCLOTRON WAVE INSTABILITIES IN A PLASMA

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ABSTRACT

Transverse electromagnetic waves that propagate along a uniform magnetic field in an unbounded, thermally anisotropic plasma are investigated. The linearized Vlasov equations are solved to obtain the dispersion relation for the electron- and ion-cyclotron waves. This dispersion relation is used to obtain the critical frequencies and wavenumbers below which the wave is unstable. The electron-cyclotron wave is unstable for $T_{\perp e} > T_{ze}$. Numerical solutions for the growth and damping rates of the waves obtained from the dispersion relation show that the maximum growth rate occurs at a frequency slightly below the marginally stable frequency. For the ion-cyclotron wave, both ion and electron thermal anisotropy are included, and cold electrons are shown to have a stabilizing influence on an unstable ion-cyclotron wave; however, when the electrons are thermally anisotropic, they have a destabilizing effect for $T_{\perp e} > T_{ze}$.

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I. INTRODUCTION

Anisotropic electron- and ion-velocity distributions occur naturally during the generation, confinement, and heating of plasma in confinement experiments,[1] and for plasma confined in the earth's magnetosphere.[2] If the mean kinetic energy or temperature of the plasma is greater transverse to the magnetic field than along it, then there is thermal anisotropy which is an energy source available to drive electromagnetic instabilities. Such instabilities cause a transfer of kinetic energy from the plasma particles into electromagnetic wave energy, and they can occur when the phase velocity of the wave is comparable to the longitudinal thermal velocity of the plasma. The wave interacts strongly with those particles that have a Doppler-shifted wave frequency in the neighborhood of their cyclotron frequency. The interaction of such resonant particles with the electromagnetic field leads to instabilities for $T_\perp > T_z$. This paper studies the damping and instabilities of transverse electromagnetic waves resulting from the thermal anisotropy of the plasma particles.

It is assumed that there are many particles in a Debye sphere ($n\lambda_D^3 \gg 1$), where n is the density of the plasma, and λ_D is the thermal Debye length. It is assumed that the thermal velocities are non-relativistic and that the plasma is not collision dominated, i.e., collision frequencies are much less than the smallest characteristic frequency of the plasma. Also, the number of plasma particles in a deBroglie sphere is negligible and quantum effects are neglected $(n(\hbar/p)^3 << 1)$ ($\hbar$ is Planck's constant and p is the thermal momentum of the plasma).

Section II contains the dispersion relation for electron- and ion-cyclotron waves. This dispersion relation, which is derived for a two-dimensional anisotropic Maxwellian particle distribution, is expressed in terms of the Fried and Conte plasma dispersion function.[3]

In Section III, the interaction of a thermally anisotropic electron plasma in a stationary-ion background and a right-hand circularly-polarized wave is considered. The stability threshold for this situation is determined from the dispersion relation. For situations in which the transverse thermal energy of the electrons is greater than the longitudinal thermal energy, wavelengths longer than a certain critical wavelength are unstable. Numerical solutions of the dispersion relation are obtained for the temporal growth and damping rates of the wave.

In Section IV, the stability of a left-hand circularly polarized wave that has a frequency close to the ion-cyclotron frequency is examined. The thermal anisotropy of the ions and electrons is included to study the effect of the electron thermal anisotropy on the ion-cyclotron wave.

II. DISPERSION RELATION

A homogeneous unbounded plasma in a uniform, static magnetic field $\underline{B}_0$ (in the z-direction) is described by a zero-order velocity distribution $f_{0j}(v_\perp^2, v_z)$, where $v_\perp$ and v_z are the zero-order velocities perpendicular and parallel to the static magnetic field. The average density is assumed constant and all perturbations vary as $\exp[-i(\omega t - kz)]$.

The dispersion relation for right- or left-hand circularly-polarized waves is derived either from the linearized collisionless Boltzmann equation or from a perturbation analysis of the particle-orbit equation. The dispersion relation for right- or left-hand circularly-polarized waves propagating along the magnetic field is[4, 5]

$$\omega^2 = k^2c^2 - \sum_j \omega_{pi}^2 \int_{-\infty}^{\infty}\int_0^{\infty} \frac{(\omega - kv_z)\dfrac{\partial f_{0j}}{\partial v_\perp} + kv_\perp \dfrac{\partial f_{0j}}{\partial v_z}}{(\omega \pm \omega_{cj} - kv_z)} \pi v_\perp^2 \, dv_\perp \, dv_z \tag{1}$$

where the wavenumber, k, is in the z-direction, j indicates the specie, $\omega_{pj}^2 = n_j e^2/m_j\epsilon_0$, and $\omega_{cj} = q_jB_0/m_j$, and carries the sign of the charge. The upper sign in the denominator of Eq. (1) refers to the right-hand circularly-polarized waves and the lower sign to left-hand circularly-polarized waves.

The two-dimensional Maxwellian used in this analysis is

$$f_{0j} = \frac{1}{\pi^{3/2}\alpha_{\perp j}^2 \alpha_{zj}} \exp - \left[\left(\frac{v_\perp}{\alpha_{\perp j}}\right)^2 + \left(\frac{v_z}{\alpha_{zj}}\right)^2\right] \quad (2)$$

where $v_\perp$ and v_z are defined with respect to the magnetic field direction, $\alpha_{\perp j}$ is the thermal half-width of the velocity distribution in the transverse direction and α_{zj} is the longitudinal velocity spread of the plasma particles. To obtain the dispersion relation, Eq. (2) is substituted into Eq. (1). After integration over $v_\perp$, Eq. (1) becomes

$$\omega^2 = k^2c^2 - \sum_j \omega_{pj}^2 \left[\int_{-\infty}^{\infty} \frac{\left(\frac{\omega}{k\alpha_{zj}} - t\right) e^{-t^2}}{\pi^{1/2}\left(t - \phi_{r_{\ell j}}\right)} dt \right.$$

$$\left. + \int_{-\infty}^{\infty} \frac{t\alpha_{\perp j}^2 e^{-t^2}}{\pi^{1/2}\alpha_{zj}^2\left(t - \phi_{r_{\ell j}}\right)} dt \right] . \quad (3)$$

where the substitutions $t = v_z/\alpha_{zj}$ and $\phi_{r_{\ell j}} = \omega \pm \omega_{cj}/k\alpha_{zj}$ are made to simplify the dispersion relation.

In terms of the Fried and Conte plasma dispersion function,[3] Eq. (3) is

$$\omega^2 = k^2c^2 - \sum_j \omega_{pj}^2 \left[\frac{\omega}{k\alpha_{zj}} Z\,\phi_{r_{\ell j}} + (\theta_j - 1)\left(1 + \phi_{r_{\ell j}} Z\left(\phi_{r_{\ell j}}\right)\right)\right] , \qquad (4)$$

where

$$Z\,\phi_{r_{\ell j}} = \frac{1}{\pi^{1/2}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{\left(t - \phi_{r_{\ell j}}\right)} dt$$

is the Fried and Conte plasma dispersion function, and $\theta_j = (\alpha_{\perp j}/\alpha_{zj})^2$ is a measure of the thermal anisotropy of the species.

The plasma dispersion function $Z\left(\phi_{r_{\ell j}}\right)$ is complex for real values of the argument $\phi_{r_{\ell j}}$. A solution of the equation for real k and complex ω determines the growth or damping of the wave for a given set of plasma parameters. These transverse electromagnetic waves propagate with the electric field vector perpendicular to the propagation vector. The dispersion relation can be separated into one for the right-hand circularly-polarized wave and one for the left-hand circularly-polarized wave. The right-hand polarized wave interacts strongly with the electrons at the electron-cyclotron frequency and the left-hand polarized wave interacts strongly with the ions at frequencies close to the ion-cyclotron frequency.

III. ELECTRON-CYCLOTRON WAVES

A. Stability

The dispersion relation for the interaction of a thermally anisotropic electron plasma in a stationary ion background and a right-hand circularly-polarized wave is

$$\omega^2 = k^2c^2 - \omega_{pe}^2 \left[\frac{\omega}{k\alpha_{ze}} Z(\phi_{re}) + (\theta_e - 1)(1 + \phi_{re} Z(\phi_{re})) \right] , \qquad (5)$$

where

$$\phi_{re} = \frac{\omega + \omega_{ce}}{k\alpha_{ze}} , \qquad \theta_e = \left(\frac{\alpha_{\perp e}}{\alpha_{ze}} \right)^2 .$$

Equation (5) is complex and, in general, must be solved numerically. To investigate the stability threshold of the wave for real wavenumbers, the imaginary part of the frequency is set to zero and the dispersion relation is separated into its real and imaginary parts. The real and imaginary parts of Eq. (5) are

$$\omega_r^2 = k^2c^2 - \omega_{pe}^2 \left[\frac{\omega_r}{k\alpha_{ze}} \operatorname{Re} Z(\phi_{re}) + (\theta_e - 1)(1 + \phi_{re} \operatorname{Re} Z(\phi_{re})) \right] \qquad (6a)$$

and

$$\omega_{pe}^2 \left[\frac{\omega_r}{k\alpha_{ze}} \operatorname{Im} Z(\phi_{re}) + (\theta_e - 1) \phi_{re} \operatorname{Im} Z(\phi_{re}) \right] = 0 \qquad (6b)$$

where $\omega = \omega_r + i\omega_i$.

The stability threshold frequency obtained from Eq. (6b) is

$$\omega_r = \frac{\theta_e - 1}{\theta_e} \left|\omega_{ce}\right| . \tag{7}$$

For cases where $\theta_e \geq 1$ the transverse thermal energy of electrons is greater than or equal to the longitudinal thermal energy. Isotropic thermal electron distributions are stable for all frequencies,[4] and it follows that solutions to the dispersion relation for which the real part of the frequency is less than the threshold frequency defined by Eq. (7) correspond to growing waves. Equation (7) is substituted into Eq. (6a) to solve for the critical wavenumber.

$$k = \frac{\left[(\theta_e - 1)\,\theta_e^2\,\omega_{pe}^2 + (\theta_e - 1)^2\,\omega_{ce}^2\right]^{1/2}}{c\,\theta_e} . \tag{8}$$

Again, by comparison with the isotropic thermal distribution, all wavenumbers less than the critical value given by Eq. (8) are unstable. Sudan[6, 7] used a normal-mode approach to obtain relations equivalent to Eqs. (7) and (8).

Equation (7) shows that the electron-cyclotron wave is never unstable at frequencies above the electron-cyclotron frequency and that all frequencies are damped for an isotropic thermal electron distribution. As the ratio of perpendicular-to-longitudinal thermal energy of the electrons is increased from unity, progressively higher frequencies become unstable, limited by the electron-cyclotron frequency.

It is convenient to rewrite Eq. (8) in terms of the new variables $U_{pe} = \omega_{pe}/k\alpha_{ze}$, $\beta_e = \alpha_{ze}/c$,

$$U_{pe}^2 = \frac{\beta_e^{-2}}{\theta_e^{-1}} - (\theta_e - 1)\left(\frac{U_{ce}}{\theta_e}\right)^2 . \tag{9}$$

This stability condition is plotted in Fig. 1 as a function of U_{pe} and $-U_{ce}$ for constant values of θ_e. As the thermal anisotropy of the electrons is increased, the critical wavelength above which the waves are unstable becomes shorter. The equation for the marginally stable density and magnetic field describes an ellipse whose ratio of major to minor axis is a function of the thermal anisotropy. In the limit of an isotropic Maxwellian temperature, the plasma is stable for all ranges of density and magnetic field. Solutions of the dispersion relation which correspond to points inside the ellipse result in a stable plasma and damping of the electromagnetic field. As the thermal anisotropy is increased further, the stable region becomes smaller. The existence of an unstable region implies that for a given thermal anisotropy and density of the plasma and impressed magnetic field, the wave has a critical wavenumber above which the wave is damped and below which the wave amplitude grows in time. If the finite extent of the system under consideration is less than the marginally stable wavelength in the plasma, then all waves which are supported in the plasma

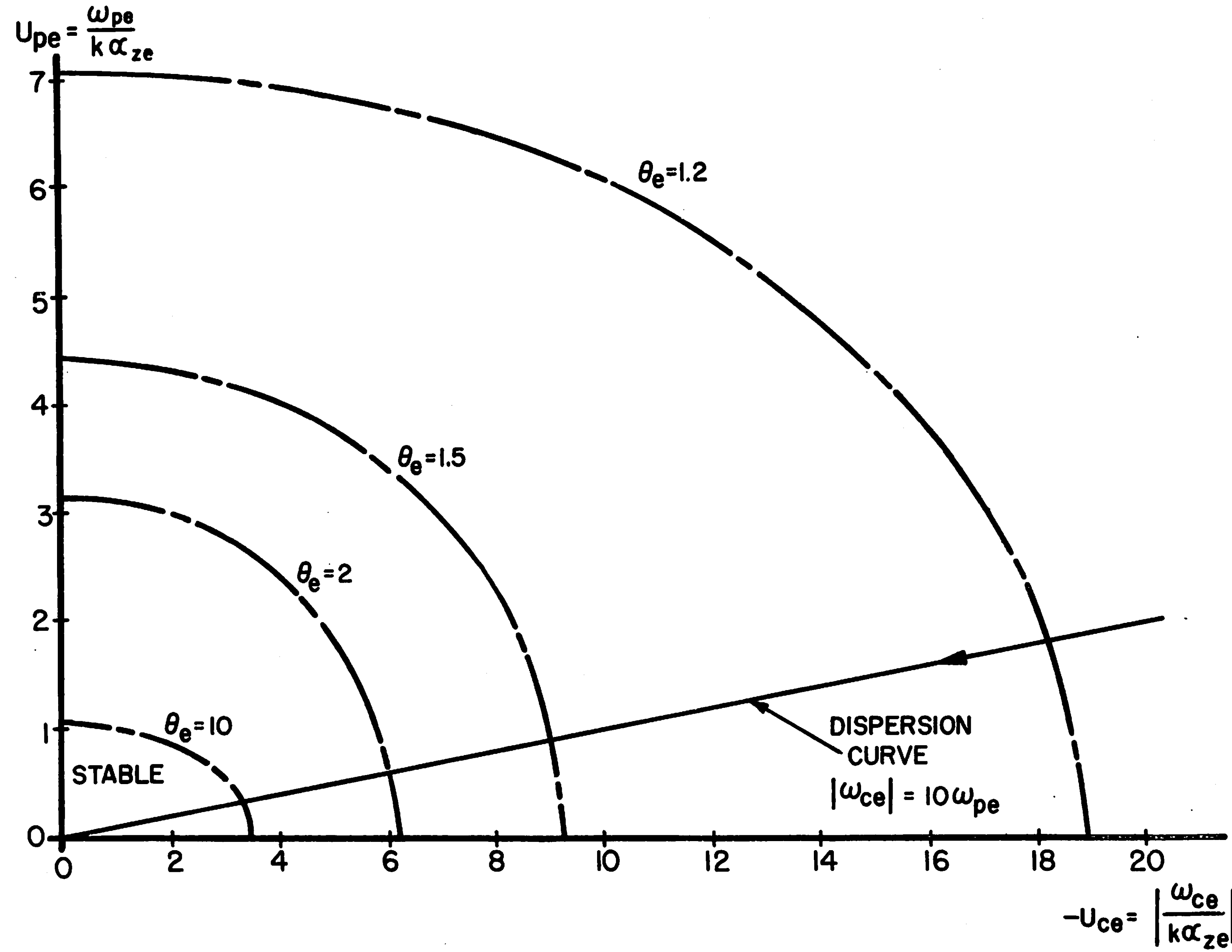

Fig. 1. Boundary between stable and unstable regions for right-hand circularly polarized waves. $\beta_e^{-2} = 10.0$; $\theta_e = T_{\perp e}/T_{ze} = (\alpha_{\perp e}/\alpha_{ze})^2$.

are damped. Hall, Heckrotte, and Kammash[8] have considered the effect of finite lengths in stabilizing instabilities arising from longitudinal oscillations.

B. Numerical Solution

The dispersion relation for electron-cyclotron waves is solved numerically. Results for $2\omega_{pe} = |\omega_{ce}|$ for an isotropic Maxwellian distribution are shown in Fig. 2. Note that there are two modes of propagation for the right-hand circularly-polarized mode. The slow-wave mode exhibits whistler-mode behavior for small wavenumbers. As the wavenumber is increased, the real part of the frequency increases and there is a resonance at the cyclotron frequency for large wavenumbers. The damping rate obtained from the imaginary part of the frequency is shown by the dashed curve. For wavelengths longer than the thermal Debye length, cyclotron damping caused by the thermal distribution of velocities along the magnetic field is negligible. As cyclotron resonance is approached, the slow-wave mode is strongly damped because the wave is in cyclotron resonance with a maximum number of particles and the wavelength is much shorter than the thermal Debye length.

The fast-wave mode is cut off above the electron-cyclotron frequency, and as the wavenumber is increased, the damping rate increases. Maximum damping occurs at the wavenumber corresponding to the velocity of light divided by the cyclotron frequency. The phase velocity

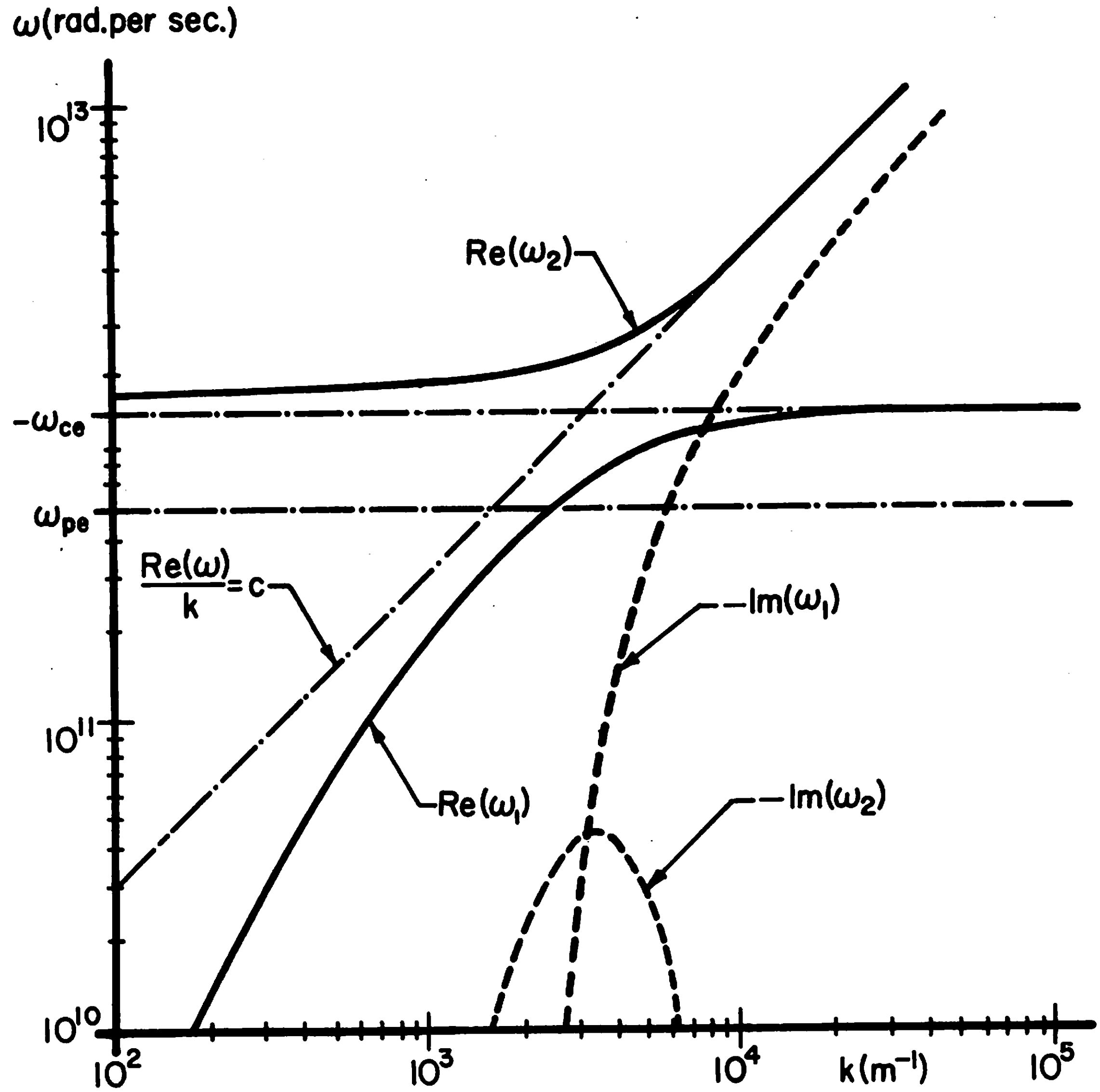

Fig. 2. Dispersion relation for right-hand circularly polarized wave. $\omega_{pe} = 5.0 \times 10^{11}$ rad/sec; $-\omega_{ce} = 1.0 \times 10^{12}$ rad/sec; $\theta_e = 1.0$; $\alpha_{ze} = 9.0 \times 10^7$ m/sec.

approaches the velocity of light, and the damping rate tends toward zero at large wavenumbers because in the tail of the Maxwellian there are few particles that are Doppler-shifted to the cyclotron-resonance frequency.

The dispersion curve for $|\omega_{ce}| = 0.5\omega_{pe}$ is shown in Fig. 3. The slow-wave mode is quite similar to the case in which the electron-cyclotron frequency is greater than the plasma frequency. The main difference is that the fast-wave mode is cut off above the plasma frequency. This means that the frequency is far from electron-cyclotron resonance for the entire mode and the maximum damping rate is less than 10^{-4} times the real part of the frequency.

The solution to the dispersion relation for an anisotropic thermal electron distribution is shown in Fig. 4. The slow-wave mode is the only one that has frequencies corresponding to unstable waves and thus is the only one discussed. The case illustrated is one for which the electrons have nine times as much transverse as longitudinal thermal energy. The plot of the imaginary part of the frequency shows that the wave is stable for short wavelengths, then is unstable for all wavelengths longer than the critical wavelength given by the threshold equation. Note that there is a maximum growth rate which exponentiates the amplitude of the wave in about five periods of oscillation of the wave.

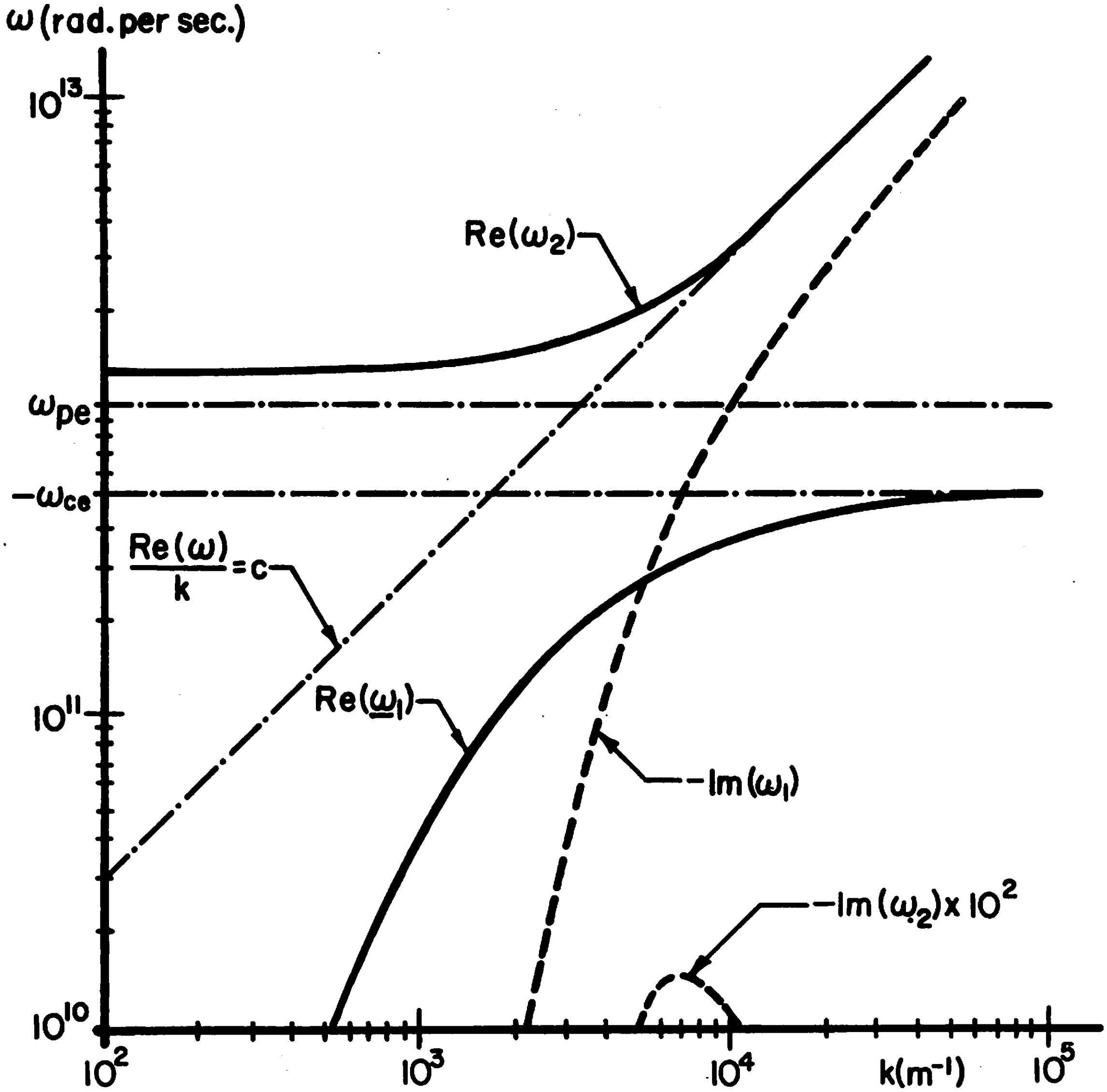

Fig. 3. Dispersion relation for right-hand circularly polarized wave. $\omega_{pe} = 1.0 \times 10^{12}$ rad/sec; $\omega_{ce} = -5.0 \times 10^{11}$ rad/sec; $\theta_e = 1.0$; $\alpha_{ze} = 9.0 \times 10^7$ m/sec.

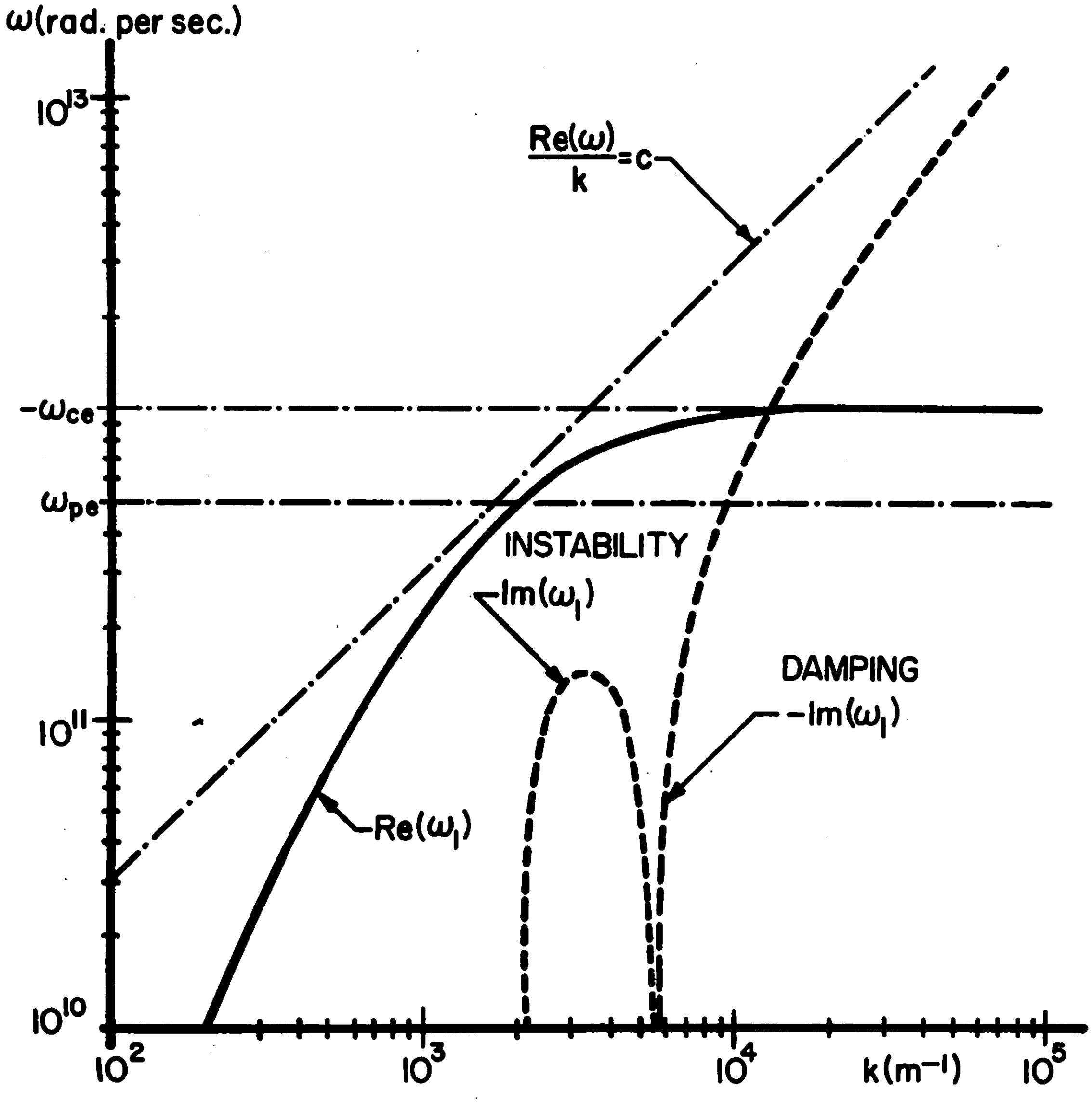

Fig. 4. Damping and instabilities for whistler mode in an anisotropic Maxwellian plasma. $\omega_{pe} = 5.0 \times 10^{11}$ rad/sec; $-\omega_{ce} = 1.0 \times 10^{12}$ rad/sec; $\theta_e = 9.0$; $\alpha_{ze} = 9.0 \times 10^7$ m/sec.

IV. ION-CYCLOTRON WAVES

The ion-cyclotron wave is left-hand circularly polarized and rotates in the direction of the ion gyration around the magnetic field. Since the electrons are mobile and can interact with the slow ion-cyclotron wave, the thermal effects of the ion and electrons are included in the analysis. From Eq. (4) the dispersion relation for a left-hand circularly polarized wave is

$$\omega^2 = k^2c^2 - \omega_{pe}^2\left[\frac{\omega}{k\alpha_{ze}} Z(\phi_{\ell e}) + (\theta_e - 1)(1 + \phi_{\ell e} Z(\phi_{\ell e}))\right]$$

$$- \omega_{pi}^2\left[\frac{\omega}{k\alpha_{zi}} Z(\phi_{\ell i}) + (\theta_i - 1)(1 + \phi_{\ell i} Z(\phi_{\ell i}))\right], \qquad (10)$$

where

$$\theta_e = \left(\frac{\alpha_{\perp e}}{\alpha_{ze}}\right)^2, \qquad \theta_i = \left(\frac{\alpha_{\perp i}}{\alpha_{zi}}\right)^2,$$

$$\phi_{\ell e} = \frac{\omega - \omega_{ce}}{k\alpha_{ze}}, \qquad \phi_{\ell i} = \frac{\omega - \omega_{ci}}{k\alpha_{zi}}.$$

If the frequency of the wave is close to the ion-cyclotron frequency and if

$$|\phi_{\ell e}| >> 1, \qquad (11a)$$

the asymptotic expansion applicable for the electron plasma dispersion function is[3]

$$\lim_{\phi_{\ell e} \to \infty} Z(\phi_{\ell e}) \to \frac{-1}{\phi_{\ell e}} , \tag{11b}$$

$$\lim_{\phi_{\ell e} \to \infty} 1/2(1 + \phi_{\ell e} Z(\phi_{\ell e})) \to \frac{1}{\phi_{\ell e}^2} .$$

The condition given by Eq. (11a) means that the wavelength of the ion-cyclotron wave is much greater than the longitudinal distance a thermal electron moves in an electron-cyclotron period. In this limit the dispersion relation becomes

$$\omega^2 = k^2 c^2 + \omega_{pe}^2 \left[\frac{\omega}{\omega - \omega_{ce}} - \frac{(\theta_e - 1)}{2} \left(\frac{k\alpha_{ze}}{\omega - \omega_{ce}} \right)^2 \right]$$

$$- \omega_{pi}^2 \left[\frac{\omega}{k\alpha_{zi}} Z(\phi_{\ell i}) + (\theta_i - 1) (1 + \phi_{\ell i} Z(\phi_{\ell i})) \right] . \tag{12}$$

To study wave stability, Eq. (12) is separated into its real and imaginary parts for Im $(\omega) = 0$.

$$\omega_r^2 = k^2c^2 + \omega_{pe}^2 \left[\frac{\omega_r}{\omega_r - \omega_{ce}} - \frac{(\theta_e - 1)}{2} \left(\frac{k\alpha_{ze}}{\omega - \omega_{ce}} \right)^2 \right]$$

$$- \omega_{pi}^2 \left[\frac{\omega_r}{k\alpha_{zi}} \operatorname{Re} Z(\phi_{\ell i}) + (\theta_i - 1) \left(1 + \frac{(\omega_r - \omega_{ci})}{k\alpha_{zi}} \operatorname{Re} Z(\phi_{\ell i}) \right) \right] ,$$

(13a)

and

$$\omega_{pi}^2 \left[\frac{\omega_r}{k\alpha_{zi}} \operatorname{Im} Z(\phi_{\ell i}) + (\theta_i - 1) \left(\frac{\omega_r - \omega_{ci}}{k\alpha_{zi}} \right) \operatorname{Im} Z(\phi_{\ell i}) \right] = 0 . \qquad (13b)$$

From Eq. (13b) the critical frequency below which the wave is unstable is

$$\omega_r = \frac{\theta_i - 1}{\theta_i} \omega_{ci} . \qquad (14)$$

The electrons do not change the critical frequency for stability. If Eq. (14) is substituted into the real part of the dispersion relation, the equation for the wavenumber stability threshold is

$$U_{pi}^2 = \frac{\beta_i^{-2}}{\theta_i - 1} + U_{pe}^2 \left[\frac{U_{ci}}{(\theta_i - 1)\, U_{ci} - \theta_i\, U_{ce}} - \left(\frac{\theta_e - 1}{\theta_i - 1} \right) \left(\frac{\alpha_{ze}}{\alpha_{zi}} \right)^2 \right.$$

$$\left. \cdot \frac{\theta_i^2}{2 \left((\theta_i - 1)\, U_{ci} - \theta_i\, U_{ce} \right)^2} \right] - (\theta_i - 1) \left(\frac{U_{ci}}{\theta_i} \right) , \qquad (15)$$

where

$$\beta_i = \frac{\alpha_{zi}}{c}, \qquad U_{ci} = \frac{\omega_{ci}}{k\alpha_{zi}}, \qquad U_{pi} = \frac{\omega_{pi}}{k\alpha_{zi}},$$

$$U_{ce} = \frac{\omega_{ce}}{k\alpha_{zi}}, \qquad U_{pe} = \frac{\omega_{pe}}{k\alpha_{zi}}.$$

In the large brackets there are two terms from the electron interaction with the ion-cyclotron wave.

The first term in the brackets is the cold electron term; the second term is a result of the electron thermal anisotropy. For charge neutrality,

$$U_{pe}^2 = \frac{m_i}{m_e} U_{pi}^2, \tag{16}$$

and Eq. (15) becomes

$$U_{pi}^2 \left[1 - \frac{m_i}{m_e} \left[\frac{U_{ci}}{(\theta_i - 1) U_{ci} - \theta_i U_{ce}} - \frac{1}{2} \left(\frac{\theta_e - 1}{\theta_i - 1} \right) \left(\frac{\alpha_{ze}}{\alpha_{zi}} \right) \frac{\theta_i^2}{\left((\theta_i - 1) U_{ci} - \theta_i U_{ce} \right)^2} \right] \right] = \frac{\beta_i^{-2}}{\theta_i - 1} - (\theta_i - 1) \left(\frac{U_{ci}}{\theta_i} \right)^2. \tag{17}$$

Since $|U_{ci}| << |U_{ce}|$ and $\theta_i \geq 1$, Eq. (17) can be simplified.

$$U_{pi}^2 \left[1 - \frac{1}{\theta_i} + \frac{1}{2} \left(\frac{\theta_e - 1}{\theta_i - 1} \right) \frac{m_e}{m_i} \left(\frac{\alpha_{ze}}{\alpha_{zi}} \right)^2 \frac{1}{U_{ci}^2} \right] = \frac{\beta_i^{-2}}{\theta_i - 1} - (\theta_i - 1) \left(\frac{U_{ci}}{\theta_i} \right)^2 . \tag{18}$$

Equation (18) is shown in Fig. 5, and the effect of electron thermal anisotropy is illustrated for several cases. The dotted curve shows the stability threshold for ions with infinite temperature electrons, which are a neutralizing background. The stabilizing effect of an isotropic zero-temperature electron distribution is shown by the alternating dashed line. Note that for $\theta_e = 9.0$ the wave is unstable for very low magnetic fields. This shows that the electrons with an excess transverse thermal energy interact with an excess transverse thermal energy interact with an ion-cyclotron wave and cause it to be more unstable. In this case the excess transverse thermal energy of the electrons is transferred into the electromagnetic energy of the wave. Thus the finite-length stabilization effects will not be able to stabilize the ion-cyclotron wave under these conditions.

From Eq. (18) it can be seen that for an isotropic electron distribution the plasma is unstable for all ion-cyclotron wavelengths greater than the critical wavelength given by

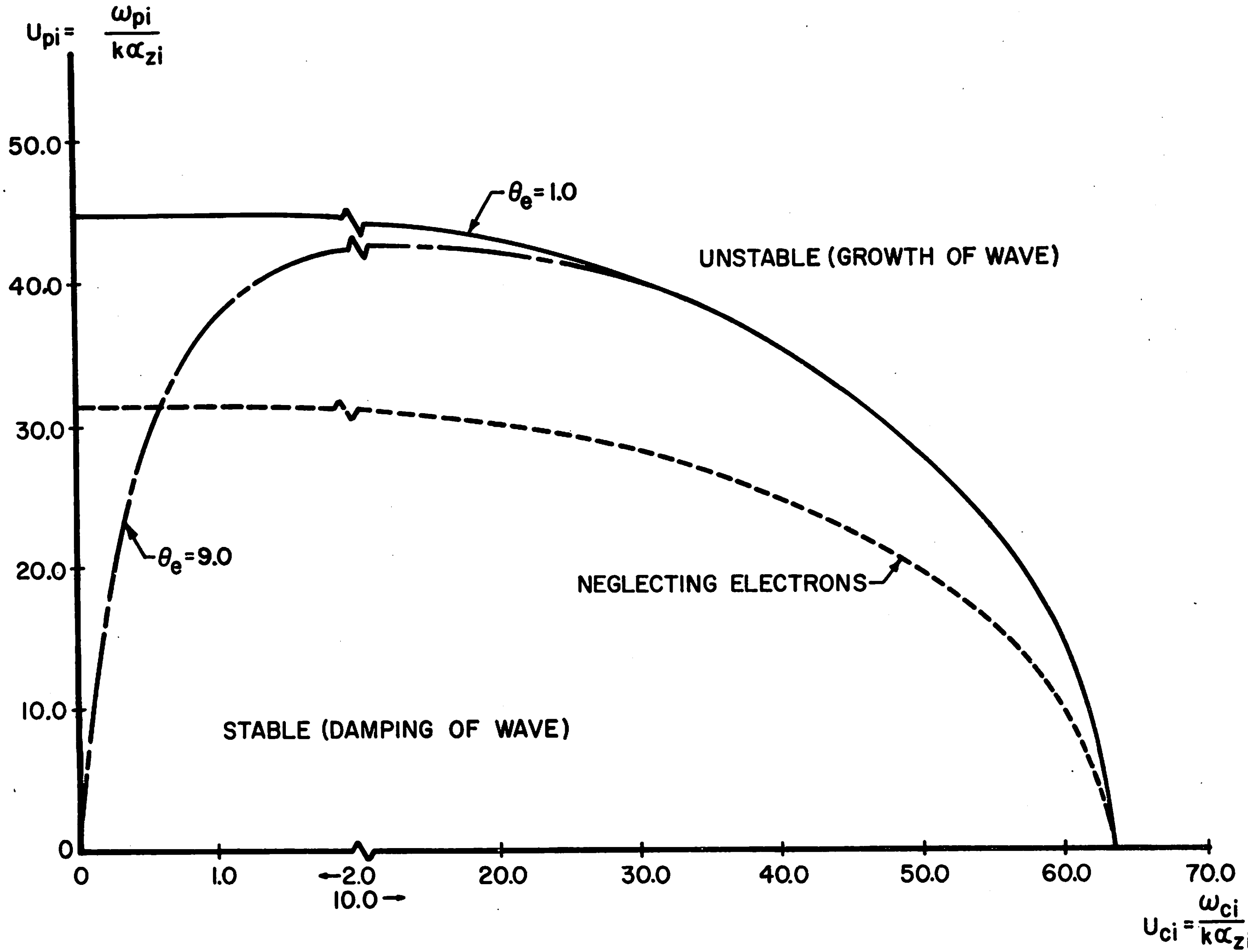

Fig. 5. Threshold of instability for ion-cyclotron waves. $\theta_i = 2.0$; $\alpha_{ze}/\alpha_{zi} = 10.0$; $\beta_i^{-2} = 1{,}000$; $n_i = n_e$.

$$\lambda = \frac{2\pi c\,\theta_i}{\left((\theta_i - 1)^2\,\omega_{ci}^2 + (\theta_i^3 - 2\theta_i^2 + \theta_i)\,\omega_{pi}^2\right)^{1/2}} \ . \tag{19}$$

V. CONCLUSIONS

The damping and instabilities of transverse electromagnetic waves in a plasma are investigated through the dispersion relation for transverse waves propagating in a plasma with anisotropic Maxwellian velocity distribution functions. For the electron-cyclotron wave it is shown that electrons with a greater transverse than longitudinal thermal energy result in a critical frequency and wavenumber below which the waves are unstable. As the thermal anisotropy of the electrons is increased, the spectrum of unstable waves is increased. The dispersion relation for an isotropic Maxwellian electron distribution function is obtained. It is found that the slow-wave mode is resonant at the cyclotron frequency and that it is strongly damped for wavelengths much shorter than the Debye length. The phase velocity of the fast-wave mode is asymptotic to the velocity of light for short wavelengths and the damping rate is peaked near the free-space wavenumber corresponding to the cyclotron frequency. The dispersion relation for an anisotropic thermal electron plasma is solved and agrees with the threshold of instability result; the slow-wave mode has a strong maximum growth rate just below the threshold frequency. The ion and electron thermal effects are included in the analysis of the ion-cyclotron wave, and it is demonstrated that zero-temperature electron distributions have a stabilizing influence on the ion-cyclotron wave as compared with the case where the electrons are assumed to have infinite temperature and only serve as a neutralizing background. In a plasma with a given ion thermal anisotropy, an electron thermal anisotropy with greater transverse than longitudinal thermal energy reduces the range of stable ion-cyclotron waves as compared with the case of an isotropic thermal distribution.

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