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INTENSITY MEASUREMENTS FOR OPTICAL MASER APPLICATIONS

by

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I. INTRODUCTION

In order to establish measurement standards, a tungsten strip lamp was used as a source of known brightness; the response of the Jarrell-Ash spectrograph used with a photomultiplier was measured over the range 3500A to 11000A. The above range was covered in parts using two different photomultipliers, RCA types 6903 and 7102. Although the former tube is quite sensitive down to about 2000A no measurements were made in this region due to the unavailability of a tungsten lamp having a quartz window. Such a lamp has been ordered and should make possible both this extension of the calibrated range and a more accurate measurement of spectrograph-photomultiplier response over the entire region.

It was found that the relative response of the spectrograph-photomultiplier combination over the range of a given tube was very nearly independent of the small changes in geometry which resulted from disassembling the apparatus and then reassembling it. However, it was learned that these uncertainties in geometry make a determination of the absolute response of the system (which can be thought of as a coefficient multiplying the above relative spectral response) to an accuracy greater than about 50 % very difficult. Methods for overcoming this problem will be discussed below.

Finally, there are in practice, two types of spectra to be measured, line spectra and continuous spectra. Methods for handling these two types are discussed.

II. THEORY

The brightness $B(\lambda)$ of a source of light is defined as the power emitted over a given wavelength interval centered at a particular wavelength into a unit solid angle per unit surface area of the source. If the source produces a line spectrum it is often more convenient to determine the total brightness of each line $B'(\lambda)$ which is given by the integral of the above quantity over a wavelength interval which includes the line. In convenient units we have

$B(\lambda)$ microwatts/cm^2 . angstrom . steradian
$B'(\lambda)$ microwatts/cm^2 . steradian

Often the total light output of a source as a function of wavelength is desired. However, once $B(\lambda)$ or $B'(\lambda)$ have been measured the total light output or any other desired quantity can easily be obtained by further integration.

In essence, therefore, the measurement of light intensities reduces to a determination of $B(\lambda)$ or $B'(\lambda)$. As explained elsewhere the method proposed for doing this involves the use of the Jarrell-Ash spectrograph and various photomultipliers. Basically, this spectrograph consists of several parts - an entrance slit of variable width, a reflection grating used as a dispersing element, an exit slit of variable width, and an external optical system which focuses the source on the entrance slit and also properly illuminates the surface of the grating. The arrangement of these elements is shown in Figure 2(c). As can be seen, a spherical mirror located at the rear of the instrument projects an image of the entrance slit onto the focal plane containing the exit slit. Since both entrance and exit slits are located in the same focal plane this image, if produced by a monochromatic source, will have the same dimensions as the entrance slit. If the source produces a continuous spectrum the image will be broad and in general illuminate the entire focal plane. It is useful to treat these two cases separately.

Consider a source producing a very narrow line. If this source is focused on the entrance slit an image ideally having the same width as the slit will be displayed on the focal plane of the instrument. As the wavelength drum is scanned over wavelengths in the neighborhood of the line, this image will move past the exit slit and light will pass through it. If the width of either the entrance or the exit slit is held fixed the amount of light passing through during the scanning process will depend upon the width of the second slit. The expected results for several cases are illustrated in Figure 1. The curves produced are in general trapezoidal with long and short sides proportional to the sum and difference, respectively, of the two slit widths. Note that the intensity maximum increases linearly with the slit opening until the two slits are equal after which the peak height remains constant.

If it were possible to reverse the above procedure and, for a given setting, λ_0, of the spectrograph wavelength drum, vary the wavelength of the source, curves very similar* to those in Figure 1 would be produced. Each such curve in analogy with electric circuits is known as the pass band of the spectrograph, $t(\lambda, \lambda_0)$, for given slit openings. The pass band, which will be used below can either be measured as above with a source of very narrow lines or determined theoretically using the actual slit widths and the reciprocal linear dispersion of the instrument.

Now suppose that instead of a line source a continuous source is used. In this case a broad continuous spectrum will be projected onto the focal plane of the spectrograph. By considering the continuous spectrum to be a superposition of narrow lines, it follows that the total intensity of light passing through the exit slit is proportional to the integral over all wavelengths of the brightness of the light incident on the entrance slit and the pass band function $t(\lambda, \lambda_0)$ characteristic of the wavelength setting of the spectrograph and of the two slit openings.

Suppose that a photomultiplier is now placed immediately behind the exit slit. If the sensitivity of the photomultiplier is given by $a \cdot S(\lambda)$ (where $S(\lambda)$ is the relative sensitivity and a is a constant determining its absolute value) the photocurrent $I(\lambda_0)$ becomes

$$I(\lambda_0) = \Omega a \int_0^\infty d\lambda\, B(\lambda) S(\lambda) t(\lambda, \lambda_0) \qquad (1)$$

$B(\lambda)$ is the brightness of that portion of the source which is focused on the entrance slit and Ω is the solid angle subtended at the source by the spectrograph and input lens. It is useful to divide $t(\lambda, \lambda_0)$ into two parts and at the same time absorb the constant Ω by writing

$$\Omega\, t(\lambda, \lambda_0) \equiv T(\lambda_0)\, \omega(\lambda, \lambda_0)$$

$T(\lambda_0)$, called the relative transmission of the spectrograph is defined to be unity at that wavelength at which it is a maximum. The remaining factor

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* The difference, if any, would be due to variations in the reciprocal linear dispersion and the transmission of the system over the small region of spectrum involved.

$\omega(\lambda, \lambda_0)$, it will be seen, is then a function of $|\lambda - \lambda_0|$ only, the nature of which depends on the entrance and exit slit widths. We then have

$$I(\lambda_0) = a \int_0^\infty d\lambda\, B(\lambda)\, S(\lambda)\, T(\lambda)\, \omega(\lambda, \lambda_0) \tag{2}$$

In practice evaluation of the above integral can be simplified because $\omega(\lambda, \lambda_0)$ is non-zero only for wavelengths very close to λ_0. We have

$$I(\lambda_0) = a\, S(\lambda_0)\, T(\lambda_0) \int B(\lambda)\, \omega(\lambda, \lambda_0)\, d\lambda \tag{3}$$

Finally, if $B(\lambda)$ changes negligibly over the range of wavelengths passed by the spectrograph we can write

$$I(\lambda_0) = a\, S(\lambda_0)\, T(\lambda_0)\, B(\lambda_0)\, W \tag{4}$$

where $W = \int d\lambda\, \omega(\lambda, \lambda_0)$.

The pass band function $\omega(\lambda, \lambda_0)$ will now be considered in more detail. As indicated above and illustrated in Figure 1 (a) and (b) the shape of the pass band depends upon the entrance and exit slit widths, d_{in} and d_{out}, used. The area of a trapezoid of altitude h whose long side is equal to $d_{in} + d_{out}$ and whose short side is equal to $|d_{in} - d_{out}|$ is given by dh where d is the larger of d_{in}, d_{out}. Since the altitude from Figure 1 is proportional to the smaller of d_{in}, d_{out} we have.

$$W = \int d\lambda\, \omega(\lambda, \lambda_0) = K\, d_{in}\, d_{out}\,. \tag{5}$$

where K is a constant to be determined. Experimentally, the dependence of W on the product $d_{in}\, d_{out}$ is found to hold to a high degree of accuracy.

If the entrance and exit slit widths are equal the pass band will be triangular with base $a\, d$, where d is the common slit width. A convenient 2d measure of the pass band is the "half-power" bandwidth equal to d. To convert this dimension into wavelength units multiply by the reciprocal linear dispersion of the spectrograph - 5.1 A/mm in first order for the grating commonly used. For several common slit openings we have, then

Slit Width	Bandwidth $\Delta\lambda$
20 μ	0.10A
50 μ	0.26A
100 μ	0.51A
400 μ	2.04A

Since spectral lines are seldom narrower than about 1-2A, it is possible to treat them as part of a continuous spectrum if a bandwidth less than, say 0.25A, is used. If the line to be measured is actually very narrow compared to the bandwidth used, Equation (3) can be used with $B(\lambda) = B'(\lambda_0)\,\delta(\lambda - \lambda_0)$. One then obtains

$$I(\lambda_0) = a\ S(\lambda_0)\ T(\lambda_0)\ B'(\lambda_0)\,\omega(\lambda_0, \lambda_0) \tag{6}$$

Following the procedure of Christensen and Ames[1] these parameters can then be measured. First the relative transmission is measured using Equation (4). The method is illustrated in Figures 2 (a) and (b). In addition to the spectrograph $(T_1 W_1)$ a monochromator (T_2, W_2) is used. With each instrument set at the same wavelength and light passing through both we have

$$I_{12}(\lambda_0) = B(\lambda_0)\ a\ S(\lambda_0)\ T_1(\lambda_0)\ T_2(\lambda_0)\ W_1 W_2$$

Measuring the light passing through only the monochromator using the same photomultiplier we obtain

$$I_2(\lambda_0) = B(\lambda_0)\ a\ S(\lambda_0)\ T_2(\lambda_0)\ W_2$$

Hence with A a constant normalizing $T_1(\lambda_0)$ to unity at maximum transmission we have

$$T_1(\lambda_0) = \frac{I_{12}}{I_1}\ A \tag{7}$$

To measure $S(\lambda_0)$ a calibrated tungsten lamp of known brightness $B(\lambda)$ is used. The calibration of such a lamp is described below. Using Equation (4) and the experimental arrangement shown in Figure 2(c) we have

$$S(\lambda_0) = \frac{K\ I(\lambda_0)}{B(\lambda_0) T(\lambda_0)} \tag{8}$$

where K is a constant normalizing $S(\lambda_0)$ to unity at peak sensitivity. This measurement must be repeated for each photomultiplier used. α can then be determined by allowing the photomultiplier to intercept a known solid angle of radiation from the calibrated source. The photocurrent is then

$$I(\text{total}) = \alpha\ a\ \Omega_0 \int S(\lambda)\, S(\lambda)\, d\lambda \tag{9}$$

where a is the area of the source used and Ω_0 is the (small) solid angle intercepted.

At this point only W_1, or if the relation $W_1 = K\, d_{in}\, d_{out}$ is to be used, K remains undetermined. It can be obtained from Equation (4) using a prescribed arrangement of the external optics, preferably that used in Figure 2(c), and a calibrated source. One then assumes that the same geometry will be employed in all successive measurements.

Unfortunately, it has been found that the value of K obtained depends critically upon both the precise alignment of the external optics and upon the nature of the source itself. For this reason only an approximate determination of K can be made. However, such a measurement is still useful for rough light intensity measurements. Another problem is α, the absolute sensitivity of the photomultiplier. Once installed in the spectrograph only a small portion of the tube is illuminated by light passing through the exit slit. Naturally α must be determined for this portion of the tube. In practice it is found that exact replacement of the tube in the spectrograph is impossible. Hence uncertainties in α, which is a function of position on the photocathode, are bound to result. One possible solution to this problem which has not been tried is to fasten a mask replacing the output slit directly to the tube itself.

Naturally if Equation (4) is to be used by itself to make intensity measurements it is not necessary to obtain separately all the factors making up this equation. Only their product together with $B(\lambda)$ is needed. For this reason the following equation equivalent to (4) has been used in the preliminary measurements made.

$$B(\lambda_0) = \frac{S_0(\lambda_0)\, I(\lambda_0)}{d_{in}\, d_{out}\, K_0} \tag{10}$$

In this equation $S_0(\lambda_0)$ accounts for all the wavelength dependent factors while d_{in}, d_{out} and K_0 account for the wavelength independent factors. S_0 is measured as the ratio $B(\lambda_0)/I(\lambda_0)$ using a source of known brightness and is normalized to unity at some wavelength. K_0 is then determined from the known brightness at that wavelength. For further discussion see the Calibration Procedure Section.

A more accurate method for brightness measurements and the one suggested by Christensen and Ames[1] is to measure the relative brightness of the source over the range of a particular photomultiplier using a modification of Equation (10).

$$B(\lambda_0) = C\, S_0(\lambda_0)\, I(\lambda_0)$$

where C is an undetermined constant, and then removing the spectrograph, expose the photomultiplier to undispersed light from the source as in the determination of α. Then using the equivalent of Equation (9) we have

$$I(\text{total}) = \alpha\, a\, \Omega_0 \int C\, S_0(\lambda)\, I(\lambda)\, S(\lambda)\, d\lambda$$

and

$$C = \frac{I\ (\text{total})}{\alpha\, a\, \Omega_0 \int S_0(\lambda)\, I(\lambda)\, S(\lambda)\, d\lambda}$$

This method of course requires that the above detailed procedure leading to a computation of $S(\lambda)$ be carried out.

III. CALIBRATION PROCEDURE

Using a tungsten strip lamp (Westinghouse EDS 18A/6V) as a source of known brightness, the response of the Jarrell-Ash spectrograph used in combination with either an RCA type 6903 or type 7102 photomultiplier was measured over the range 3500A to 11000A. The optical arrangement is shown in Figure 2(c). As outlined in the manufacturer's instruction manual[2], in order to properly fill the internal optics of the spectrograph and obtain maximum light utilization, the source must be placed about 48 cm from the entrance slit. For focusing, two cylindrical lenses were placed as indicated. L_1 focuses light in the horizontal plane thereby producing a vertical image of the source on the entrance slit. L_2 focuses light in the vertical plane and insures that the spectrograph internal optics will be properly filled. Experiments have shown that a source larger in diameter than about 1/3 inch, or one located off the center line of the external optics will cause large errors due to the presence of unwanted stray light in the instrument. For this reason a circular aperture of diameter 7.5 mm was placed in front of the tungsten lamp to reduce the apparent size of the source.

The tungsten lamp was adjusted so that only light from a central portion of the filament was focused on the spectrograph slit. The brightness temperature of this portion of the lamp was then measured with a Leeds and Northrup disappearing filament type optical pyrometer. The supply voltage which was regulated with a Sola constant voltage transformer, was adjusted to provide a brightness temperature of 2366°C. Using a nomograph prepared by DeVos[3] the true temperature was determined to be 2627°C or 2900°K. The brightness of the lamp was then computed from blackbody radiation tables[4] and the known emissivity of tungsten also measured by DeVos[5]. We have

$$B_\lambda = N_\lambda \quad \epsilon_\lambda \, \tau$$

where B_λ, N_λ are the brightnesses of the lamp and a blackbody, respectively, ϵ_λ is the emissivity and τ is the transmission of the glass envelope of the lamp. τ was assumed to be equal to 0.92 throughout the spectral region considered (3500-11000Å).

With the lamp maintained at 2366°C the RCA 6903 photomultiplier was placed in front of the exit slit and the photocurrent recorded with the Jarrell-Ash amplifier and recorder over the range 3500-7000A. Above about 6200A second order effects were removed using an orange filter. The photomultiplier was operated at 1000V. Photocurrent recordings were made for various entrance and exit slit widths, and in all cases an input slit height of 20mm was employed.

Using the RCA 7102 photomultiplier the same procedure was repeated over the wavelength range 4000-11000A. The tube was operated at 750V in order to reduce the dark current to a value small enough to permit the use of the zero suppression control provided. Both tubes were mildly aged for about one hour at light levels larger than those to be used in order to stabilize their response.

It was found that the relative response of the spectrograph-photomultiplier combination at various wavelengths was, for a given tube, independent of either the entrance or the exit slit widths. In addition, experiments at a given wavelength and given exit slit width showed that the photocurrent was very nearly proportional to the entrance slit width. This was found to be true for both phototubes at all wavelengths studied in agreement with the theory.

In order to determine the absolute response of the system photocurrent measurements were made for specific entrance and exit slit widths at wavelengths near the peak response of each tube. 5000A was chosen for the 6903 tube operated at 1000V while 7500A was selected for the 7102 photomultiplier operated at 750V. The results given below also illustrate the proportionality of the photocurrent to entrance slit width.

	Photomultiplier Current μa					
	RCA 6903			RCA 7102		
	Exit Slit Width			Exit Slit Width		
Entrance Slit Width	50μ	200μ	400μ	50μ	200μ	400μ
10μ	0.037	0.18	0.35	0.00033	0.0013	0.00275
20μ	0.072	0.35	0.72	0.0006	0.0026	0.0056
50μ	0.182	0.84	1.80	0.0017	0.0071	0.0145
100μ	0.365	1.65	3.55	0.00338	0.014	0.029
200μ	0.735			0.0068	0.029	0.059
400μ	1.48			0.0175	0.059	0.118

$S_0(\lambda)$ was computed from Equation (10) and normalized to unity at 5000A for the 6903 tube and 7500A for the 7102 tube. The resulting curves for $S_0(\lambda)$ called the attenuation characteristic of the spectrograph-photomultiplier combination are given in Figures 3 and 4. Finally, using values for K_0 computed from the above table*, we have

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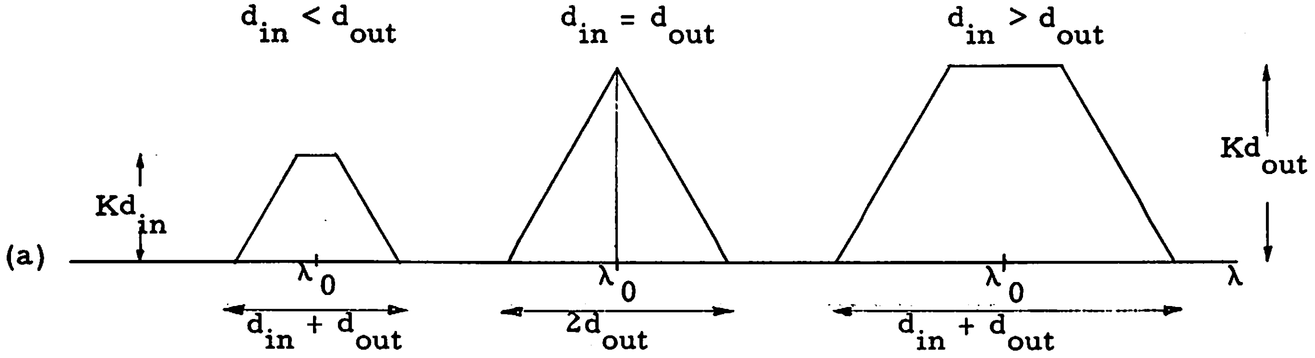
* As explained above, due to variations in sensitivity over the surface of the photomultiplier cathode $I(\lambda)$ is not strictly proportional to d_{out}. Approximate values for K_0 were obtained using slit widths of $d_{in} = d_{out} = 50\mu$ for the 6903 tube and $d_{in} = d_{out} = 200\mu$ for the 7102 tube.

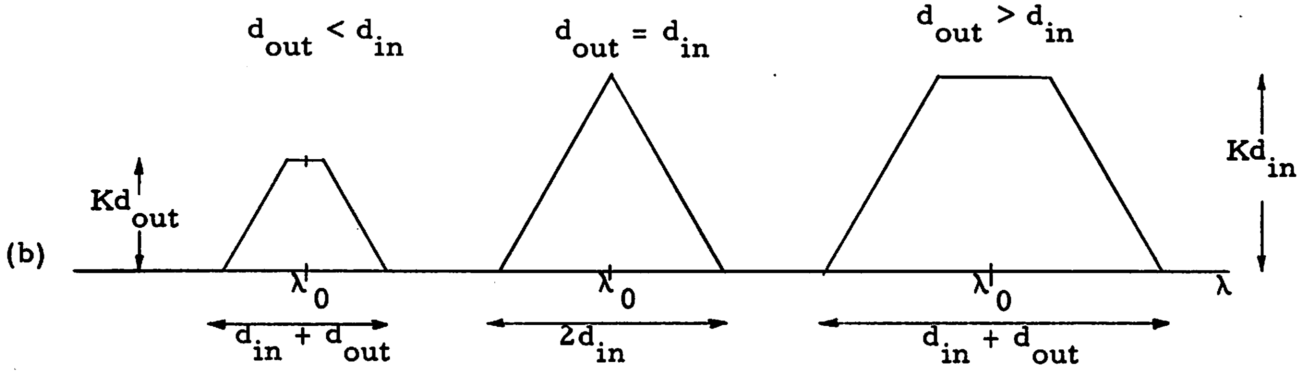
$$B(\lambda) = \frac{S_0(\lambda)\, I(\lambda)}{d_{in}\, d_{out}\, K_0} \quad \text{(microwatts/cm}^2 \text{ . A . steradian)}$$

where $K_0 = 9.4 \times 10^{-7}$ for the RCA 6903 tube at 1000V
$= 2.9 \times 10^{-10}$ for the RCA 7102 tube at 750V.

In all cases a slit length of 20mm and a "rack setting" of 56 were used. The slit widths are measured in microns and the photocurrent in microamperes.



Varying entrance slit width



Varying exit slit width

Figure 1. Effect of slit widths on passband of spectrograph

d_{in} Entrance slit width

d_{out} Exit slit width

K Undetermined constant

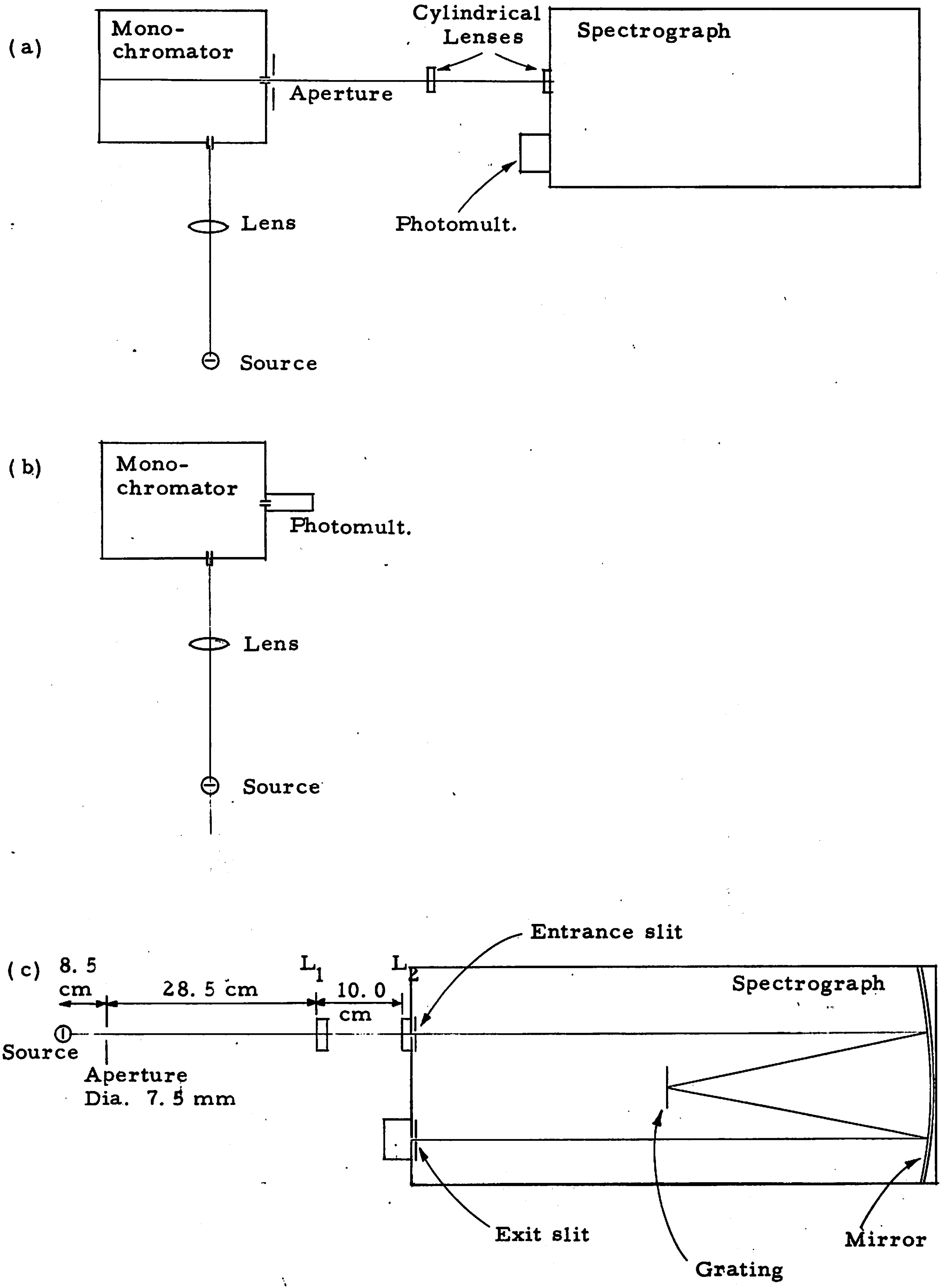
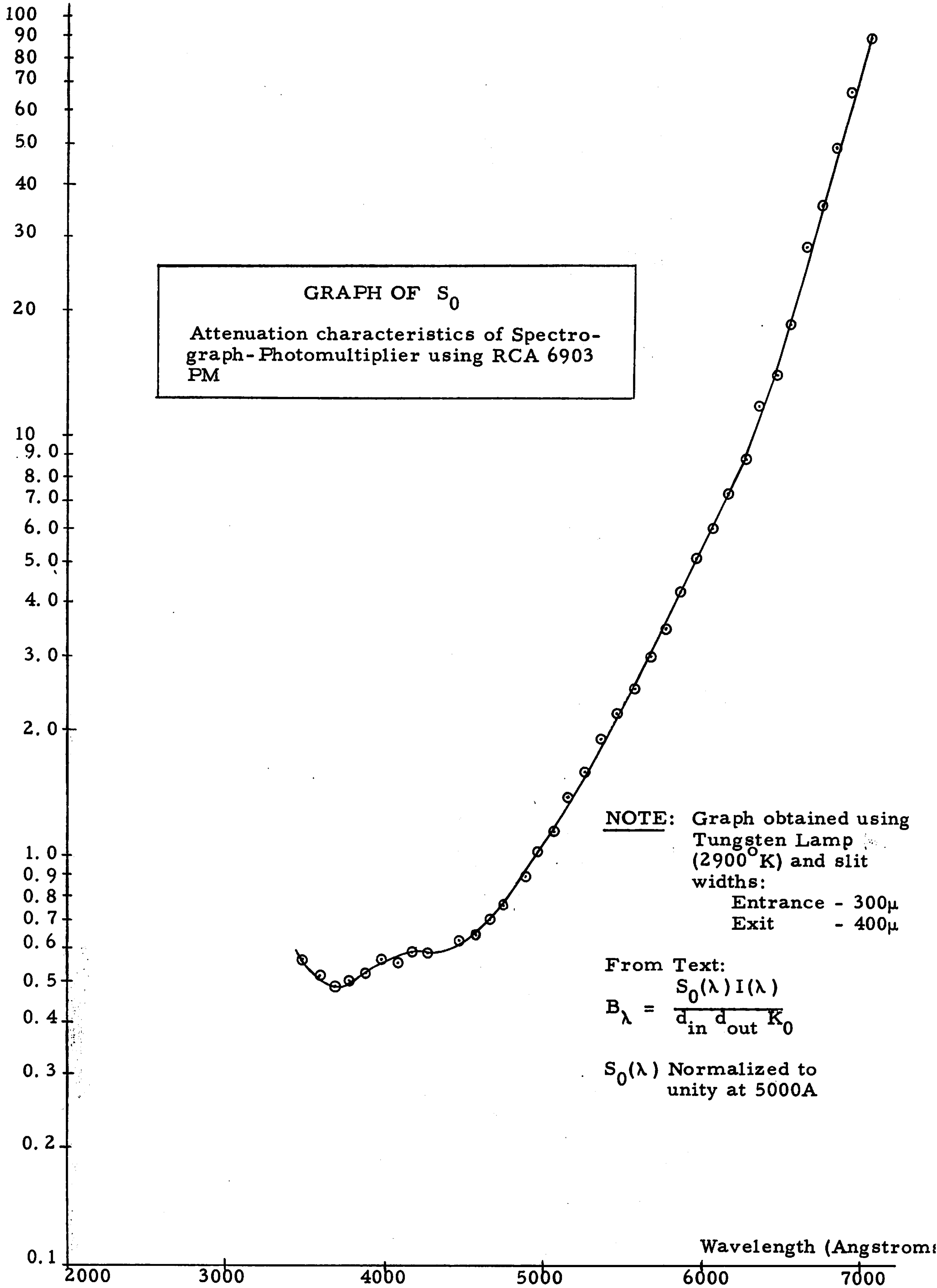


Figure 2. Arrangement of spectrograph and associated equipment for making brightness measurements.



GRAPH OF S_0

Attenuation characteristics of Spectrograph-Photomultiplier using RCA 6903 PM

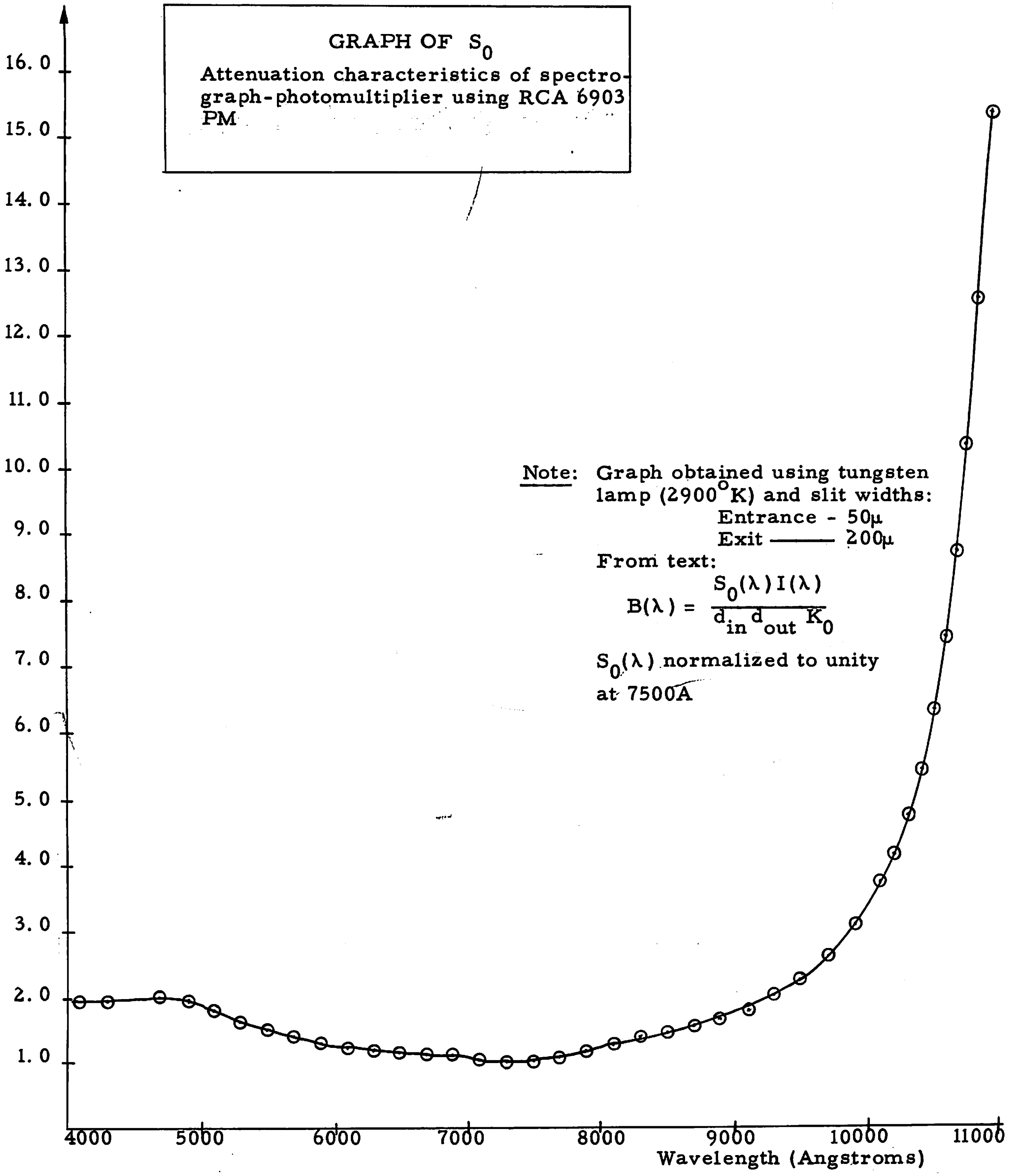
NOTE: Graph obtained using Tungsten Lamp (2900°K) and slit widths:
Entrance - 300μ
Exit - 400μ

From Text:

$$B_\lambda = \frac{S_0(\lambda)\,I(\lambda)}{d_{in}\,d_{out}\,K_0}$$

$S_0(\lambda)$ Normalized to unity at 5000A

Figure 3.



GRAPH OF S_0

Attenuation characteristics of spectrograph-photomultiplier using RCA 6903 PM

Note: Graph obtained using tungsten lamp (2900°K) and slit widths:

Entrance - 50μ

Exit ——— 200μ

From text:

$$B(\lambda) = \frac{S_0(\lambda)\, I(\lambda)}{d_{in}\, d_{out}\, K_0}$$

$S_0(\lambda)$ normalized to unity at 7500A

Figure 4.

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